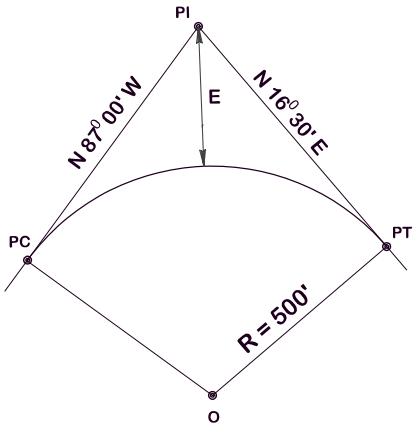
The Horizontal Curves Primer

Preparing for the Civil PE Exam

Solutions

1. Horizontal Curve – Find Curve Length.

Determine the angle I from the bearings. It may help to rotate the figure so that the bearings are correct relative to due north, but since both the back and the ahead tangent bearings have northerly headings, I is equal to the sum of the bearing angles.



$$I = 87^0 + 16^0\ 30' = 103^0\ 30' = 103.50^0$$

Use the equation for E to determine the External.

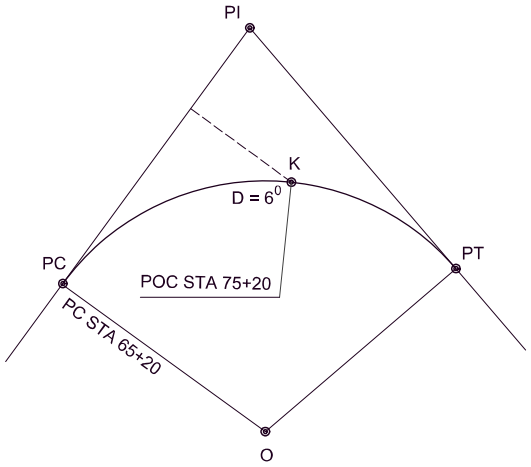
$$E = R\tan\frac{I}{2}\tan\frac{I}{4} \qquad \text{(CERM Equa 79.5)}$$

$$= 500\tan\frac{103.50}{2}\tan\frac{103.50}{4} = 500(1.268)(0.485) = \underline{\underline{307.61\,ft}}$$

THE CORRECT ANSWER IS (D)

2. Horizontal Curve – Find Tangent Offset.

See CERM page 79-5.



Determine R (Use CERM Equation 79.8)

$$R = \frac{5729.578}{D} = \frac{5729.578}{6} = 954.930 \text{ ft}$$

$$L = 75 + 20 - 65{+}20 = 1000 \text{ ft}$$

Determine the angel β.

$$\beta = \frac{L}{2\pi R}\times 360^0 = \frac{1000}{2\pi 954.930}\times 360^0 = \frac{1000}{6000}\times 360^0 = 60^0$$

Determine y, the tangent offset.

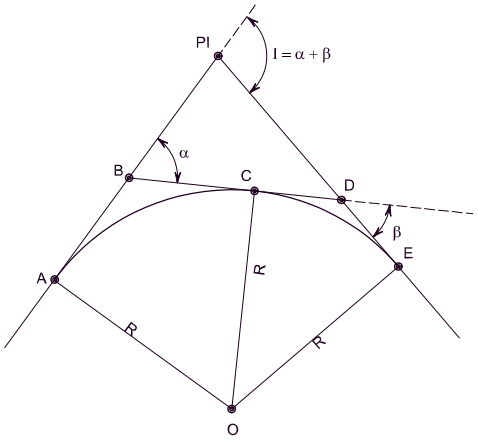
$$y = R(1-\cos\beta) = 954.93(1-\cos 60^0) = \underline{\underline{477.465 \text{ ft}}}$$

THE CORRECT ANSWER IS (B)

3. Horizontal Curve along River - PI Inaccessible.

The sum of the two curve arc deflections, α and β = I = 59.8^0 + 43.48^0 = 103.28^0.

Let Points A, C, and E be the points of tangency.



$$AB = BC = R\tan\frac{\alpha}{2} \quad \text{and} \quad CD = DE = R\tan\frac{\beta}{2}$$

Combine:

$$BD = BC + CD = R\left(\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}\right) = 1{,}168.54 \text{ ft}$$

Rearrange:

$$R = \frac{BD}{\left(\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}\right)} = \frac{1{,}168.54}{\left(\tan\frac{59.8}{2} + \tan\frac{43.48}{2}\right)}$$

$$= \frac{1{,}168.54}{(0.575 + 0.399)} = 1199.733 \text{ say } \underline{\underline{1200 \text{ ft}}}$$

THE CORRECT ANSWER IS (D)

4. Horizontal Curve - Find Coordinates.

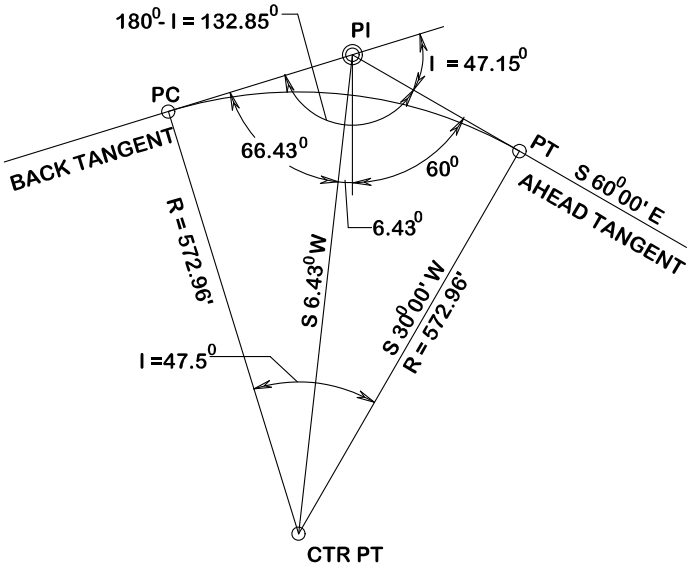
Convert degree of curvature to radius. R = 572.9578 ft

Find Tangent length, T.

T = PI Sta. – PC Sta. = 89000.00 – 88750.00 = 250.00 ft.

I = 2 $\tan^{-1}$(T/R) = 2 $\tan^{-1}$ (250.00/572.9578) = 47.15^0

L = 2πRI/360^0 = 2π(572.9578)(47.15)/360^0 = 471.46 ft



Back Tangent Bearing: $180^0 - 47.15^0 - 60^0 = 72.8535^0 =$ **N 72.85^0 E**

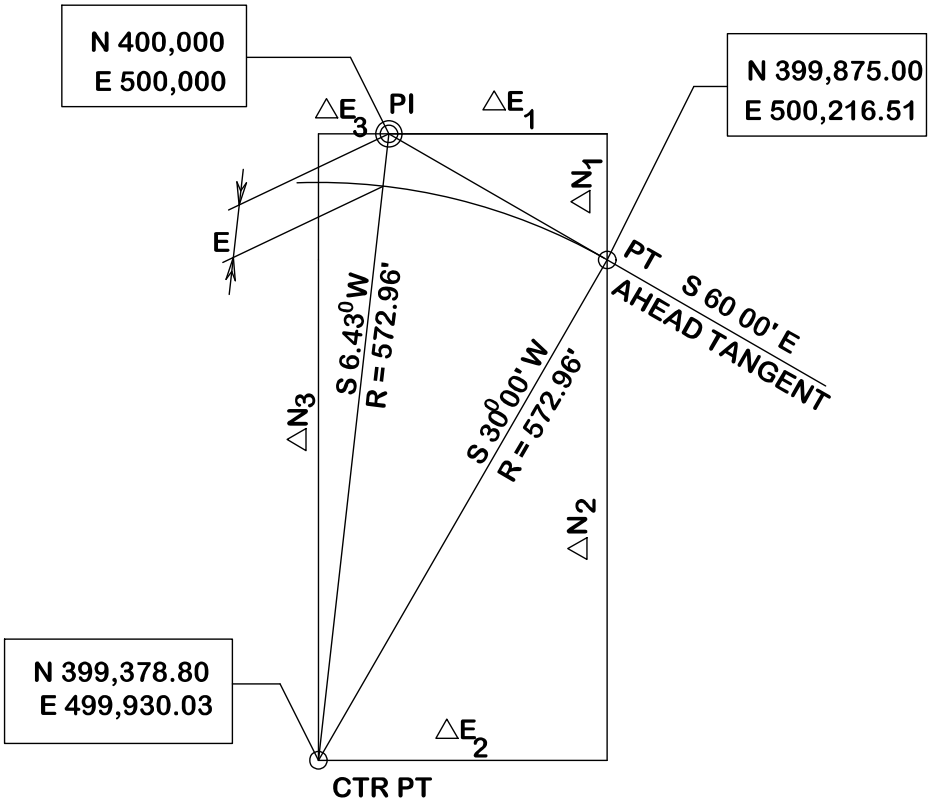
PT Station = PC Station + L = Sta. 887+50.00 + 471.46 = **Sta. 892+21.46**

PT Coordinates: see sketch below.
Use Latitude and Departure of ahead tangent, S 60^0 00' E

Latitude = – 250 cos 60^0 = – 125.00 ft
N_{PT} = 400,000 – 125 = **399,875.00**
Departure = 250 sin 60^0 = 216.51 ft
E_{PT} = 500,000 + 216.51 = **500,216.51**

Center Point Coordinates: see sketch below.
Use Latitude and Departure of radial line, S 30^0 00' W

Latitude = –572.9578 cos 30^0 = – 496.1960 ft
N_{CC} = 399,875.0000 – 496.1960 = **399,378.804**
Departure = –572.9578 sin 30^0 = –286.479 ft
E_{CC} = 500,216.50635 –286.479 = **499,930.0274**



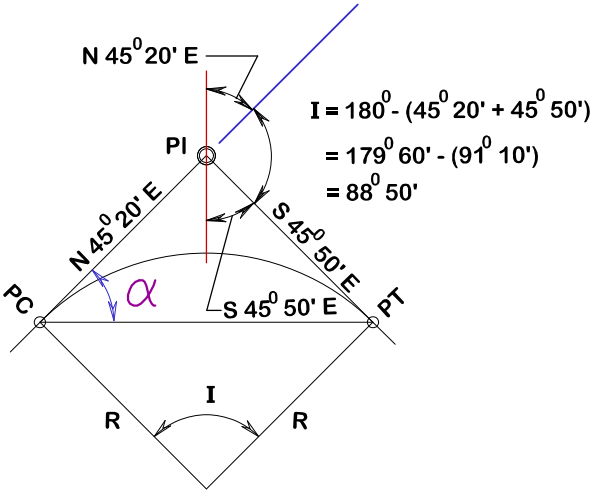
THE CORRECT ANSWER IS (B)

5. Horizontal Curve - Find Angle Alpha.

To find the angle alpha, we need to first find the angle I.
As indicated in CERM Figure 70-1, alpha is equal to I/2.

As shown in the figure below, finding the angle I involves working with the back and ahead tangent bearing angles.

First draw the blue line by extending the back tangent to the right and upward from the PI.
Then draw the red vertical line directly through the PI.



Notice that:
The angle between the red and blue lines is the back tangent bearing angle, or 45^0 20', and that the angle between the ahead tangent and the red line is the ahead tangent bearing angle, or 45^0 50'.

$I = 180^0 - (45^0\ 20' + 45^0\ 50')$

$= 180^0 - (90^0\ 70')$

$= 179^0\ 60' - (91^0\ 10')$

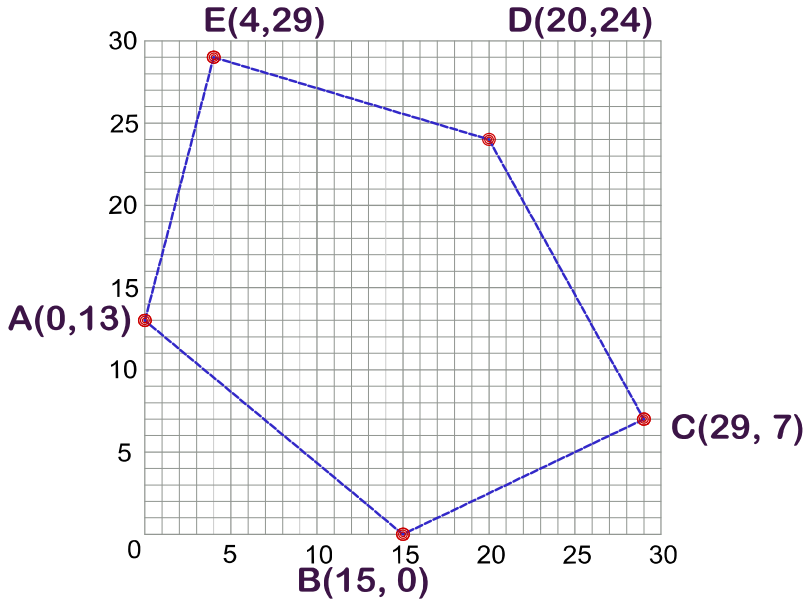
$= 88^0\ 50'$

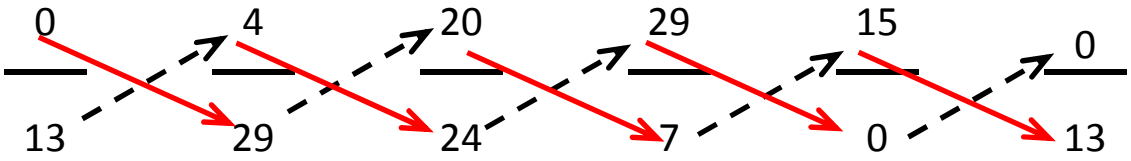
Therefore $\alpha = \frac{I}{2} = \frac{88^0\ 50'}{2} = 44^0\ 25'$

1. THE CORRECT ANSWER IS (A)

Note also that the bearing of the Main Chord (from PC to PT) is:
N 89^0 25' E (= 45^0 20' + 44^0 25'),
which is very close to, but not quite due east.
So looks can be deceiving. Even though the line appears to be running due east/west, the bearing is actually slightly to the north of due east.

6. Area by Coordinates. The *Area by Coordinates* method is shown on CERM page 78-17.





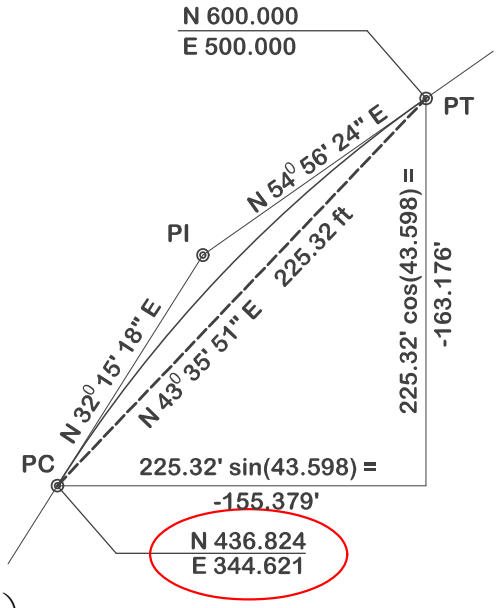
$$A = \frac{1}{2}\left|\sum \text{of solid line products} - \sum \text{of dashed line products}\right|$$

$$= \frac{1}{2}\left|(0+96+140+0+195)-(52+580+696+105+0)\right|$$

$$= \frac{1}{2}\left|(431)-(1,433)\right| = \frac{1}{2}\left|-1,002\right| = 501$$

THE CORRECT ANSWER IS (A)

7. Horizontal Curve - Find Coordinates.



Find angle I: $54^0\ 56'\ 24"$

$-\ 32^0\ 15'\ 18"$

$22^0\ 41'\ 06"$

Angle $\alpha = I/2$: $11^0\ 20'\ 33"$

$+\ 32^0\ 15'\ 18"$

Chord Bearing: N $43^0\ 35'\ 51"$ E

(from PC to PT)

Chord Length: $C = 2R\sin\frac{I}{2} = 2(572.958)\sin\left(\frac{22.685^0}{2}\right) = \underline{225.371'}$

PC Point Coordinates: see sketch above.

Find Latitude and Departure of chord (from PT to PC), S 43⁰ 35’ 51” W

Latitude = 225.32 cos 43.598⁰ = – 163.176 ft

N_{PC} = 600.000 – 163.176 = **436.824**

Departure = 225.32 sin 43.598⁰ = –155.379 ft

E_{PC} = 500.000 –155.379= **344.621**

Latitude of Chord:

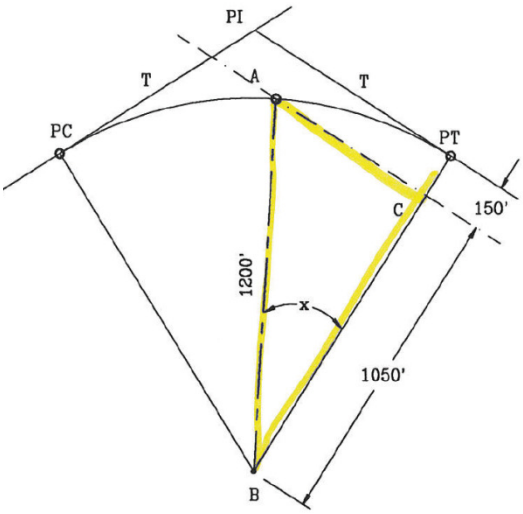
$-\ 225.371'\cos(43.598) = -163.176'$

Departure of Chord:

$-\ 225.371'\sin(43.598) = -155.379'$

THE CORRECT ANSWER IS (B)

8. Horizontal Curve on Grade Crossing.



Calculation Sequence

1. Calculate curve data
2. PI Sta – Tangent = PC Sta
3. PC Sta + Curve Length = PT Sta
4. Calculate Right angled Δ ABC
5. Calculate Curve Length for A to PT.
6. PT Sta – Curve Length = Station of Point A
7. Calculate M & E as shown

Curve Data

$R = 1200.0$ ft

$\Delta = 55°30' = 0.9686598$ radian

$L = 1200\ (\Delta) = 1200.0\ (0.9686598) = 1162.39$ ft

$T = R \tan \frac{\Delta}{2} = 1200.0\ (0.526125) = 631.35$ ft

PC Station = PI Station – T

= (182 + 27.52) – (6 + 31.35)

PC Sta = 175 + 96.17

PT Sta = PC + L of curve

= (175 + 96.17) + (11 + 62.39)

= 187 + 58.56

In right Δ, ABC

$$\cos x = \frac{1050}{1200} = 0.875$$

$$x = 28.955°$$

Curve length from A to PT

A to PT = R × x (x in radians)

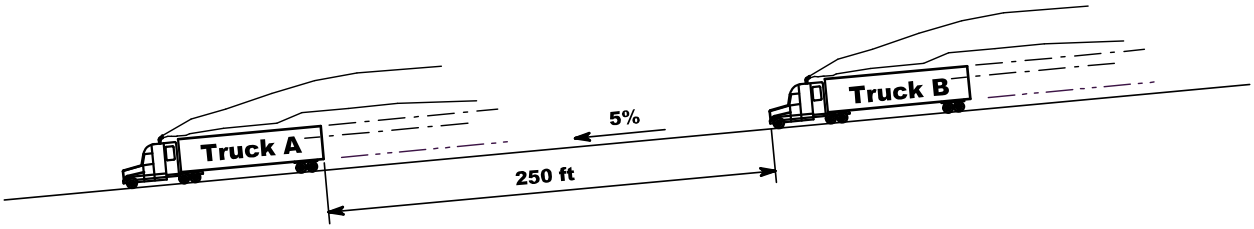
= 1200.0 (0.5053612) = 606.43 ft

Station of Point A = (187 + 58.56) – 606.43 = 181 + 52.13

$M = R \text{ vers } \frac{\Delta}{2} = 1200\ (1 - 0.884988) = 138.01$ ft (vers = 1 – cos)

$E = T (\tan \frac{\Delta}{4}) = 631.35 \times 0.247012 = 155.95$ ft

9. Stopping Distance.



$$S_A = 1.47\ t_p\ V_{mph} + \frac{V_{mph}^{\ 2}}{30(f + G)}$$

$$S_A = 1.47\ (0)\ (40) + \frac{(40)^2}{30(0.76\ -\ 0.05)} = 0 = \frac{1600}{21.30} = 75.12'$$

$$S_B = 1.47\ t_p\ V_{mph} + \frac{V_{mph}^{\ 2}}{30(f + G)}$$

$$S_B = 1.47\ (1.50)\ (40) + \frac{(40)^2}{30(0.33\ -\ 0.05)} = 88.20' + \frac{1600}{8.40} = 278.68'$$

Final distance between trucks:

$75.12' + 250' - 278.68' = \underline{\underline{46.44'}}$

THE CORRECT ANSWER IS (D)