NCEES
advancing licensure for engineers and surveyors

PE | Civil

Reference Handbook
Version 2.0

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Version 2.0

INTRODUCTION

About the Handbook

The Principles and Practice of Engineering (PE) Civil exam is computer-based, and NCEES will supply all the resource material that you are allowed to use during the exam. Reviewing the *PE Civil Reference Handbook* before exam day will help you become familiar with the charts, formulas, tables, and other reference information provided. You will not be allowed to bring your personal copy of the *PE Civil Reference Handbook* into the exam room. Instead, the computer-based exam will include a PDF version of the handbook for your use. No printed copies of the handbook will be allowed in the exam room.

The PDF version of the *PE Civil Reference Handbook* that you use on exam day will be very similar to this one. However, pages not needed to solve exam questions—such as the cover and introductory material—may not be included in the exam version. In addition, NCEES will periodically revise and update the handbook, and each PE Civil exam will be administered using the updated version.

The *PE Civil Reference Handbook* does not contain all the information required to answer every question on the exam. Theories, conversions, formulas, and definitions that examinees are expected to know have not been included. The handbook is intended solely for use on the NCEES PE Civil exam.

Other Supplied Exam Material

In addition to the *PE Civil Reference Handbook*, the exam will include searchable PDF versions of codes and standards. A list of the material that will be included in your exam is shown on the exam specifications. Any additional material required for the solution of a particular exam question will be included in the question itself. You will not be allowed to bring personal copies of any material into the exam room.

To familiarize yourself with the format, style, and navigation of a computer-based exam, visit the Exam Resources section on the NCEES YouTube channel.

Updates on Exam Content and Procedures

NCEES.org is our home on the web. Visit us there for updates on everything exam-related, including specifications, exam-day policies, scoring, and practice tests.

Errata

To report errata in this book, send your correction through your MyNCEES account. Examinees are not penalized for any errors in the *Handbook* that affect an exam question.

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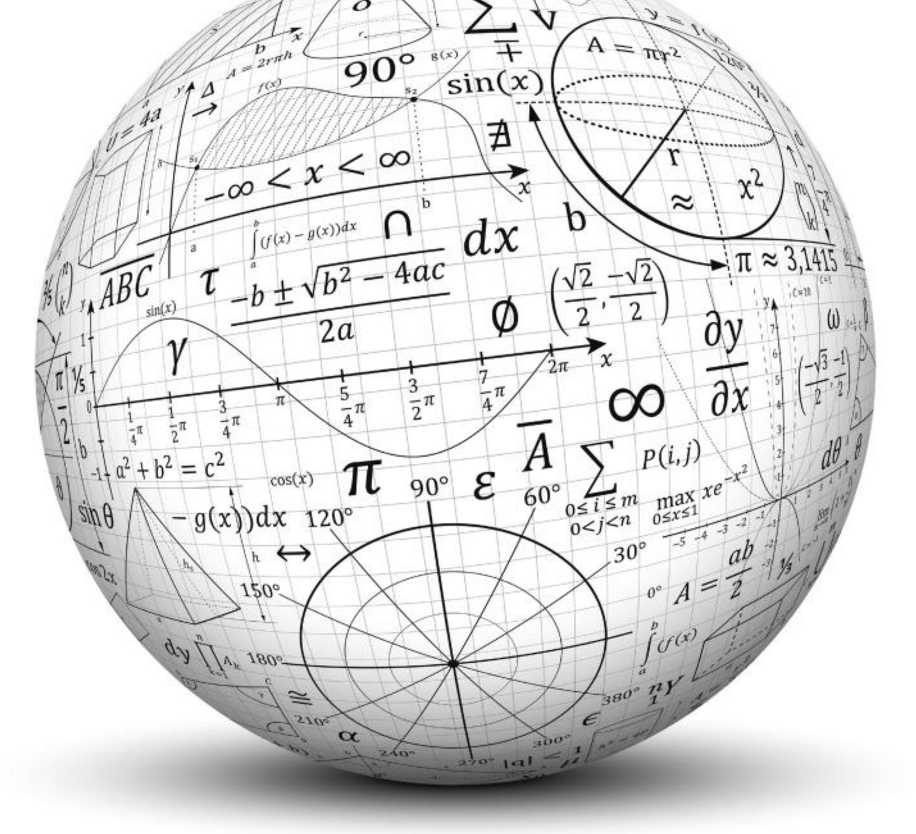


1 GENERAL ENGINEERING

1.1 Units

1.1.1 Distinguishing Pound-Force from Pound-Mass

This handbook uses the International Systems of Units (SI) (metric) and the U.S. Customary System (USCS). In USCS units, both force and mass are called pounds. Therefore, one must distinguish the pound-force (lbf) from the pound-mass (lbm). If the exam question is presented in USCS units, it may be necessary to use the constant g_c in the equation to have a consistent set of units.

$$1 \text{ lbf} = 32.174 \frac{\text{lbm-ft}}{\text{sec}^2}$$

$$F = \frac{ma}{g_c}$$

where

F is in lbf

m is in lbm

a is in $\frac{\text{ft}}{\text{sec}^2}$

$g_c = 32.174 \frac{\text{lbm-ft}}{\text{lbf-sec}^2}$

Kinetic Energy: $KE = \frac{mv^2}{2g_c}$ with KE in ft-lbf

Potential Energy: $PE = \frac{mgh}{g_c}$ with PE in ft-lbf

Fluid Pressure: $p = \frac{\rho gh}{g_c}$ with p in $\frac{\text{lbf}}{\text{ft}^2}$

Specific Weight: $SW = \frac{\rho g}{g_c}$ with SW in $\frac{\text{lbf}}{\text{ft}^3}$

Shear Stress: $\tau = \left(\frac{\mu}{g_c}\right)\left(\frac{dv}{dy}\right)$ with τ in $\frac{\text{lbf}}{\text{ft}^2}$

METRIC PREFIXES		
Multiple	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deka	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

COMMONLY USED EQUIVALENTS	
1 gallon of water weighs	8.34 lbf
1 cubic foot of water weighs	62.4 lbf
1 cubic inch of mercury weighs	0.491 lbf
The mass of 1 cubic meter of water is	1,000 kilograms
1 mg/L is	8.34×10^{-6} lbf/gal

TEMPERATURE CONVERSIONS

$°F = 1.8\,(°C) + 32$

$°C = (°F - 32)/1.8$

$°R = °F + 459.69$

$K = °C + 273.15$

1.1.2 Fundamental Constants

Quantity		Symbol	Value	Units
gravity acceleration (standard)	metric	g	9.807	m/s^2
gravity acceleration (standard)	USCS	g	32.174	ft/sec^2
molar volume (ideal gas), T = 273.15 K, p = 101.3 kPa		Vm	22,414	L/kmol

1.2 Conversion Factors

Multiply	By	To Obtain
acre	43,560	square feet (ft^2)
atmosphere (atm)	760	mm, mercury (Hg)
atm, std	29.92	in., mercury (Hg)
atm, std	14.70	lbf/in^2 abs (psia)
atm, std	33.90	ft, water
atm, std	1.013×10^5	pascal (Pa)
bar	1×10^5	Pa
bar	0.987	atm
barrels–oil	42	gallons–oil
board-ft	144	in^3 timber
Btu	1,055	joule (J)
Btu	2.928×10^{-4}	kilowatt-hr (kWh)
Btu	778	ft-lbf
Btu/hr	3.930×10^{-4}	horsepower (hp)
Btu/hr	0.293	watt (W)
Btu/hr	0.216	ft-lbf/sec
calorie (g-cal)	3.968×10^{-3}	Btu
cal	1.560×10^{-6}	hp-hr
cal	4.184	joule (J)
cal/sec	4.184	watt (W)
centimeter (cm)	3.281×10^{-2}	foot (ft)
cm	0.394	inch (in)
centipoise (cP)	0.001	pascal•sec (Pa•s)
centipoise (cP)	1	g/(m•s)
centipoise (cP)	2.419	lbm/hr-ft
centistoke (cSt)	1×10^{-6}	m^2/sec (m^2/s)
cubic feet/second (cfs)	0.646317	million gallons/day (MGD)
cubic foot (ft^3)	7.481	gallon
cubic meters (m^3)	1,000	liters
foot (ft)	30.48	cm
ft	0.3048	meter (m)
ft of H_2O	0.4332	psi
ft-pound (ft-lbf)	1.285×10^{-3}	Btu
ft-lbf	3.766×10^{-7}	kilowatt-hr (kWh)
ft-lbf	0.324	calorie (g-cal)
ft-lbf	1.356	joule (J)
ft-lbf/sec	1.818×10^{-3}	horsepower (hp)
gallon (U.S. Liq)	3.785	liter (L)
gallon (U.S. Liq)	0.134	ft^3
gallons of water	8.3453	pounds of water
gram (g)	2.205×10^{-3}	pound (lbm)
hectare	1×10^4	square meters (m^2)
hectare	2.47104	acres
horsepower (hp)	42.4	Btu/min
hp	745.7	watt (W)
hp	33,000	(ft-lbf)/min
hp	550	(ft-lbf)/sec
hp-hr	2,545	Btu
hp-hr	1.98×10^6	ft-lbf
hp-hr	2.68×10^6	joule (J)
hp-hr	0.746	kWh
inch (in.)	2.540	centimeter (cm)
in. of Hg	0.0334	atm
in. of Hg	13.60	in. of H_2O
in. of H_2O	0.0361	lbf/in^2 (psi)
in. of H_2O	0.002458	atm

Multiply	By	To Obtain
joule (J)	9.478×10^{-4}	Btu
J	0.7376	ft-lbf
J	1	newton•m (N•m)
J/s	1	watt (W)
kilogram (kg)	2.205	pound-mass (lbm)
kgf	9.8066	newton (N)
kilometer (km)	3,281	feet (ft)
km/hr	0.621	mph
kilopascal (kPa)	0.145	lbf/in^2 (psi)
kilowatt (kW)	1.341	horsepower (hp)
kW	3,413	Btu/hr
kW	737.6	(ft-lbf)/sec
kW-hour (kWh)	3,413	Btu
kWh	1.341	hp-hr
kWh	3.6×10^6	joule (J)
kip (K)	1,000	lbf
K	4,448	newton (N)
liter (L)	61.02	in^3
L	0.264	gal (U.S. Liq)
L	10^{-3}	m^3
L/second (L/s)	2.119	ft^3/min (cfm)
L/s	15.85	gal (U.S.)/min (gpm)
meter (m)	3.281	feet (ft)
m	1.094	yard
m/second (m/s)	196.8	feet/min (ft/min)
mile (statute)	5,280	feet (ft)
mile (statute)	1.609	kilometer (km)
mile/hour (mph)	88.0	ft/min (fpm)
mph	1.609	km/h
mm of Hg	1.316×10^{-3}	atm
mm of H_2O	9.678×10^{-5}	atm
newton (N)	0.225	lbf
newton (N)	1	kg•m/s^2
N•m	0.7376	ft-lbf
N•m	1	joule (J)
pascal (Pa)	9.869×10^{-6}	atmosphere (atm)
Pa	1	newton/m^2 (N/m^2)
Pa•sec (Pa•s)	10	poise (P)
pound (lbm, avdp)	0.454	kilogram (kg)
lbf	4.448	N
lbf-ft	1.356	N•m
lbf/in^2 (psi)	0.068	atm
psi	2.307	ft of H_2O
psi	2.036	in. of Hg
psi	6,895	Pa
radian (rad)	$180/\pi$	degree
revolution (rev)	$2 \times \pi$	radian
rpm (revolutions per minute)	$2 \times \pi/60$	radian/second
slug	32.174	pound-mass (lbm)
stokes	1×10^{-4}	m^2/s
tesla	1.0	weber/m^2
therm	1×10^5	Btu
ton (metric)	1,000	kilogram (kg)
ton (short)	2,000	pound-force (lbf)
watt (W)	3.413	Btu/hr
W	1.341×10^{-3}	horsepower (hp)

1.3 Mathematics

1.3.1 Straight Line

The general form of the equation is

$$Ax + By + C = 0$$

The standard form of the equation is

$$y = mx + b$$

which is also known as the slope-intercept form.

The *point-slope* form is

$$y - y_1 = m(x - x_1)$$

Given two points, the slope is

$$m = (y_2 - y_1)/(x_2 - x_1)$$

The angle between lines with slopes m_1 and m_2 is

$$\alpha = \arctan\left[(m_2 - m_1)/(1 + m_2 \cdot m_1)\right]$$

Two lines are perpendicular if $m_1 = -1/m_2$

The distance between two points is

$$d = \sqrt{\left(y_2 - y_1\right)^2 + \left(x_2 - x_1\right)^2}$$

1.3.2 Quadratic Equation

$$ax^2 + bx + c = 0$$

$$x = \text{Roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.3.3 Quadric Surface (SPHERE)

The standard form of the equation is

$$(x - h)^2 + (y - k)^2 + (z - m)^2 = r^2$$

with center at (h, k, m).

In a three-dimensional space, the distance between two points is

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$$

1.3.4 Difference Equations

Any system whose input $v(t)$ and output $y(t)$ are defined only at the equally spaced intervals

$$f(t) = y' = \frac{y_{i+1} - y_i}{t_{i+1} - t_i}$$

can be described by a difference equation.

First-Order Linear Difference Equation

$$\Delta t = t_{i+1} - t_i$$

$$y_{i+1} = y_i + y'(\Delta t)$$

1.3.5 Logarithms

The logarithm of x to the Base b is defined by

$$\log_b(x) = c$$

where $b^c = x$

Special definitions for b = e or b = 10 are:

ln x, Base = e

log x, Base = 10

To change from one Base to another:

$$\log_b x = (\log_a x)/(\log_a b)$$

e.g., $\ln x = (\log_{10} x)/(\log_{10} \mathrm{e}) = 2.302585 (\log_{10} x)$

Identities

$$\log_b b^n = n$$

$$\log x^c = c \log x;\ x^c = \text{antilog}(c \log x)$$

$$\log xy = \log x + \log y$$

$$\log_b b = 1$$

$$\log 1 = 0$$

$$\log x/y = \log x - \log y$$

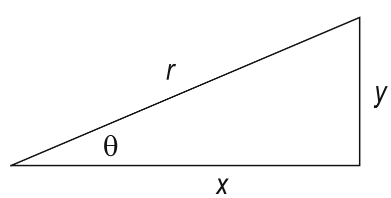
1.3.6 Trigonometry

Trigonometric functions are defined using a right triangle.

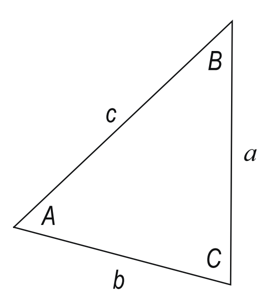
$\sin\theta = y/r$, $\cos\theta = x/r$

$\tan\theta = y/x$, $\cot\theta = x/y$

$\csc\theta = r/y$, $\sec\theta = r/x$



1.3.6.1 Law of Sines $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

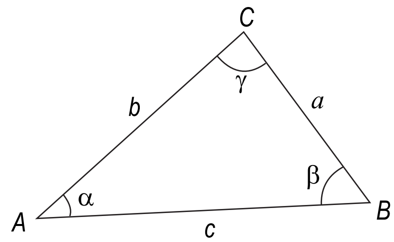


1.3.6.2 Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

1.3.6.3 Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$



1.3.6.4 Identities

$\cos\theta = \sin(\theta + \pi/2) = -\sin(\theta - \pi/2)$

$\sin\theta = \cos(\theta - \pi/2) = -\cos(\theta + \pi/2)$

$\csc\theta = 1/\sin\theta$

$\sec\theta = 1/\cos\theta$

$\tan\theta = \sin\theta/\cos\theta$

$\cot\theta = 1/\tan\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

$\tan 2\alpha = (2\tan\alpha)/(1 - \tan^2\alpha)$

$\cot 2\alpha = (\cot^2\alpha - 1)/(2\cot\alpha)$

$\tan(\alpha + \beta) = (\tan\alpha + \tan\beta)/(1 - \tan\alpha\tan\beta)$

$\cot(\alpha + \beta) = (\cot\alpha\cot\beta - 1)/(\cot\alpha + \cot\beta)$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha - \beta) = (\tan\alpha - \tan\beta)/(1 + \tan\alpha\tan\beta)$

$\cot(\alpha - \beta) = (\cot\alpha\cot\beta + 1)/(\cot\beta - \cot\alpha)$

$\sin(\alpha/2) = \pm\sqrt{(1-\cos\alpha)/2}$

$\cos(\alpha/2) = \pm\sqrt{(1+\cos\alpha)/2}$

$\tan(\alpha/2) = \pm\sqrt{(1-\cos\alpha)/(1+\cos\alpha)}$

$\cot(\alpha/2) = \pm\sqrt{(1+\cos\alpha)/(1-\cos\alpha)}$

$\sin\alpha\sin\beta = (1/2)[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos\alpha\cos\beta = (1/2)[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha\cos\beta = (1/2)[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\sin\alpha + \sin\beta = 2\sin[(1/2)(\alpha+\beta)]\cos[(1/2)(\alpha-\beta)]$

$\sin\alpha - \sin\beta = 2\cos[(1/2)(\alpha+\beta)]\sin[(1/2)(\alpha-\beta)]$

$\cos\alpha + \cos\beta = 2\cos[(1/2)(\alpha+\beta)]\cos[(1/2)(\alpha-\beta)]$

$\cos\alpha - \cos\beta = -2\sin[(1/2)(\alpha+\beta)]\sin[(1/2)(\alpha-\beta)]$

1.3.7 Mensuration of Areas and Volumes

1.3.7.1 Nomenclature

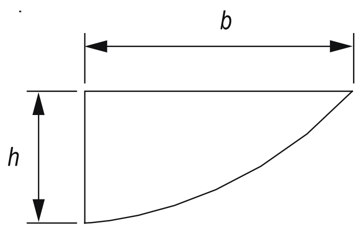
A = total surface area

P = perimeter

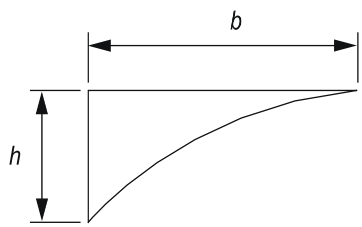
V = volume

1.3.7.2 Parabola

$A = 2bh/3$

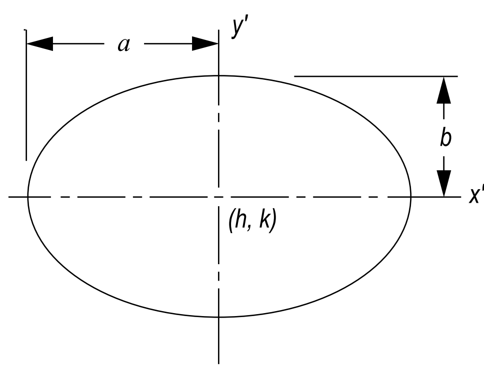


$A = bh/3$



Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.3 Ellipse



$A = \pi ab$

$P_{approx} = 2\pi\sqrt{\left(a^2+b^2\right)/2}$

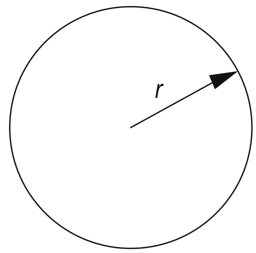
$$P = \pi(a+b)\begin{bmatrix} 1 + (1/2)^2\lambda^2 + (1/2\times 1/4)^2\lambda^4 \\ +(1/2\times 1/4\times 3/6)^2\lambda^6 + (1/2\times 1/4\times 3/6\times 5/8)^2\lambda^8 \\ +(1/2\times 1/4\times 3/6\times 5/8\times 7/10)^2\lambda^{10} + \ldots \end{bmatrix}$$

where

$\lambda = (a-b)/(a+b)$

Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.4 Circle



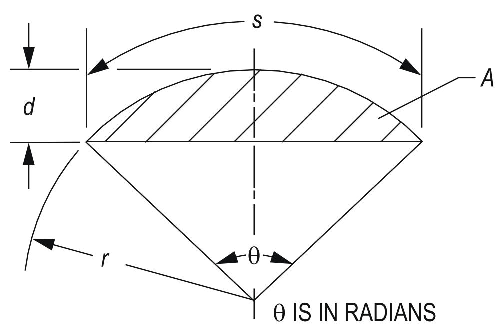
A = area of a circle = πr^2

C = circumference = $2\pi r = \pi d$

1.3.7.5 Circular Segment

$A = \left[r^2(\theta - \sin\theta)\right]/2$

$\theta = s/r = 2\left\{\arccos\left[(r-d)/r\right]\right\}$

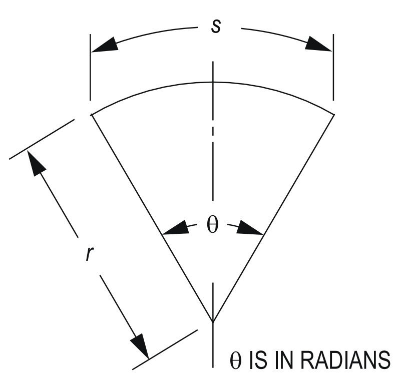


Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.6 Circular Sector

$A = \theta r^2/2 = sr/2$

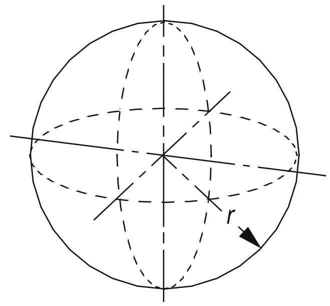
$\theta = s/r$



Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.7 Sphere

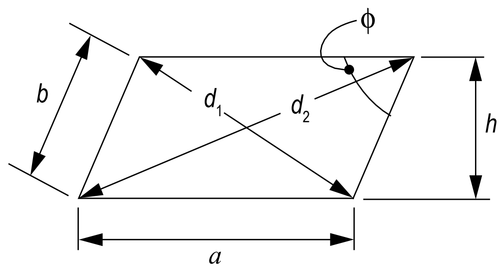
$$V = 4\pi r^3/3 = \pi d^3/6$$
$$A = 4\pi r^2 = \pi d^2$$



Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.8 Parallelogram

$$P = 2(a + b)$$
$$d_1 = \sqrt{a^2 + b^2 - 2ab(\cos\phi)}$$
$$d_2 = \sqrt{a^2 + b^2 + 2ab(\cos\phi)}$$
$$d_1^2 + d_2^2 = 2(a^2 + b^2)$$
$$A = ah = ab(\sin\phi)$$

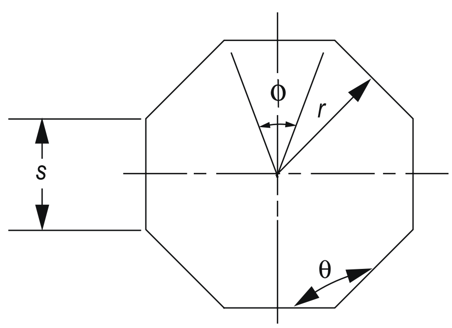


If $a = b$, the parallelogram is a rhombus.

Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.9 Regular Polygon (n equal sides)

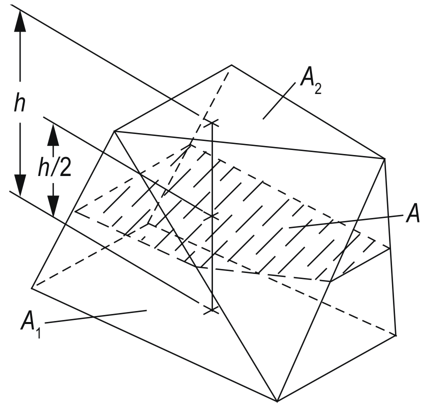
$$\phi = 2\pi/n$$
$$\theta = \left[\frac{\pi(n-2)}{n}\right] = \pi\left(1 - \frac{2}{n}\right)$$
$$P = ns$$
$$s = 2r\left[\tan(\phi/2)\right]$$
$$A = (nsr)/2$$



Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.10 Prismoid

$V = (h/6)(A_1 + A_2 + 4A)$



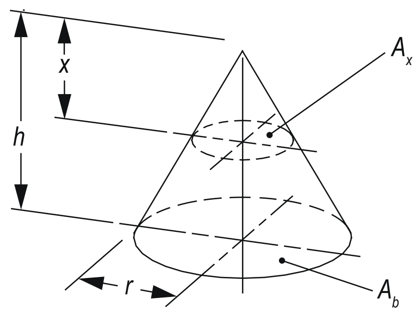
Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.11 Right Circular Cone

$V = (\pi r^2 h)/3$

$A = \text{side area} + \text{base area} = \pi r\left(r + \sqrt{r^2 + h^2}\right)$

$A_x : A_b = x^2 : h^2$

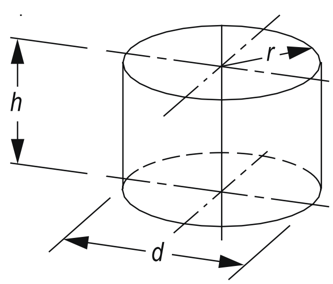


Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.12 Right Circular Cylinder

$V = \pi r^2 h = \dfrac{\pi d^2 h}{4}$

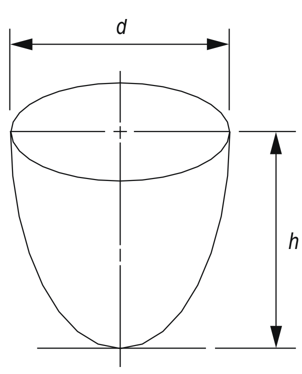
$A = \text{side area} + \text{end areas} = 2\pi r(h + r)$



Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.7.13 Paraboloid of Revolution

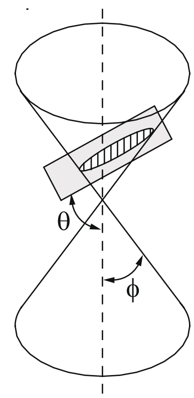
$V = \dfrac{\pi d^2 h}{8}$



Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

1.3.8 Conic Sections

e = eccentricity = $\cos\theta/(\cos\phi)$

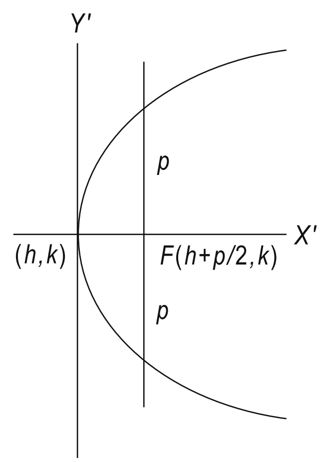


Source: Gieck, K. and R. Gieck. *Engineering Formulas*. Gieck Publishing, 1967.

[Note: X' and Y', in the following cases, are translated axes.]

1.3.8.1 Case 1: Parabola e = 1

$(y - k)^2 = 2p(x - h)$

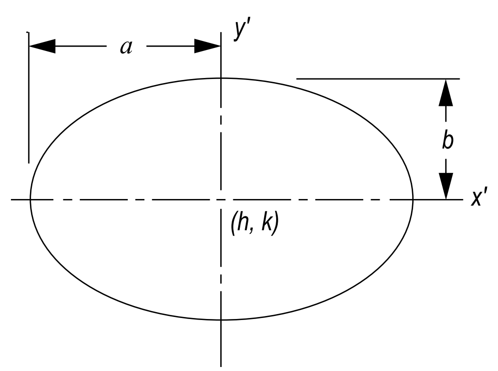


Center at (h, k) is the standard form of the equation. When $h = k = 0$, Focus: $(p/2, 0)$; Directrix: $x = -p/2$

Source: Brink, R.W. *A First Year of College Mathematics*. D. Appleton-Century Company, Inc. (Prentice Hall), 1937.

1.3.8.2 Case 2: Ellipse e < 1

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



Center at (h, k) is the standard form of the equation. When $h = k = 0$:

Eccentricity: $e = \sqrt{1 - \left(b^2/a^2\right)} = c/a$

$b = a\sqrt{1 - e^2}$

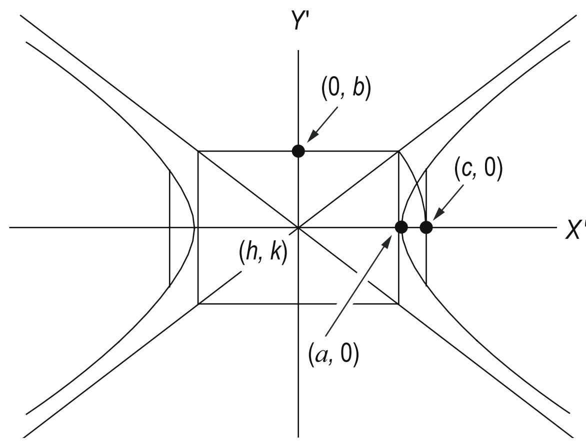
Focus: ($\pm$ ae, 0)

Directrix: $x = \pm\ a/e$

Source: Brink, R.W. *A First Year of College Mathematics*. D. Appleton-Century Company, Inc. (Prentice Hall), 1937.

1.3.8.3 Case 3: Hyperbola e > 1

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$



Center at (h, k) is the standard form of the equation. When $h = k = 0$:

Eccentricity: $e = \sqrt{1 + \left(b^2/a^2\right)} = c/a$

$b = a\sqrt{e^2 - 1}$

Focus: ($\pm$ ae, 0)

Directrix: $x = \pm\ a/e$

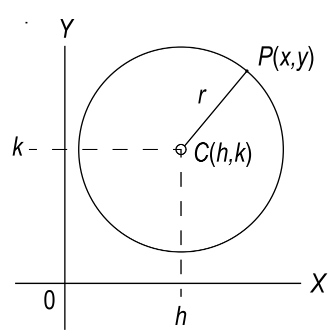
Source: Brink, R.W. *A First Year of College Mathematics*. D. Appleton-Century Company, Inc. (Prentice Hall), 1937.

1.3.8.4 Case 4: Circle e = 0

$(x - h)^2 + (y - k)^2 = r^2$

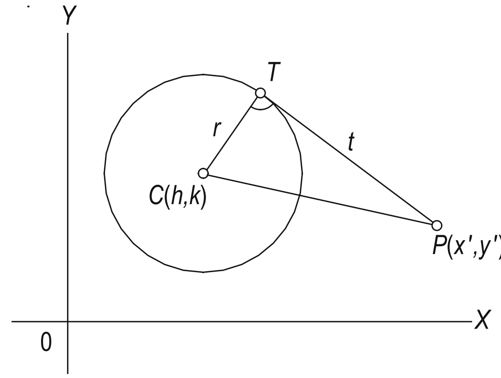
Center at (h, k) is the standard form of the equation with radius

$r = \sqrt{(x-h)^2 + (y-k)^2}$



Length of the tangent line from a point on a circle to a point (x', y'):

$$t^2 = (x' - h)^2 + (y' - k)^2 - r^2$$



Source: Brink, R.W. *A First Year of College Mathematics*. D. Appleton-Century Company, Inc. (Prentice Hall), 1937.

1.4 Engineering Probability and Statistics

1.4.1 Dispersion, Mean, Median, and Mode Values

If $X_1, X_2, \ldots, X_n$ represent the values of a random sample of n items or observations, the *arithmetic mean* of these items or observations, denoted $\overline{X}$, is defined as

$$\overline{X} = (1/n)(X_1 + X_2 + \ldots + X_n) = (1/n)\sum_{i=1}^{n} X_i$$

$\overline{X} \rightarrow \mu$ for sufficiently large values of n.

The *weighted arithmetic mean* is

$$\overline{X} = \frac{\sum w_i X_i}{\sum w_i}$$

where

X_i = value of the ith observation, and

w_i = weight applied to X_i

The *variance* of the population is the *arithmetic mean* of the *squared deviations from the population mean*. If μ is the arithmetic mean of a discrete population of size N, the *population variance* is defined by

$$\sigma^2 = (1/N)\left[(X_1 - \mu)^2 + (X_2 - \mu)^2 + \ldots + (X_N - \mu)^2\right]$$

$$= (1/N)\sum_{i=1}^{N} (X_i - \mu)^2$$

Standard deviation formulas (assuming statistical independence) are

$$\sigma_{\text{population}} = \sqrt{(1/N)\sum (X_i - \mu)^2}$$

$$\sigma_{\text{sum}} = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2}$$

$$\sigma_{\text{series}} = \sigma\sqrt{n}$$

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\text{product}} = \sqrt{A^2\sigma_b^2 + B^2\sigma_a^2}$$

The *sample variance* is

$$s^2 = \left[1/(n-1)\right]\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$$

The *sample standard deviation* is

$$s = \sqrt{\left[1/(n-1)\right]\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}$$

The *sample coefficient of variation* = $CV = s/\overline{X}$

The *sample geometric mean* = $\sqrt[n]{X_1 X_2 X_3 \ldots X_n}$

The *sample root-mean-square value* = $\sqrt{(1/n)\sum X_i^2}$

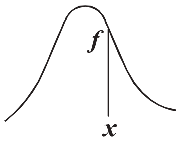
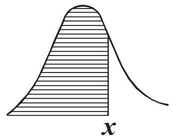
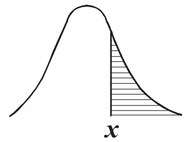
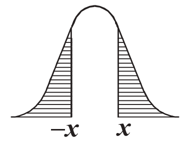
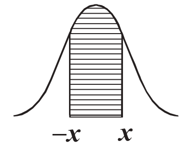
When the discrete data are rearranged in increasing order and n is odd, the median is the value of the $\left(\frac{n+1}{2}\right)^{\text{th}}$ item.

When n is even, the median is the average of the $\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2}+1\right)^{\text{th}}$ items.

The *mode* of a set of data is the value that occurs with the greatest frequency.

The *sample range R* is the largest sample value minus the smallest sample value.

Unit Normal Distribution $(\mu = 0,\ \sigma = 1)$

					
x	***f*(*x*)**	***F*(*x*)**	***R*(*x*)**	**2*R*(*x*)**	***W*(*x*)**
0.0	0.3989	0.5000	0.5000	1.0000	0.0000
0.1	0.3970	0.5398	0.4602	0.9203	0.0797
0.2	0.3910	0.5793	0.4207	0.8415	0.1585
0.3	0.3814	0.6179	0.3821	0.7642	0.2358
0.4	0.3683	0.6554	0.3446	0.6892	0.3108
0.5	0.3521	0.6915	0.3085	0.6171	0.3829
0.6	0.3332	0.7257	0.2743	0.5485	0.4515
0.7	0.3123	0.7580	0.2420	0.4839	0.5161
0.8	0.2897	0.7881	0.2119	0.4237	0.5763
0.9	0.2661	0.8159	0.1841	0.3681	0.6319
1.0	0.2420	0.8413	0.1587	0.3173	0.6827
1.1	0.2179	0.8643	0.1357	0.2713	0.7287
1.2	0.1942	0.8849	0.1151	0.2301	0.7699
1.3	0.1714	0.9032	0.0968	0.1936	0.8064
1.4	0.1497	0.9192	0.0808	0.1615	0.8385
1.5	0.1295	0.9332	0.0668	0.1336	0.8664
1.6	0.1109	0.9452	0.0548	0.1096	0.8904
1.7	0.0940	0.9554	0.0446	0.0891	0.9109
1.8	0.0790	0.9641	0.0359	0.0719	0.9281
1.9	0.0656	0.9713	0.0287	0.0574	0.9426
2.0	0.0540	0.9772	0.0228	0.0455	0.9545
2.1	0.0440	0.9821	0.0179	0.0357	0.9643
2.2	0.0355	0.9861	0.0139	0.0278	0.9722
2.3	0.0283	0.9893	0.0107	0.0214	0.9786
2.4	0.0224	0.9918	0.0082	0.0164	0.9836
2.5	0.0175	0.9938	0.0062	0.0124	0.9876
2.6	0.0136	0.9953	0.0047	0.0093	0.9907
2.7	0.0104	0.9965	0.0035	0.0069	0.9931
2.8	0.0079	0.9974	0.0026	0.0051	0.9949
2.9	0.0060	0.9981	0.0019	0.0037	0.9963
3.0	0.0044	0.9987	0.0013	0.0027	0.9973
Fractiles					
1.2816	0.1755	0.9000	0.1000	0.2000	0.8000
1.6449	0.1031	0.9500	0.0500	0.1000	0.9000
1.9600	0.0584	0.9750	0.0250	0.0500	0.9500
2.0537	0.0484	0.9800	0.0200	0.0400	0.9600
2.3263	0.0267	0.9900	0.0100	0.0200	0.9800
2.5758	0.0145	0.9950	0.0050	0.0100	0.9900

1.4.2 Statistical Quality Control

1.4.2.1 Average and Range Charts

n	A_2	D_3	D_4
2	1.880	0	3.268
3	1.023	0	2.574
4	0.729	0	2.282
5	0.577	0	2.114
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777

$$\overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$$

$$\overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \ldots + \overline{X}_k}{k}$$

$$\overline{R} = \frac{R_1 + R_2 + \ldots + R_k}{k}$$

where

X_i = an individual observation

n = sample size of a group

k = number of groups

R = range, the difference between the largest and smallest observations in a sample of size n

The R Chart formulas are:

$$CL_R = \overline{R}$$

$$UCL_R = D_4\overline{R}$$

$$LCL_R = D_3\overline{R}$$

The $\overline{X}$ Chart formulas are:

$$CL_X = \overline{\overline{X}}$$

$$UCL_X = \overline{\overline{X}} + A_2\overline{R}$$

$$LCL_X = \overline{\overline{X}} - A_2\overline{R}$$

1.4.2.2 Standard Deviation Charts

$$UCL_X = \overline{\overline{X}} + A_3\overline{S}$$

$$CL_X = \overline{\overline{X}}$$

$$LCL_X = \overline{\overline{X}} - A_3\overline{S}$$

$$UCL_S = B_4\overline{S}$$

$$CL_S = \overline{S}$$

$$LCL_S = B_3\overline{S}$$

n	A_3	B_3	B_4
2	2.659	0	3.267
3	1.954	0	2.568
4	1.628	0	2.266
5	1.427	0	2.089
6	1.287	0.030	1.970
7	1.182	0.119	1.882
8	1.099	0.185	1.815
9	1.032	0.239	1.761
10	0.975	0.284	1.716

1.4.2.3 Approximations

The following table and equations may be used to generate initial approximations of the items indicated.

$$\hat{\sigma} = \overline{R}/d_2$$

$$\hat{\sigma} = \overline{S}/c_4$$

$$\sigma_R = d_3\hat{\sigma}$$

$$\sigma_S = \hat{\sigma}\sqrt{1 - c_4^2}$$

n	c_4	d_2	d_3
2	0.7979	1.128	0.853
3	0.8862	1.693	0.888
4	0.9213	2.059	0.880
5	0.9400	2.326	0.864
6	0.9515	2.534	0.848
7	0.9594	2.704	0.833
8	0.9650	2.847	0.820
9	0.9693	2.970	0.808
10	0.9727	3.078	0.797

where

$\hat{\sigma}$ = an estimate of σ

σ_R = an estimate of the standard deviation of the ranges of the samples

σ_S = an estimate of the standard deviation of the standard deviations of the samples

Probability and Density Functions: Means and Variances

Variable	Equation	Mean	Variance
Binomial Coefficient	$\binom{n}{x} = \frac{n!}{x!(n-x)!}$		
Binomial	$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Hyper Geometric	$h(x; n, r, N) = \binom{r}{x} \frac{\binom{N-r}{n-x}}{\binom{N}{n}}$	$\frac{nr}{N}$	$\frac{r(N-r)n(N-n)}{N^2(N-1)}$
Poisson	$f(x; \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ
Geometric	$g(x; p) = p\,(1-p)^{x-1}$	$1/p$	$(1-p)/p^2$
Negative Binomial	$f(y; r, p) = \binom{y+r-1}{r-1} p^r (1-p)^y$	r/p	$r\,(1-p)/p^2$
Multinomial	$f(x_1, \ldots, x_k) = \frac{n!}{x_1!, \ldots, x_k!} p_1^{x_1} \ldots p_k^{x_k}$	np_i	$np_i\,(1-p_i)$
Uniform	$f(x) = 1/(b-a)$	$(a+b)/2$	$(b-a)^2/12$
Gamma	$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}; \quad \alpha > 0,\ \beta > 0$	$\alpha\beta$	$\alpha\beta^2$
Exponential	$f(x) = \frac{1}{\beta} e^{-x/\beta}$	β	β^2
Weibull	$f(x) = \frac{\alpha}{\beta} x^{\alpha-1} e^{-x^\alpha/\beta}$	$\beta^{1/\alpha}\Gamma[(\alpha+1)/\alpha]$	$\beta^{2/\alpha}\left[\Gamma\left(\frac{\alpha+1}{\alpha}\right) - \Gamma^2\left(\frac{\alpha+1}{\alpha}\right)\right]$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2
Triangular	$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(m-a)} \text{ if } a \le x \le m \\ \frac{2(b-x)}{(b-a)(b-m)} \text{ if } m < x \le b \end{cases}$	$\frac{a+b+m}{3}$	$\frac{a^2 + b^2 + m^2 - ab - am - bm}{18}$

1.5 Statics

1.5.1 Force

A *force* is a *vector* quantity. It is defined when its **(1)** magnitude, **(2)** point of application, and **(3)** direction are known.

The vector form of a force is

$$\boldsymbol{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

1.5.2 Resultant (Two Dimensions)

The *resultant*, F, of n forces with components $F_{x,i}$ and $F_{y,i}$ has the magnitude of

$$F = \left[\left(\sum_{i=1}^{n} F_{x,i}\right)^2 + \left(\sum_{i=1}^{n} F_{y,i}\right)^2\right]^{1/2}$$

The resultant direction with respect to the x-axis is

$$\theta = \arctan\left(\sum_{i=1}^{n} F_{y,i} \Big/ \sum_{i=1}^{n} F_{x,i}\right)$$

1.5.3 Resolution of a Force

$$F_x = F\cos\theta_x \qquad F_y = F\cos\theta_y \qquad F_z = F\cos\theta_z$$

$$\cos\theta_x = F_x/F \qquad \cos\theta_y = F_y/F \qquad \cos\theta_z = F_z/F$$

Separating a force into components when the geometry of force is known and $R = \sqrt{x^2 + y^2 + z^2}$:

$$F_x = F(x/R) \qquad F_y = F(y/R) \qquad F_z = F(z/R)$$

1.5.4 Moments (Couples)

A system of two forces that are equal in magnitude, opposite in direction, and parallel to each other is called a *couple*. A *moment* $\boldsymbol{M}$ is defined as the cross-product of the *radius vector* $\boldsymbol{r}$ and the *force* $\boldsymbol{F}$ from a point to the line of action of the force.

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F} \qquad M_x = yF_z - zF_y$$

$$M_y = zF_x - xF_z$$

$$M_z = xF_y - yF_x$$

1.5.5 Systems of Forces

$$\boldsymbol{F} = \Sigma\,\boldsymbol{F}_n$$

$$\boldsymbol{M} = \Sigma\,(\boldsymbol{r}_n \times \boldsymbol{F}_n)$$

Equilibrium Requirements

$$\sum \boldsymbol{F}_n = 0$$
$$\sum \boldsymbol{M}_n = 0$$

1.5.6 Centroids of Masses, Areas, Lengths, and Volumes

The following formulas are for discrete masses, areas, lengths, and volumes:

$$\boldsymbol{r}_c = \frac{\sum m_n \boldsymbol{r}_n}{\sum m_n}$$

where

m_n = mass of each particle making up the system

$\boldsymbol{r}_n$ = radius vector to each particle from a selected reference point

$\boldsymbol{r}_c$ = radius vector to the centroid of the total mass from the selected reference point

The *moment of area* (M_a) is defined as

$$M_{ay} = \sum x_n a_n$$

$$M_{ax} = \sum y_n a_n$$

The *centroid of area* is defined as

$$x_{ac} = M_{ay}/A = \sum x_n a_n / A$$

$$y_{ac} = M_{ax}/A = \sum y_n a_n / A$$

where $A = \sum a_n$

The following equations are for an area, bounded by the axes and the function $y = f(x)$. The centroid of area is defined as

$$x_c = \frac{\int x dA}{A}$$

$$y_c = \frac{\int y dA}{A}$$

$$A = \int f(x)\, dx$$

$$dA = f(x)\, dx = g(y)\, dy$$

The *first moment of area* with respect to the *y*-axis and the *x*-axis, respectively, are

$$M_y = \int x\, dA = x_c A$$

$$M_x = \int y\, dA = y_c A$$

1.5.7 Moment of Inertia

The *moment of inertia*, or the *second moment of area*, is defined as

$$I_y = \int x^2\, dA$$

$$I_x = \int y^2\, dA$$

The *polar moment of inertia J* of an area about a point is equal to the sum of the moments of inertia of the area about any two perpendicular axes in the area and passing through the same point:

$$I_z = J = I_y + I_x = \int (x^2 + y^2)\, dA$$

$$= r_p^{\,2} A$$

where r_p = radius of gyration (as defined below)

1.5.7.1 Moment of Inertia Parallel Axis Theorem

The moment of inertia of an area about any axis is defined as the moment of inertia of the area about a parallel centroidal axis plus a term equal to the area multiplied by the square of the perpendicular distance d from the centroidal axis to the axis in question.

$$I_x = I_{x_c} + d_y^2 A$$

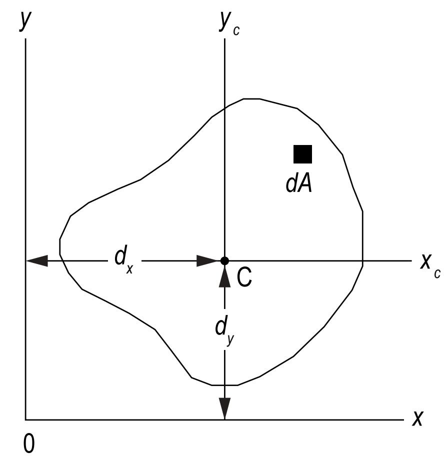
$$I_y = I_{y_c} + d_x^2 A$$

where

d_x, d_y = distance between the two axes in question

I_{x_c}, I_{y_c} = moment of inertia about the centroidal axis

I_x, I_y = moment of inertia about the new axis



Source: Hibbeler, Russell C. *Engineering Mechanics: Statics and Dynamics*. 10 ed. Pearson, 2004, p. 501.

1.5.7.2 Radius of Gyration

The *radius of gyration* r_p, r_x, r_y is the distance from a reference axis at which all of the area can be considered to be concentrated to produce the moment of inertia.

$$r_x = \sqrt{I_x / A} \qquad r_y = \sqrt{I_y / A} \qquad r_p = \sqrt{J / A}$$

1.5.7.3 Product of Inertia

The *product of inertia* (I_{xy}, etc.) is defined as

$I_{xy} = \int xy\,dA$, with respect to the xy-coordinate system

The *parallel-axis theorem* also applies:

$I'_{xy} = I_{x_c y_C} + d_x d_y A$ for the xy-coordinate system, etc.

where

d_x = x-axis distance between the two axes in question

d_y = y-axis distance between the two axes in question

1.5.8 Friction

The largest frictional force is called the *limiting friction*.

Any further increase in applied forces will cause motion.

$$F \le \mu_s N$$

where

F = friction force

μ_s = coefficient of static friction

N = normal force between surfaces in contact

1.5.9 Screw Thread

For a *screw-jack*, *square thread:*

$$M = Pr \tan(\alpha \pm \phi)$$

where

+ is for screw tightening

– is for screw loosening

M = external moment applied to axis of screw

P = load on jack applied along and on the line of the axis

r = mean thread radius

α = pitch angle of the thread

μ = $\tan \phi$ = appropriate coefficient of friction

1.5.10 Belt Friction

$$F_1 = F_2 e^{\mu\theta}$$

where

F_1 = force being applied in the direction of impending motion

F_2 = force applied to resist impending motion

μ = coefficient of static friction

θ = total angle of contact between the surfaces expressed in radians

1.5.11 Statically Determinate Truss

1.5.11.1 Plane Truss: Method of Joints

The method consists of solving for the forces in the members by writing the two equilibrium equations for each joint of the truss.

$$\sum F_H = 0 \text{ and } \sum F_V = 0$$

where

F_H = horizontal forces and member components

F_V = vertical forces and member components

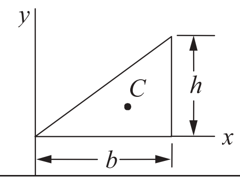
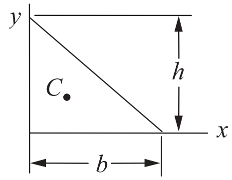
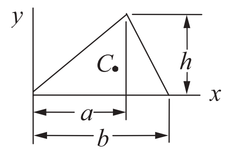
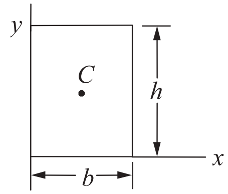
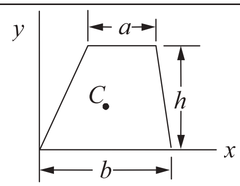
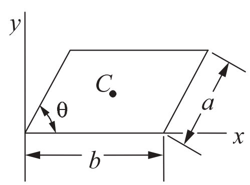
1.5.11.2 Plane Truss: Method of Sections

The method consists of drawing a free-body diagram of a portion of the truss in such a way that the unknown truss member force is exposed as an external force.

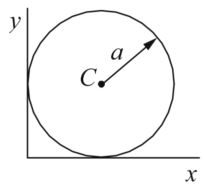
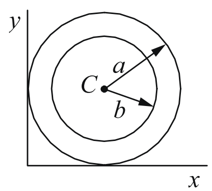
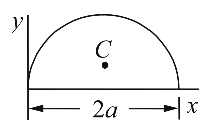
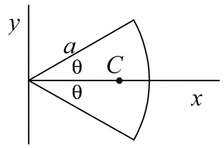
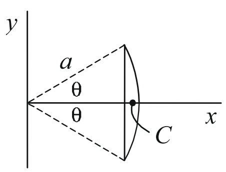
1.5.12 Concurrent Forces

A concurrent-force system is one in which the lines of action of the applied forces all meet at one point.

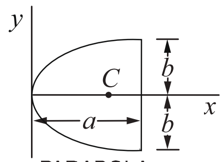
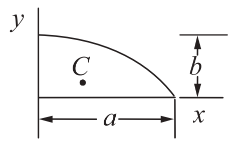
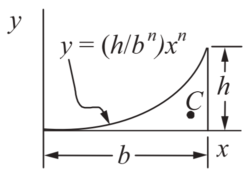
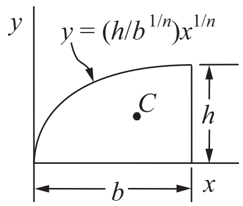
A two-force body in static equilibrium has two applied forces that are equal in magnitude, opposite in direction, and collinear.

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a + b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = \left[bh\left(b^2 - ab + a^2\right)\right]/36$ $I_x = bh^3/12$ $I_y = \left[bh\left(b^2 + ab + a^2\right)\right]/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = \left(b^2 - ab + a^2\right)/18$ $r_x^2 = h^2/6$ $r_y^2 = \left(b^2 + ab + a^2\right)/6$	$I_{x_c y_c} = \left[Ah(2a - b)\right]/36$ $= \left[bh^2(2a - b)\right]/72$ $I_{xy} = \left[Ah(2a + b)\right]/12$ $= \left[bh^2(2a + b)\right]/24$
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ $I_x = bh^3/3$ $I_y = b^3h/3$ $J = \left[bh\left(b^2 + h^2\right)\right]/12$	$r_{x_c}^2 = h^2/12$ $r_{y_c}^2 = b^2/12$ $r_x^2 = h^2/3$ $r_y^2 = b^2/3$ $r_p^2 = \left(b^2 + h^2\right)/12$	$I_{x_c y_c} = 0$ $I_{xy} = Abh/4 = b^2h^2/4$
	$A = h(a + b)/2$ $y_c = \dfrac{h(2a + b)}{3(a + b)}$	$I_{x_c} = \dfrac{h^3\left(a^2 + 4ab + b^2\right)}{36(a + b)}$ $I_x = \dfrac{h^3(3a + b)}{12}$	$r_{x_c}^2 = \dfrac{h^2\left(a^2 + 4ab + b^2\right)}{18(a + b)}$ $r_x^2 = \dfrac{h^2(3a + b)}{6(a + b)}$	
	$A = ab \sin\theta$ $x_c = (b + a\cos\theta)/2$ $y_c = (a\sin\theta)/2$	$I_{x_c} = \left(a^3 b \sin^3\theta\right)/12$ $I_{y_c} = \left[ab\sin\theta\left(b^2 + a^2\cos^2\theta\right)\right]/12$ $I_x = \left(a^3 b \sin^3\theta\right)/3$ $I_y = \left[ab\sin\theta\,(b + a\cos\theta)^2\right]/3$ $-\left(a^2b^2\sin\theta\cos\theta\right)/6$	$r_{x_c}^2 = (a\sin\theta)^2/12$ $r_{y_c}^2 = \left(b^2 + a^2\cos^2\theta\right)/12$ $r_x^2 = (a\sin\theta)^2/3$ $r_y^2 = (b + a\cos\theta)^2/3$ $-(ab\cos\theta)/6$	$I_{x_c y_c} = \left(a^3 b \sin^2\theta\cos\theta\right)/12$

Source: Housner, George W. and Hudson, Donald E. (1980) *Applied Mechanics Dynamics*. California Institute of Technology, Pasadena, CA. https://resolver.caltech.edu/CaltechBOOK:1980.001

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = \pi a^2$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi a^4/4$ $I_x = I_y = 5\pi a^4/4$ $J = \pi a^4/2$	$r_{x_c}^2 = r_{y_c}^2 = a^2/4$ $r_x^2 = r_y^2 = 5a^2/4$ $r_p^2 = a^2/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$
	$A = \pi(a^2 - b^2)$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi\left(a^4 - b^4\right)/4$ $I_x = I_y = \dfrac{5\pi a^4}{4} - \pi a^2 b^2 - \dfrac{\pi b^4}{4}$ $J = \pi\left(a^4 - b^4\right)/2$	$r_{x_c}^2 = r_{y_c}^2 = \left(a^2 + b^2\right)/4$ $r_x^2 = r_y^2 = \left(5a^2 + b^2\right)/4$ $r_p^2 = \left(a^2 + b^2\right)/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$ $= \pi a^2\left(a^2 - b^2\right)$
	$A = \pi a^2/2$ $x_c = a$ $y_c = 4a/(3\pi)$	$I_{x_c} = \dfrac{a^4\left(9\pi^2 - 64\right)}{72\pi}$ $I_{y_c} = \pi a^4/8$ $I_x = \pi a^4/8$ $I_y = 5\pi a^4/8$	$r_{x_c}^2 = \dfrac{a^2\left(9\pi^2 - 64\right)}{36\pi^2}$ $r_{y_c}^2 = a^2/4$ $r_x^2 = a^2/4$ $r_y^2 = 5a^2/4$	$I_{x_c y_c} = 0$ $I_{xy} = 2a^4/3$
CIRCULAR SECTOR	$A = a^2\theta$ $x_c = \dfrac{2a}{3}\dfrac{\sin\theta}{\theta}$ $y_c = 0$	$I_x = a^4(\theta - \sin\theta\cos\theta)/4$ $I_y = a^4(\theta + \sin\theta\cos\theta)/4$	$r_x^2 = \dfrac{a^2}{4}\dfrac{(\theta - \sin\theta\cos\theta)}{\theta}$ $r_y^2 = \dfrac{a^2}{4}\dfrac{(\theta + \sin\theta\cos\theta)}{\theta}$	$I_{x_c y_c} = 0$ $I_{xy} = 0$
CIRCULAR SEGMENT	$A = a^2\left[\theta - \dfrac{\sin 2\theta}{2}\right]$ $x_c = \dfrac{2a}{3}\dfrac{\sin^3\theta}{\theta - \sin\theta\cos\theta}$ $y_c = 0$	$I_x = \dfrac{Aa^2}{4}\left[1 - \dfrac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$ $I_y = \dfrac{Aa^2}{4}\left[1 + \dfrac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$	$r_x^2 = \dfrac{a^2}{4}\left[1 - \dfrac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$ $r_y^2 = \dfrac{a^2}{4}\left[1 + \dfrac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$	$I_{x_c y_c} = 0$ $I_{xy} = 0$

Source: Housner, George W. and Hudson, Donald E. (1980) *Applied Mechanics Dynamics*. California Institute of Technology, Pasadena, CA. https://resolver.caltech.edu/CaltechBOOK:1980.001

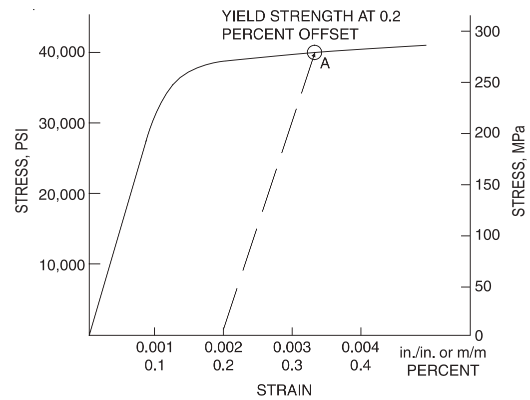
Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
 PARABOLA	$A = 4ab/3$ $x_c = 3a/5$ $y_c = 0$	$I_{x_c} = I_x = 4ab^3/15$ $I_{y_c} = 16a^3b/175$ $I_y = 4a^3b/7$	$r_{x_c}^2 = r_x^2 = b^2/5$ $r_{y_c}^2 = 12a^2/175$ $r_y^2 = 3a^2/7$	$I_{x_c y_c} = 0$ $I_{xy} = 0$
 HALF A PARABOLA	$A = 2ab/3$ $x_c = 3a/5$ $y_c = 3b/8$	$I_x = 2ab^3/15$ $I_y = 2ba^3/7$	$r_x^2 = b^2/5$ $r_y^2 = 3a^2/7$	$I_{xy} = Aab/4 = a^2b^2$
 n^{th} DEGREE PARABOLA	$A = bh/(n+1)$ $x_c = \dfrac{n+1}{n+2}b$ $y_c = \dfrac{h}{2}\dfrac{n+1}{2n+1}$	$I_x = \dfrac{bh^3}{3(3n+1)}$ $I_y = \dfrac{hb^3}{n+3}$	$r_x^2 = \dfrac{h^2(n+1)}{3(3n+1)}$ $r_y^2 = \dfrac{n+1}{n+3}b^2$	
 n^{th} DEGREE PARABOLA	$A = \dfrac{n}{n+1}bh$ $x_c = \dfrac{n+1}{2n+1}b$ $y_c = \dfrac{n+1}{2(n+2)}h$	$I_x = \dfrac{n}{3(n+3)}bh^3$ $I_y = \dfrac{n}{3n+1}b^3h$	$r_x^2 = \dfrac{n+1}{3(n+1)}h^2$ $r_y^2 = \dfrac{n+1}{3n+1}b^2$	

Source: Housner, George W. and Hudson, Donald E. (1980) *Applied Mechanics Dynamics*. California Institute of Technology, Pasadena, CA.
https://resolver.caltech.edu/CaltechBOOK:1980.001

1.6 Mechanics of Materials

1.6.1 Uniaxial Stress-Strain

Stress-Strain Curve for Mild Steel



Source: Flinn, Richard, and Paul K. Trojan. *Engineering Materials and Their Applications*. 4th ed. John Wiley and Sons Ltd., 1990.

The slope of the linear portion of the curve equals the modulus of elasticity.

1.6.2 Definitions

1.6.2.1 Engineering Strain

$\varepsilon = \Delta L / L_o$

where

ε = engineering strain (units per unit)

ΔL = change in length (units) of member

L_o = original length (units) of member

1.6.2.2 Percent Elongation

$$\% \text{ Elongation} = \left(\frac{\Delta L}{L_o}\right) \times 100$$

1.6.2.3 Percent Reduction in Area (RA)

The % reduction in area from initial area, A_i, to final area, A_f, is:

$$\% RA = \left(\frac{A_i - A_f}{A_i}\right) \times 100$$

1.6.2.4 Shear Stress-Strain

$\gamma = \tau / G$

where

γ = shear strain

τ = shear stress

G = shear modulus (constant in linear torsion-rotation relationship)

$$G = \frac{E}{2(1+\nu)}$$

where

E = modulus of elasticity (Young's modulus)

ν = Poisson's ratio

ν = – (lateral strain)/(longitudinal strain)

1.6.2.5 Bulk (Volume) Modulus of Elasticity

$$K = \frac{E}{3(1-2\nu)}$$

where

K = bulk modulus

E = modulus of elasticity

ν = Poisson's ratio

1.6.2.6 Uniaxial Loading and Deformation

$\sigma = P/A$

where

σ = stress on the cross section

P = loading

A = cross-sectional area

$\varepsilon = \delta/L$

where

δ = elastic longitudinal deformation

L = length of member

$$E = \sigma/\varepsilon = \frac{P/A}{\delta/L}$$

$$\delta = \frac{PL}{AE}$$

True stress is load divided by actual cross-sectional area whereas engineering stress is load divided by the initial area.

1.6.3 Thermal Deformations

$\delta_t = \alpha L(T - T_o)$

where

δ_t = deformation caused by a change in temperature

α = temperature coefficient of expansion

L = length of member

T = final temperature

T_o = initial temperature

1.6.4 Stress and Strain

1.6.4.1 Principal Stresses

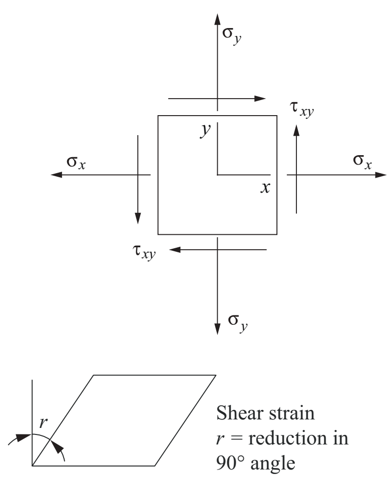
For the special case of a *two-dimensional* stress state, the equations for principal stress reduce to

$$\sigma_a, \sigma_b = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_c = 0$$

The two nonzero values calculated from this equation are temporarily labeled σ_a and σ_b and the third value σ_c is always zero in this case. Depending on their values, the three roots are then labeled according to the convention:

algebraically largest = σ_1, *algebraically smallest* = σ_3, *other* = σ_2. A typical 2D stress element is shown below with all indicated components shown in their positive sense.



Source: Crandall, Stephen H. and Norman C. Dahl, *An Introduction to the Mechanics of Solids*. New York: McGraw-Hill, 1999.

1.6.4.2 Mohr's Circle—Stress, 2D

To construct a Mohr's circle, use the following sign conventions:

1. Tensile normal stress components are plotted on the horizontal axis and are considered positive. Compressive normal stress components are negative.
2. For constructing Mohr's circle only, shearing stresses are plotted above the normal stress axis when the pair of shearing stresses, acting on opposite and parallel faces of an element, forms a clockwise couple. Shearing stresses are plotted below the normal axis when the shear stresses form a counterclockwise couple.

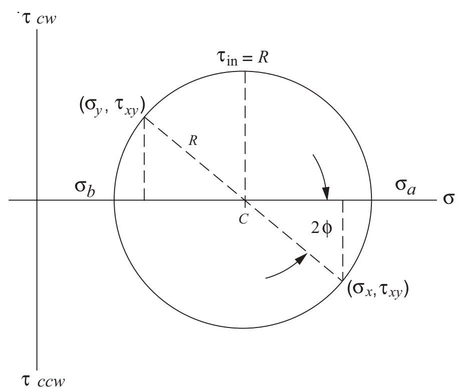
The circle drawn with the center on the normal stress (horizontal) axis with center, C, and radius, R, where

$$C = \frac{\sigma_x + \sigma_y}{2}, \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The two nonzero principal stresses are then:

$$\sigma_a = C + R$$

$$\sigma_b = C - R$$



Source: Crandall, Stephen H. and Norman C. Dahl, *An Introduction to the Mechanics of Solids*. New York: McGraw-Hill, 1999.

The maximum *inplane* shear stress is $\tau_{in} = R$. However, the maximum shear stress considering three dimensions is always

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

1.6.5 Torsion

Torsion stress in circular solid or thick-walled ($t > 0.1\ r$) shafts:

$$\tau = \frac{Tr}{J}$$

where J = polar moment of inertia

1.6.6 Torsional Strain

$$\gamma_{\phi z} = \lim_{\Delta z \to 0} r(\Delta\phi/\Delta z) = r(d\phi/dz)$$

The shear strain varies in direct proportion to the radius, from zero strain at the center to the greatest strain at the outside of the shaft. $d\phi/dz$ is the twist per unit length or the rate of twist.

$$\tau_{\phi z} = G\gamma_{\phi z} = Gr(d\phi/dz)$$

$$T = G(d\phi/dz)\int_A r^2\, dA = GJ(d\phi/dz)$$

$$\phi = \int_o^L \frac{T}{GJ} dz = \frac{TL}{GJ}$$

where

ϕ = total angle (radians) of twist

T = torque

L = length of shaft

T/ϕ gives the *twisting moment per radian of twist*. This is called the *torsional stiffness* and is often denoted by the symbol k or c.

For Hollow, Thin-Walled Shafts

$$\tau = \frac{T}{2A_m t}$$

where

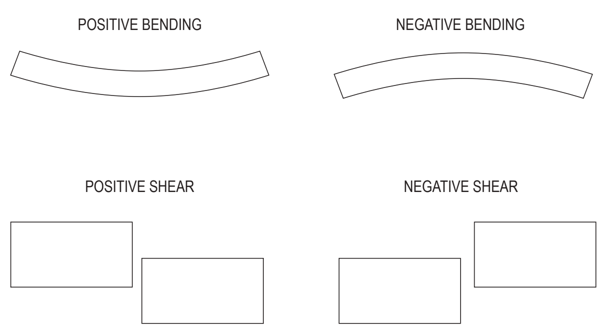
t = thickness of shaft wall

A_m = area of a solid shaft of radius equal to the mean radius of the hollow shaft

1.6.7 Beams

1.6.7.1 Shearing Force and Bending Moment Sign Conventions

1. The bending moment is *positive* if it produces bending of the beam *concave upward* (compression in top fibers and tension in bottom fibers).
2. The shearing force is *positive* if the *right portion of the beam tends to shear downward with respect to the left*.



Source: Timoshenko, S. and Gleason H. MacCullough. *Elements of Strength of Materials*. 3rd ed. Van Nostrand (Wadsworth), 1954.

The relationship between the load (w), shear (V), and moment (M) equations are:

$$w(x) = -\frac{dV(x)}{dx}$$

$$V = \frac{dM(x)}{dx}$$

$$V_2 - V_1 = \int_{x1}^{x2} [-w(x)]dx$$

$$M_2 - M_1 = \int_{x1}^{x2} V(x)dx$$

1.6.7.2 Stresses in Beams

The normal stress in a beam due to bending:

$\sigma = -My/I$

where

M = moment at the section

I = moment of inertia of the cross section

y = distance from the neutral axis to the fiber location above or below the neutral axis

The maximum normal stresses in a beam due to bending:

$\sigma = \pm Mc/I$

where

c = distance from the neutral axis to the outermost fiber of a symmetrical beam section

$\sigma = M/s$

where

s = elastic section modulus of the beam

$s = I/c$

Transverse shear stress:

$\tau_{xy} = VQ/(Ib)$

where

V = shear force

$Q = A'\overline{y'}$ = first moment of area above or below the point where shear stress is to be determined

Source: Hibbeler, Russell C. *Mechanics of Materials*. 10th ed. Pearson, 2015, pp. 386-387.

where

A' = area above the layer (or plane) upon which the desired transverse shear stress acts

$\overline{y'}$ = distance from neutral axis to area centroid

b = width or thickness of the cross section

Transverse shear flow:

$q = VQ/I$

1.6.7.3 Composite Sections

The bending stresses in a beam composed of dissimilar materials (Material 1 and Material 2) where $E_1 > E_2$ are:

$\sigma_1 = -nMy/I_T$

$\sigma_2 = -My/I_T$

where

I_T = moment of inertia of the transformed section

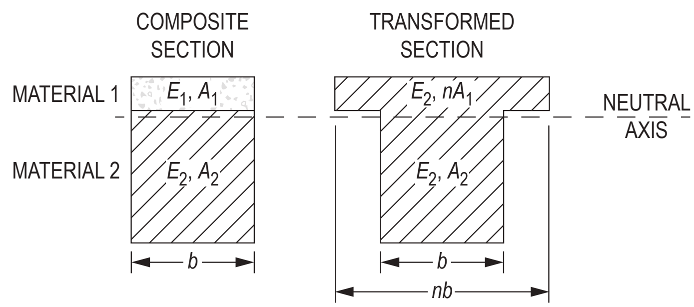
n = modular ratio E_1/E_2

E_1 = elastic modulus of Material 1

E_2 = elastic modulus of Material 2

y = distance from the neutral axis to the fiber location above or below the neutral axis

The composite section is transformed into a section composed of a single material. The centroid and then the moment of inertia are found on the transformed section for use in the bending stress equations.



1.6.8 Columns

Critical axial load for long column subject to buckling:

Euler's Formula

$$P_{cr} = \frac{\pi^2 EI}{(K\ell)^2}$$

where

ℓ = unbraced column length

K = effective-length factor to account for end supports

Theoretical effective-length factors for columns include:

Pinned-pinned, $K = 1.0$

Fixed-fixed, $K = 0.5$

Fixed-pinned, $K = 0.7$

Fixed-free, $K = 2.0$

Critical buckling stress for long columns:

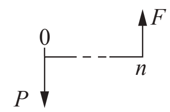
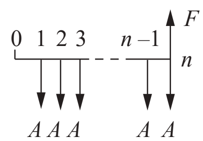
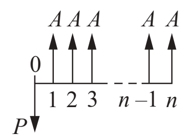
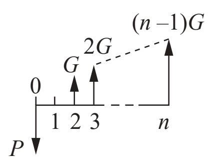
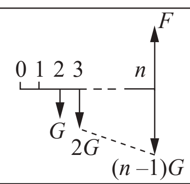
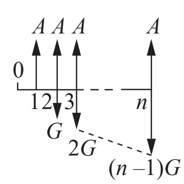
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(K\ell/r)^2}$$

where

r = radius of gyration = $\sqrt{I/A}$

$K\ell/r$ = effective slenderness ratio for the column

1.7 Engineering Economics

Factor Name	Converts	Cash Flow Diagram	Symbol	Formula
Single Payment Compound Amount	to F given P		$(F/P, i\%, n)$	$P(1+i)^n$
Single Payment Present Worth	to P given F		$(P/F, i\%, n)$	$F(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F		$(A/F, i\%, n)$	$F\dfrac{i}{(1+i)^n-1}$
Uniform Series Compound Amount	to F given A		$(F/A, i\%, n)$	$A\dfrac{(1+i)^n-1}{i}$
Uniform Series Present Worth	to P given A		$(P/A, i\%, n)$	$A\dfrac{(1+i)^n-1}{i(1+i)^n}$
Capital Recovery	to A given P		$(A/P, i\%, n)$	$P\dfrac{i(1+i)^n}{(1+i)^n-1}$
Uniform Gradient Present Worth	to P given G		$(P/G, i\%, n)$	$G\left[\dfrac{(1+i)^n-1}{i^2(1+i)^n}-\dfrac{n}{i(1+i)^n}\right]$
Uniform Gradient Future Worth †	to F given G		$(F/G, i\%, n)$	$G\left[\dfrac{(1+i)^n-1}{i^2}-\dfrac{n}{i}\right]$
Uniform Gradient Uniform Series	to A given G		$(A/G, i\%, n)$	$G\left[\dfrac{1}{i}-\dfrac{n}{(1+i)^n-1}\right]$

† $F/G = (F/A - n)/i = (F/A) \times (A/G)$

1.7.1 Nomenclature and Definitions

A Uniform amount per interest period

B Benefit

BV Book value

C Cost

d Inflation adjusted interest rate per interest period

D_j Depreciation in year j

EV Expected value

F Future worth, value, or amount

f General inflation rate per interest period

G Uniform gradient amount per interest period

i Interest rate per interest period

i_e Annual effective interest rate

MARR Minimum acceptable/attractive rate of return

m Number of compounding periods per year

n Number of compounding periods; or the expected life of an asset

P Present worth, value, or amount

r Nominal annual interest rate

S_n Expected salvage value in year n

Subscripts

j at time j

n at time n

1.7.2 Nonannual Compounding

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

1.7.3 Breakeven Analysis

By altering the value of any one of the variables in a situation, holding all the other values constant, it is possible to find a value for that variable that makes the two alternatives equally economical. This value is the *breakeven point*.

Breakeven analysis is used to describe the percentage of capacity of operation for a manufacturing plant at which income will just cover expenses.

The *payback period* is the period of time required for the profit or other benefits of an investment to equal the cost of the investment.

1.7.4 Inflation

To account for inflation, dollars are deflated by the general inflation rate per interest period f, and then are shifted over the time scale using the interest rate per interest period i. Use an inflation adjusted interest rate per interest period d for computing present worth values P.

The formula for d is $d = i + f + (i \times f)$

1.7.5 Depreciation

Straight Line

$$D_j = \frac{C - S_n}{n}$$

1.7.6 Book Value

$BV = \text{initial cost} - \sum D_j$

1.7.7 Capitalized Costs

Capitalized costs are present worth values using an assumed perpetual period of time:

$$\text{Capitalized Costs} = P = \frac{A}{i}$$

1.7.8 Rate-of-Return

The minimum acceptable rate-of-return (MARR) is that interest rate that one is willing to accept, or the rate one desires to earn on investments. The rate-of-return on an investment is the interest rate that makes the benefits and costs equal.

1.7.9 Benefit-Cost Analysis

In a benefit-cost analysis, the benefits B of a project should exceed the estimated costs C.

$B - C \geq 0$, or $B/C \geq 1$

1.7.10 Interest Rate Tables

Interest Rate Tables
Factor Table: i = 0.50%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9950	0.9950	0.0000	1.0050	1.0000	1.0050	1.0000	0.0000
2	0.9901	1.9851	0.9901	1.0100	2.0050	0.5038	0.4988	0.4988
3	0.9851	2.9702	2.9604	1.0151	3.0150	0.3367	0.3317	0.9967
4	0.9802	3.9505	5.9011	1.0202	4.0301	0.2531	0.2481	1.4938
5	**0.9754**	**4.9259**	**9.8026**	**1.0253**	**5.0503**	**0.2030**	**0.1980**	**1.9900**
6	0.9705	5.8964	14.6552	1.0304	6.0755	0.1696	0.1646	2.4855
7	0.9657	6.8621	20.4493	1.0355	7.1059	0.1457	0.1407	2.9801
8	0.9609	7.8230	27.1755	1.0407	8.1414	0.1278	0.1228	3.4738
9	0.9561	8.7791	34.8244	1.0459	9.1821	0.1139	0.1089	3.9668
10	**0.9513**	**9.7304**	**43.3865**	**1.0511**	**10.2280**	**0.1028**	**0.0978**	**4.4589**
11	0.9466	10.6770	52.8526	1.0564	11.2792	0.0937	0.0887	4.9501
12	0.9419	11.6189	63.2136	1.0617	12.3356	0.0861	0.0811	5.4406
13	0.9372	12.5562	74.4602	1.0670	13.3972	0.0796	0.0746	5.9302
14	0.9326	13.4887	86.5835	1.0723	14.4642	0.0741	0.0691	6.4190
15	**0.9279**	**14.4166**	**99.5743**	**1.0777**	**15.5365**	**0.0694**	**0.0644**	**6.9069**
16	0.9233	15.3399	113.4238	1.0831	16.6142	0.0652	0.0602	7.3940
17	0.9187	16.2586	128.1231	1.0885	17.6973	0.0615	0.0565	7.8803
18	0.9141	17.1728	143.6634	1.0939	18.7858	0.0582	0.0532	8.3658
19	0.9096	18.0824	160.0360	1.0994	19.8797	0.0553	0.0503	8.8504
20	**0.9051**	**18.9874**	**177.2322**	**1.1049**	**20.9791**	**0.0527**	**0.0477**	**9.3342**
21	0.9006	19.8880	195.2434	1.1104	22.0840	0.0503	0.0453	9.8172
22	0.8961	20.7841	214.0611	1.1160	23.1944	0.0481	0.0431	10.2993
23	0.8916	21.6757	233.6768	1.1216	24.3104	0.0461	0.0411	10.7806
24	0.8872	22.5629	254.0820	1.1272	25.4320	0.0443	0.0393	11.2611
25	**0.8828**	**23.4456**	**275.2686**	**1.1328**	**26.5591**	**0.0427**	**0.0377**	**11.7407**
30	0.8610	27.7941	392.6324	1.1614	32.2800	0.0360	0.0310	14.1265
40	0.8191	36.1722	681.3347	1.2208	44.1588	0.0276	0.0226	18.8359
50	0.7793	44.1428	1,035.6966	1.2832	56.6452	0.0227	0.0177	23.4624
60	0.7414	51.7256	1,448.6458	1.3489	69.7700	0.0193	0.0143	28.0064
100	**0.6073**	**78.5426**	**3,562.7934**	**1.6467**	**129.3337**	**0.0127**	**0.0077**	**45.3613**

Factor Table: i = 1.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9901	0.9901	0.0000	1.0100	1.0000	1.0100	1.0000	0.0000
2	0.9803	1.9704	0.9803	1.0201	2.0100	0.5075	0.4975	0.4975
3	0.9706	2.9410	2.9215	1.0303	3.0301	0.3400	0.3300	0.9934
4	0.9610	3.9020	5.8044	1.0406	4.0604	0.2563	0.2463	1.4876
5	**0.9515**	**4.8534**	**9.6103**	**1.0510**	**5.1010**	**0.2060**	**0.1960**	**1.9801**
6	0.9420	5.7955	14.3205	1.0615	6.1520	0.1725	0.1625	2.4710
7	0.9327	6.7282	19.9168	1.0721	7.2135	0.1486	0.1386	2.9602
8	0.9235	7.6517	26.3812	1.0829	8.2857	0.1307	0.1207	3.4478
9	0.9143	8.5650	33.6959	1.0937	9.3685	0.1167	0.1067	3.9337
10	**0.9053**	**9.4713**	**41.8435**	**1.1046**	**10.4622**	**0.1056**	**0.0956**	**4.4179**
11	0.8963	10.3676	50.8067	1.1157	11.5668	0.0965	0.0865	4.9005
12	0.8874	11.2551	60.5687	1.1268	12.6825	0.0888	0.0788	5.3815
13	0.8787	12.1337	71.1126	1.1381	13.8093	0.0824	0.0724	5.8607
14	0.8700	13.0037	82.4221	1.1495	14.9474	0.0769	0.0669	6.3384
15	**0.8613**	**13.8651**	**94.4810**	**1.1610**	**16.0969**	**0.0721**	**0.0621**	**6.8143**
16	0.8528	14.7179	107.2734	1.1726	17.2579	0.0679	0.0579	7.2886
17	0.8444	15.5623	120.7834	1.1843	18.4304	0.0643	0.0543	7.7613
18	0.8360	16.3983	134.9957	1.1961	19.6147	0.0610	0.0510	8.2323
19	0.8277	17.2260	149.8950	1.2081	20.8109	0.0581	0.0481	8.7017
20	**0.8195**	**18.0456**	**165.4664**	**1.2202**	**22.0190**	**0.0554**	**0.0454**	**9.1694**
21	0.8114	18.8570	181.6950	1.2324	23.2392	0.0530	0.0430	9.6354
22	0.8034	19.6604	198.5663	1.2447	24.4716	0.0509	0.0409	10.0998
23	0.7954	20.4558	216.0660	1.2572	25.7163	0.0489	0.0389	10.5626
24	0.7876	21.2434	234.1800	1.2697	26.9735	0.0471	0.0371	11.0237
25	**0.7798**	**22.0232**	**252.8945**	**1.2824**	**28.2432**	**0.0454**	**0.0354**	**11.4831**
30	0.7419	25.8077	355.0021	1.3478	34.7849	0.0387	0.0277	13.7557
40	0.6717	32.8347	596.8561	1.4889	48.8864	0.0305	0.0205	18.1776
50	0.6080	39.1961	879.4176	1.6446	64.4632	0.0255	0.0155	22.4363
60	0.5504	44.9550	1,192.8061	1.8167	81.6697	0.0222	0.0122	26.5333
100	**0.3697**	**63.0289**	**2,605.7758**	**2.7048**	**170.4814**	**0.0159**	**0.0059**	**41.3426**

Interest Rate Tables

Factor Table: i = 1.50%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9852	0.9852	0.0000	1.0150	1.0000	1.0150	1.0000	0.0000
2	0.9707	1.9559	0.9707	1.0302	2.0150	0.5113	0.4963	0.4963
3	0.9563	2.9122	2.8833	1.0457	3.0452	0.3434	0.3284	0.9901
4	0.9422	3.8544	5.7098	1.0614	4.0909	0.2594	0.2444	1.4814
5	**0.9283**	**4.7826**	**9.4229**	**1.0773**	**5.1523**	**0.2091**	**0.1941**	**1.9702**
6	0.9145	5.6972	13.9956	1.0934	6.2296	0.1755	0.1605	2.4566
7	0.9010	6.5982	19.4018	1.1098	7.3230	0.1516	0.1366	2.9405
8	0.8877	7.4859	26.6157	1.1265	8.4328	0.1336	0.1186	3.4219
9	0.8746	8.3605	32.6125	1.1434	9.5593	0.1196	0.1046	3.9008
10	**0.8617**	**9.2222**	**40.3675**	**1.1605**	**10.7027**	**0.1084**	**0.0934**	**4.3772**
11	0.8489	10.0711	48.8568	1.1779	11.8633	0.0993	0.0843	4.8512
12	0.8364	10.9075	58.0571	1.1956	13.0412	0.0917	0.0767	5.3227
13	0.8240	11.7315	67.9454	1.2136	14.2368	0.0852	0.0702	5.7917
14	0.8118	12.5434	78.4994	1.2318	15.4504	0.0797	0.0647	6.2582
15	**0.7999**	**13.3432**	**89.6974**	**1.2502**	**16.6821**	**0.0749**	**0.0599**	**6.7223**
16	0.7880	14.1313	101.5178	1.2690	17.9324	0.0708	0.0558	7.1839
17	0.7764	14.9076	113.9400	1.2880	19.2014	0.0671	0.0521	7.6431
18	0.7649	15.6726	126.9435	1.3073	20.4894	0.0638	0.0488	8.0997
19	0.7536	16.4262	140.5084	1.3270	21.7967	0.0609	0.0459	8.5539
20	**0.7425**	**17.1686**	**154.6154**	**1.3469**	**23.1237**	**0.0582**	**0.0432**	**9.0057**
21	0.7315	17.9001	169.2453	1.3671	24.4705	0.0559	0.0409	9.4550
22	0.7207	18.6208	184.3798	1.3876	25.8376	0.0537	0.0387	9.9018
23	0.7100	19.3309	200.0006	1.4084	27.2251	0.0517	0.0367	10.3462
24	0.6995	20.0304	216.0901	1.4295	28.6335	0.0499	0.0349	10.7881
25	**0.6892**	**20.7196**	**232.6310**	**1.4509**	**30.0630**	**0.0483**	**0.0333**	**11.2276**
30	0.6398	24.0158	321.5310	1.5631	37.5387	0.0416	0.0266	13.3883
40	0.5513	29.9158	524.3568	1.8140	54.2679	0.0334	0.0184	17.5277
50	0.4750	34.9997	749.9636	2.1052	73.6828	0.0286	0.0136	21.4277
60	0.4093	39.3803	988.1674	2.4432	96.2147	0.0254	0.0104	25.0930
100	**0.2256**	**51.6247**	**1,937.4506**	**4.4320**	**228.8030**	**0.0194**	**0.0044**	**37.5295**

Factor Table: i = 2.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9804	0.9804	0.0000	1.0200	1.0000	1.0200	1.0000	0.0000
2	0.9612	1.9416	0.9612	1.0404	2.0200	0.5150	0.4950	0.4950
3	0.9423	2.8839	2.8458	1.0612	3.0604	0.3468	0.3268	0.9868
4	0.9238	3.8077	5.6173	1.0824	4.1216	0.2626	0.2426	1.4752
5	**0.9057**	**4.7135**	**9.2403**	**1.1041**	**5.2040**	**0.2122**	**0.1922**	**1.9604**
6	0.8880	5.6014	13.6801	1.1262	6.3081	0.1785	0.1585	2.4423
7	0.8706	6.4720	18.9035	1.1487	7.4343	0.1545	0.1345	2.9208
8	0.8535	7.3255	24.8779	1.1717	8.5830	0.1365	0.1165	3.3961
9	0.8368	8.1622	31.5720	1.1951	9.7546	0.1225	0.1025	3.8681
10	**0.8203**	**8.9826**	**38.9551**	**1.2190**	**10.9497**	**0.1113**	**0.0913**	**4.3367**
11	0.8043	9.7868	46.9977	1.2434	12.1687	0.1022	0.0822	4.8021
12	0.7885	10.5753	55.6712	1.2682	13.4121	0.0946	0.0746	5.2642
13	0.7730	11.3484	64.9475	1.2936	14.6803	0.0881	0.0681	5.7231
14	0.7579	12.1062	74.7999	1.3195	15.9739	0.0826	0.0626	6.1786
15	**0.7430**	**12.8493**	**85.2021**	**1.3459**	**17.2934**	**0.0778**	**0.0578**	**6.6309**
16	0.7284	13.5777	96.1288	1.3728	18.6393	0.0737	0.0537	7.0799
17	0.7142	14.2919	107.5554	1.4002	20.0121	0.0700	0.0500	7.5256
18	0.7002	14.9920	119.4581	1.4282	21.4123	0.0667	0.0467	7.9681
19	0.6864	15.6785	131.8139	1.4568	22.8406	0.0638	0.0438	8.4073
20	**0.6730**	**16.3514**	**144.6003**	**1.4859**	**24.2974**	**0.0612**	**0.0412**	**8.8433**
21	0.6598	17.0112	157.7959	1.5157	25.7833	0.0588	0.0388	9.2760
22	0.6468	17.6580	171.3795	1.5460	27.2990	0.0566	0.0366	9.7055
23	0.6342	18.2922	185.3309	1.5769	28.8450	0.0547	0.0347	10.1317
24	0.6217	18.9139	199.6305	1.6084	30.4219	0.0529	0.0329	10.5547
25	**0.6095**	**19.5235**	**214.2592**	**1.6406**	**32.0303**	**0.0512**	**0.0312**	**10.9745**
30	0.5521	22.3965	291.7164	1.8114	40.5681	0.0446	0.0246	13.0251
40	0.4529	27.3555	461.9931	2.2080	60.4020	0.0366	0.0166	16.8885
50	0.3715	31.4236	642.3606	2.6916	84.5794	0.0318	0.0118	20.4420
60	0.3048	34.7609	823.6975	3.2810	114.0515	0.0288	0.0088	23.6961
100	**0.1380**	**43.0984**	**1,464.7527**	**7.2446**	**312.2323**	**0.0232**	**0.0032**	**33.9863**

Interest Rate Tables

Factor Table: i = 3.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9709	0.9709	0.0000	1.0300	1.0000	1.0300	1.0000	0.0000
2	0.9426	1.9135	0.9426	1.0609	2.0300	0.5226	0.4926	0.4926
3	0.9151	2.8286	2.7729	1.0927	3.0909	0.3535	0.3235	0.9803
4	0.8885	3.7171	5.4383	1.1255	4.1836	0.2690	0.2390	1.4631
5	**0.8626**	**4.5797**	**8.8888**	**1.1593**	**5.3091**	**0.2184**	**0.1884**	**1.9409**
6	0.8375	5.4172	13.0762	1.1941	6.4684	0.1846	0.1546	2.4138
7	0.8131	6.2303	17.9547	1.2299	7.6625	0.1605	0.1305	2.8819
8	0.7894	7.0197	23.4806	1.2668	8.8923	0.1425	0.1125	3.3450
9	0.7664	7.7861	29.6119	1.3048	10.1591	0.1284	0.0984	3.8032
10	**0.7441**	**8.5302**	**36.3088**	**1.3439**	**11.4639**	**0.1172**	**0.0872**	**4.2565**
11	0.7224	9.2526	43.5330	1.3842	12.8078	0.1081	0.0781	4.7049
12	0.7014	9.9540	51.2482	1.4258	14.1920	0.1005	0.0705	5.1485
13	0.6810	10.6350	59.4196	1.4685	15.6178	0.0940	0.0640	5.5872
14	0.6611	11.2961	68.0141	1.5126	17.0863	0.0885	0.0585	6.0210
15	**0.6419**	**11.9379**	**77.0002**	**1.5580**	**18.5989**	**0.0838**	**0.0538**	**6.4500**
16	0.6232	12.5611	86.3477	1.6047	20.1569	0.0796	0.0496	6.8742
17	0.6050	13.1661	96.0280	1.6528	21.7616	0.0760	0.0460	7.2936
18	0.5874	13.7535	106.0137	1.7024	23.4144	0.0727	0.0427	7.7081
19	0.5703	14.3238	116.2788	1.7535	25.1169	0.0698	0.0398	8.1179
20	**0.5537**	**14.8775**	**126.7987**	**1.8061**	**26.8704**	**0.0672**	**0.0372**	**8.5229**
21	0.5375	15.4150	137.5496	1.8603	28.6765	0.0649	0.0349	8.9231
22	0.5219	15.9396	148.5094	1.9161	30.5368	0.0627	0.0327	9.3186
23	0.5067	16.4436	159.6566	1.9736	32.4529	0.0608	0.0308	9.7093
24	0.4919	16.9355	170.9711	2.0328	34.4265	0.0590	0.0290	10.0954
25	**0.4776**	**17.4131**	**182.4336**	**2.0938**	**36.4593**	**0.0574**	**0.0274**	**10.4768**
30	0.4120	19.6004	241.3613	2.4273	47.5754	0.0510	0.0210	12.3141
40	0.3066	23.1148	361.7499	3.2620	75.4013	0.0433	0.0133	15.6502
50	0.2281	25.7298	477.4803	4.3839	112.7969	0.0389	0.0089	18.5575
60	0.1697	27.6756	583.0526	5.8916	163.0534	0.0361	0.0061	21.0674
100	**0.0520**	**31.5989**	**879.8540**	**19.2186**	**607.2877**	**0.0316**	**0.0016**	**27.8444**

Factor Table: i = 4.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9615	0.9615	0.0000	1.0400	1.0000	1.0400	1.0000	0.0000
2	0.9246	1.8861	0.9246	1.0816	2.0400	0.5302	0.4902	0.4902
3	0.8890	2.7751	2.7025	1.1249	3.1216	0.3603	0.3203	0.9739
4	0.8548	3.6299	5.2670	1.1699	4.2465	0.2755	0.2355	1.4510
5	**0.8219**	**4.4518**	**8.5547**	**1.2167**	**5.4163**	**0.2246**	**0.1846**	**1.9216**
6	0.7903	5.2421	12.5062	1.2653	6.6330	0.1908	0.1508	2.3857
7	0.7599	6.0021	17.0657	1.3159	7.8983	0.1666	0.1266	2.8433
8	0.7307	6.7327	22.1806	1.3686	9.2142	0.1485	0.1085	3.2944
9	0.7026	7.4353	27.8013	1.4233	10.5828	0.1345	0.0945	3.7391
10	**0.6756**	**8.1109**	**33.8814**	**1.4802**	**12.0061**	**0.1233**	**0.0833**	**4.1773**
11	0.6496	8.7605	40.3772	1.5395	13.4864	0.1141	0.0741	4.6090
12	0.6246	9.3851	47.2477	1.6010	15.0258	0.1066	0.0666	5.0343
13	0.6006	9.9856	54.4546	1.6651	16.6268	0.1001	0.0601	5.4533
14	0.5775	10.5631	61.9618	1.7317	18.2919	0.0947	0.0547	5.8659
15	**0.5553**	**11.1184**	**69.7355**	**1.8009**	**20.0236**	**0.0899**	**0.0499**	**6.2721**
16	0.5339	11.6523	77.7441	1.8730	21.8245	0.0858	0.0458	6.6720
17	0.5134	12.1657	85.9581	1.9479	23.6975	0.0822	0.0422	7.0656
18	0.4936	12.6593	94.3498	2.0258	25.6454	0.0790	0.0390	7.4530
19	0.4746	13.1339	102.8933	2.1068	27.6712	0.0761	0.0361	7.8342
20	**0.4564**	**13.5903**	**111.5647**	**2.1911**	**29.7781**	**0.0736**	**0.0336**	**8.2091**
21	0.4388	14.0292	120.3414	2.2788	31.9692	0.0713	0.0313	8.5779
22	0.4220	14.4511	129.2024	2.3699	34.2480	0.0692	0.0292	8.9407
23	0.4057	14.8568	138.1284	2.4647	36.6179	0.0673	0.0273	9.2973
24	0.3901	15.2470	147.1012	2.5633	39.0826	0.0656	0.0256	9.6479
25	**0.3751**	**15.6221**	**156.1040**	**2.6658**	**41.6459**	**0.0640**	**0.0240**	**9.9925**
30	0.3083	17.2920	201.0618	3.2434	56.0849	0.0578	0.0178	11.6274
40	0.2083	19.7928	286.5303	4.8010	95.0255	0.0505	0.0105	14.4765
50	0.1407	21.4822	361.1638	7.1067	152.6671	0.0466	0.0066	16.8122
60	0.0951	22.6235	422.9966	10.5196	237.9907	0.0442	0.0042	18.6972
100	**0.0198**	**24.5050**	**563.1249**	**50.5049**	**1,237.6237**	**0.0408**	**0.0008**	**22.9800**

Interest Rate Tables

Factor Table: *i* = 5.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9524	0.9524	0.0000	1.0500	1.0000	1.0500	1.0000	0.0000
2	0.9070	1.8594	0.9070	1.1025	2.0500	0.5378	0.4878	0.4878
3	0.8638	2.7232	2.6347	1.1576	3.1525	0.3672	0.3172	0.9675
4	0.8227	3.5460	5.1028	1.2155	4.3101	0.2820	0.2320	1.4391
5	**0.7835**	**4.3295**	**8.2369**	**1.2763**	**5.5256**	**0.2310**	**0.1810**	**1.9025**
6	0.7462	5.0757	11.9680	1.3401	6.8019	0.1970	0.1470	2.3579
7	0.7107	5.7864	16.2321	1.4071	8.1420	0.1728	0.1228	2.8052
8	0.6768	6.4632	20.9700	1.4775	9.5491	0.1547	0.1047	3.2445
9	0.6446	7.1078	26.1268	1.5513	11.0266	0.1407	0.0907	3.6758
10	**0.6139**	**7.7217**	**31.6520**	**1.6289**	**12.5779**	**0.1295**	**0.0795**	**4.0991**
11	0.5847	8.3064	37.4988	1.7103	14.2068	0.1204	0.0704	4.5144
12	0.5568	8.8633	43.6241	1.7959	15.9171	0.1128	0.0628	4.9219
13	0.5303	9.3936	49.9879	1.8856	17.7130	0.1065	0.0565	5.3215
14	0.5051	9.8986	56.5538	1.9799	19.5986	0.1010	0.0510	5.7133
15	**0.4810**	**10.3797**	**63.2880**	**2.0789**	**21.5786**	**0.0963**	**0.0463**	**6.0973**
16	0.4581	10.8378	70.1597	2.1829	23.6575	0.0923	0.0423	6.4736
17	0.4363	11.2741	77.1405	2.2920	25.8404	0.0887	0.0387	6.8423
18	0.4155	11.6896	84.2043	2.4066	28.1324	0.0855	0.0355	7.2034
19	0.3957	12.0853	91.3275	2.5270	30.5390	0.0827	0.0327	7.5569
20	**0.3769**	**12.4622**	**98.4884**	**2.6533**	**33.0660**	**0.0802**	**0.0302**	**7.9030**
21	0.3589	12.8212	105.6673	2.7860	35.7193	0.0780	0.0280	8.2416
22	0.3418	13.1630	112.8461	2.9253	38.5052	0.0760	0.0260	8.5730
23	0.3256	13.4886	120.0087	3.0715	41.4305	0.0741	0.0241	8.8971
24	0.3101	13.7986	127.1402	3.2251	44.5020	0.0725	0.0225	9.2140
25	**0.2953**	**14.0939**	**134.2275**	**3.3864**	**47.7271**	**0.0710**	**0.0210**	**9.5238**
30	0.2314	15.3725	168.6226	4.3219	66.4388	0.0651	0.0151	10.9691
40	0.1420	17.1591	229.5452	7.0400	120.7998	0.0583	0.0083	13.3775
50	0.0872	18.2559	277.9148	11.4674	209.3480	0.0548	0.0048	15.2233
60	0.0535	18.9293	314.3432	18.6792	353.5837	0.0528	0.0028	16.6062
100	**0.0076**	**19.8479**	**381.7492**	**131.5013**	**2610.0252**	**0.0504**	**0.0004**	**19.2337**

Factor Table: *i* = 6.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9434	0.9434	0.0000	1.0600	1.0000	1.0600	1.0000	0.0000
2	0.8900	1.8334	0.8900	1.1236	2.0600	0.5454	0.4854	0.4854
3	0.8396	2.6730	2.5692	1.1910	3.1836	0.3741	0.3141	0.9612
4	0.7921	3.4651	4.9455	1.2625	4.3746	0.2886	0.2286	1.4272
5	**0.7473**	**4.2124**	**7.9345**	**1.3382**	**5.6371**	**0.2374**	**0.1774**	**1.8836**
6	0.7050	4.9173	11.4594	1.4185	6.9753	0.2034	0.1434	2.3304
7	0.6651	5.5824	15.4497	1.5036	8.3938	0.1791	0.1191	2.7676
8	0.6274	6.2098	19.8416	1.5938	9.8975	0.1610	0.1010	3.1952
9	0.5919	6.8017	24.5768	1.6895	11.4913	0.1470	0.0870	3.6133
10	**0.5584**	**7.3601**	**29.6023**	**1.7908**	**13.1808**	**0.1359**	**0.0759**	**4.0220**
11	0.5268	7.8869	34.8702	1.8983	14.9716	0.1268	0.0668	4.4213
12	0.4970	8.3838	40.3369	2.0122	16.8699	0.1193	0.0593	4.8113
13	0.4688	8.8527	45.9629	2.1329	18.8821	0.1130	0.0530	5.1920
14	0.4423	9.2950	51.7128	2.2609	21.0151	0.1076	0.0476	5.5635
15	**0.4173**	**9.7122**	**57.5546**	**2.3966**	**23.2760**	**0.1030**	**0.0430**	**5.9260**
16	0.3936	10.1059	63.4592	2.5404	25.6725	0.0990	0.0390	6.2794
17	0.3714	10.4773	69.4011	2.6928	28.2129	0.0954	0.0354	6.6240
18	0.3505	10.8276	75.3569	2.8543	30.9057	0.0924	0.0324	6.9597
19	0.3305	11.1581	81.3062	3.0256	33.7600	0.0896	0.0296	7.2867
20	**0.3118**	**11.4699**	**87.2304**	**3.2071**	**36.7856**	**0.0872**	**0.0272**	**7.6051**
21	0.2942	11.7641	93.1136	3.3996	39.9927	0.0850	0.0250	7.9151
22	0.2775	12.0416	98.9412	3.6035	43.3923	0.0830	0.0230	8.2166
23	0.2618	12.3034	104.7007	3.8197	46.9958	0.0813	0.0213	8.5099
24	0.2470	12.5504	110.3812	4.0489	50.8156	0.0797	0.0197	8.7951
25	**0.2330**	**12.7834**	**115.9732**	**4.2919**	**54.8645**	**0.0782**	**0.0182**	**9.0722**
30	0.1741	13.7648	142.3588	5.7435	79.0582	0.0726	0.0126	10.3422
40	0.0972	15.0463	185.9568	10.2857	154.7620	0.0665	0.0065	12.3590
50	0.0543	15.7619	217.4574	18.4202	290.3359	0.0634	0.0034	13.7964
60	0.0303	16.1614	239.0428	32.9877	533.1282	0.0619	0.0019	14.7909
100	**0.0029**	**16.6175**	**272.0471**	**339.3021**	**5,638.3681**	**0.0602**	**0.0002**	**16.3711**

Interest Rate Tables

Factor Table: *i* = 7.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9346	0.9346	0.0000	1.0700	1.0000	1.0700	1.0000	0.0000
2	0.8734	1.8080	0.8734	1.1449	2.0700	0.5531	0.4831	0.4831
3	0.8163	2.6243	2.5060	1.2250	3.2149	0.3811	0.3111	0.9549
4	0.7629	3.3872	4.7947	1.3108	4.4399	0.2952	0.2252	1.4155
5	**0.7130**	**4.1002**	**7.6467**	**1.4026**	**5.7507**	**0.2439**	**0.1739**	**1.8650**
6	0.6663	4.7665	10.9784	1.5007	7.1533	0.2098	0.1398	2.3032
7	0.6227	5.3893	14.7149	1.6058	8.6540	0.1856	0.1156	2.7304
8	0.5820	5.9713	18.7889	1.7182	10.2598	0.1675	0.0975	3.1465
9	0.5439	6.5152	23.1404	1.8385	11.9780	0.1535	0.0835	3.5517
10	**0.5083**	**7.0236**	**27.7156**	**1.9672**	**13.8164**	**0.1424**	**0.0724**	**3.9461**
11	0.4751	7.4987	32.4665	2.1049	15.7836	0.1334	0.0634	4.3296
12	0.4440	7.9427	37.3506	2.2522	17.8885	0.1259	0.0559	4.7025
13	0.4150	8.3577	42.3302	2.4098	20.1406	0.1197	0.0497	5.0648
14	0.3878	8.7455	47.3718	2.5785	22.5505	0.1143	0.0443	5.4167
15	**0.3624**	**9.1079**	**52.4461**	**2.7590**	**25.1290**	**0.1098**	**0.0398**	**5.7583**
16	0.3387	9.4466	57.5271	2.9522	27.8881	0.1059	0.0359	6.0897
17	0.3166	9.7632	62.5923	3.1588	30.8402	0.1024	0.0324	6.4110
18	0.2959	10.0591	67.6219	3.3799	33.9990	0.0994	0.0294	6.7225
19	0.2765	10.3356	72.5991	3.6165	37.3790	0.0968	0.0268	7.0242
20	**0.2584**	**10.5940**	**77.5091**	**3.8697**	**40.9955**	**0.0944**	**0.0244**	**7.3163**
21	0.2415	10.8355	82.3393	4.1406	44.8652	0.0923	0.0223	7.5990
22	0.2257	11.0612	87.0793	4.4304	49.0057	0.0904	0.0204	7.8725
23	0.2109	11.2722	91.7201	4.7405	53.4361	0.0887	0.0187	8.1369
24	0.1971	11.4693	96.2545	5.0724	58.1767	0.0872	0.0172	8.3923
25	**0.1842**	**11.6536**	**100.6765**	**5.4274**	**63.2490**	**0.0858**	**0.0158**	**8.6391**
30	0.1314	12.4090	120.9718	7.6123	94.4608	0.0806	0.0106	9.7487
40	0.0668	13.3317	152.2928	14.9745	199.6351	0.0750	0.0050	11.4233
50	0.0339	13.8007	172.9051	29.4570	406.5289	0.0725	0.0025	12.5287
60	0.0173	14.0392	185.7677	57.9464	813.5204	0.0712	0.0012	13.2321
100	**0.0012**	**14.2693**	**202.2001**	**867.7163**	**12381.6618**	**0.0701**	**0.0001**	**14.1703**

Factor Table: *i* = 8.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9259	0.9259	0.0000	1.0800	1.0000	1.0800	1.0000	0.0000
2	0.8573	1.7833	0.8573	1.1664	2.0800	0.5608	0.4808	0.4808
3	0.7938	2.5771	2.4450	1.2597	3.2464	0.3880	0.3080	0.9487
4	0.7350	3.3121	4.6501	1.3605	4.5061	0.3019	0.2219	1.4040
5	**0.6806**	**3.9927**	**7.3724**	**1.4693**	**5.8666**	**0.2505**	**0.1705**	**1.8465**
6	0.6302	4.6229	10.5233	1.5869	7.3359	0.2163	0.1363	2.2763
7	0.5835	5.2064	14.0242	1.7138	8.9228	0.1921	0.1121	2.6937
8	0.5403	5.7466	17.8061	1.8509	10.6366	0.1740	0.0940	3.0985
9	0.5002	6.2469	21.8081	1.9990	12.4876	0.1601	0.0801	3.4910
10	**0.4632**	**6.7101**	**25.9768**	**2.1589**	**14.4866**	**0.1490**	**0.0690**	**3.8713**
11	0.4289	7.1390	30.2657	2.3316	16.6455	0.1401	0.0601	4.2395
12	0.3971	7.5361	34.6339	2.5182	18.9771	0.1327	0.0527	4.5957
13	0.3677	7.9038	39.0463	2.7196	21.4953	0.1265	0.0465	4.9402
14	0.3405	8.2442	43.4723	2.9372	24.2149	0.1213	0.0413	5.2731
15	**0.3152**	**8.5595**	**47.8857**	**3.1722**	**27.1521**	**0.1168**	**0.0368**	**5.5945**
16	0.2919	8.8514	52.2640	3.4259	30.3243	0.1130	0.0330	5.9046
17	0.2703	9.1216	56.5883	3.7000	33.7502	0.1096	0.0296	6.2037
18	0.2502	9.3719	60.8426	3.9960	37.4502	0.1067	0.0267	6.4920
19	0.2317	9.6036	65.0134	4.3157	41.4463	0.1041	0.0241	6.7697
20	**0.2145**	**9.8181**	**69.0898**	**4.6610**	**45.7620**	**0.1019**	**0.0219**	**7.0369**
21	0.1987	10.0168	73.0629	5.0338	50.4229	0.0998	0.0198	7.2940
22	0.1839	10.2007	76.9257	5.4365	55.4568	0.0980	0.0180	7.5412
23	0.1703	10.3711	80.6726	5.8715	60.8933	0.0964	0.0164	7.7786
24	0.1577	10.5288	84.2997	6.3412	66.7648	0.0950	0.0150	8.0066
25	**0.1460**	**10.6748**	**87.8041**	**6.8485**	**73.1059**	**0.0937**	**0.0137**	**8.2254**
30	0.0994	11.2578	103.4558	10.0627	113.2832	0.0888	0.0088	9.1897
40	0.0460	11.9246	126.0422	21.7245	259.0565	0.0839	0.0039	10.5699
50	0.0213	12.2335	139.5928	46.9016	573.7702	0.0817	0.0017	11.4107
60	0.0099	12.3766	147.3000	101.2571	1,253.2133	0.0808	0.0008	11.9015
100	**0.0005**	**12.4943**	**155.6107**	**2,199.7613**	**27,484.5157**	**0.0800**		**12.4545**

Interest Rate Tables

Factor Table: i = 9.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9174	0.9174	0.0000	1.0900	1.0000	1.0900	1.0000	0.0000
2	0.8417	1.7591	0.8417	1.1881	2.0900	0.5685	0.4785	0.4785
3	0.7722	2.5313	2.3860	1.2950	3.2781	0.3951	0.3051	0.9426
4	0.7084	3.2397	4.5113	1.4116	4.5731	0.3087	0.2187	1.3925
5	**0.6499**	**3.8897**	**7.1110**	**1.5386**	**5.9847**	**0.2571**	**0.1671**	**1.8282**
6	0.5963	4.4859	10.0924	1.6771	7.5233	0.2229	0.1329	2.2498
7	0.5470	5.0330	13.3746	1.8280	9.2004	0.1987	0.1087	2.6574
8	0.5019	5.5348	16.8877	1.9926	11.0285	0.1807	0.0907	3.0512
9	0.4604	5.9952	20.5711	2.1719	13.0210	0.1668	0.0768	3.4312
10	**0.4224**	**6.4177**	**24.3728**	**2.3674**	**15.1929**	**0.1558**	**0.0658**	**3.7978**
11	0.3875	6.8052	28.2481	2.5804	17.5603	0.1469	0.0569	4.1510
12	0.3555	7.1607	32.1590	2.8127	20.1407	0.1397	0.0497	4.4910
13	0.3262	7.4869	36.0731	3.0658	22.9534	0.1336	0.0436	4.8182
14	0.2992	7.7862	39.9633	3.3417	26.0192	0.1284	0.0384	5.1326
15	**0.2745**	**8.0607**	**43.8069**	**3.6425**	**29.3609**	**0.1241**	**0.0341**	**5.4346**
16	0.2519	8.3126	47.5849	3.9703	33.0034	0.1203	0.0303	5.7245
17	0.2311	8.5436	51.2821	4.3276	36.9737	0.1170	0.0270	6.0024
18	0.2120	8.7556	54.8860	4.7171	41.3013	0.1142	0.0242	6.2687
19	0.1945	8.9501	58.3868	5.1417	46.0185	0.1117	0.0217	6.5236
20	**0.1784**	**9.1285**	**61.7770**	**5.6044**	**51.1601**	**0.1095**	**0.0195**	**6.7674**
21	0.1637	9.2922	65.0509	6.1088	56.7645	0.1076	0.0176	7.0006
22	0.1502	9.4424	68.2048	6.6586	62.8733	0.1059	0.0159	7.2232
23	0.1378	9.5802	71.2359	7.2579	69.5319	0.1044	0.0144	7.4357
24	0.1264	9.7066	74.1433	7.9111	76.7898	0.1030	0.0130	7.6384
25	**0.1160**	**9.8226**	**76.9265**	**8.6231**	**84.7009**	**0.1018**	**0.0118**	**7.8316**
30	0.0754	10.2737	89.0280	13.2677	136.3075	0.0973	0.0073	8.6657
40	0.0318	10.7574	105.3762	31.4094	337.8824	0.0930	0.0030	9.7957
50	0.0134	10.9617	114.3251	74.3575	815.0836	0.0912	0.0012	10.4295
60	0.0057	11.0480	118.9683	176.0313	1944.7921	0.0905	0.0005	10.7683
100	**0.0002**	**11.1091**	**123.2335**	**5529.0408**	**61422.6755**	**0.0900**	**0.0000**	**11.0930**

Factor Table: i = 10.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.9091	0.9091	0.0000	1.1000	1.0000	1.1000	1.0000	0.0000
2	0.8264	1.7355	0.8264	1.2100	2.1000	0.5762	0.4762	0.4762
3	0.7513	2.4869	2.3291	1.3310	3.3100	0.4021	0.3021	0.9366
4	0.6830	3.1699	4.3781	1.4641	4.6410	0.3155	0.2155	1.3812
5	**0.6209**	**3.7908**	**6.8618**	**1.6105**	**6.1051**	**0.2638**	**0.1638**	**1.8101**
6	0.5645	4.3553	9.6842	1.7716	7.7156	0.2296	0.1296	2.2236
7	0.5132	4.8684	12.7631	1.9487	9.4872	0.2054	0.1054	2.6216
8	0.4665	5.3349	16.0287	2.1436	11.4359	0.1874	0.0874	3.0045
9	0.4241	5.7590	19.4215	2.3579	13.5735	0.1736	0.0736	3.3724
10	**0.3855**	**6.1446**	**22.8913**	**2.5937**	**15.9374**	**0.1627**	**0.0627**	**3.7255**
11	0.3505	6.4951	26.3962	2.8531	18.5312	0.1540	0.0540	4.0641
12	0.3186	6.8137	29.9012	3.1384	21.3843	0.1468	0.0468	4.3884
13	0.2897	7.1034	33.3772	3.4523	24.5227	0.1408	0.0408	4.6988
14	0.2633	7.3667	36.8005	3.7975	27.9750	0.1357	0.0357	4.9955
15	**0.2394**	**7.6061**	**40.1520**	**4.1772**	**31.7725**	**0.1315**	**0.0315**	**5.2789**
16	0.2176	7.8237	43.4164	4.5950	35.9497	0.1278	0.0278	5.5493
17	0.1978	8.0216	46.5819	5.0545	40.5447	0.1247	0.0247	5.8071
18	0.1799	8.2014	49.6395	5.5599	45.5992	0.1219	0.0219	6.0526
19	0.1635	8.3649	52.5827	6.1159	51.1591	0.1195	0.0195	6.2861
20	**0.1486**	**8.5136**	**55.4069**	**6.7275**	**57.2750**	**0.1175**	**0.0175**	**6.5081**
21	0.1351	8.6487	58.1095	7.4002	64.0025	0.1156	0.0156	6.7189
22	0.1228	8.7715	60.6893	8.1403	71.4027	0.1140	0.0140	6.9189
23	0.1117	8.8832	63.1462	8.9543	79.5430	0.1126	0.0126	7.1085
24	0.1015	8.9847	65.4813	9.8497	88.4973	0.1113	0.0113	7.2881
25	**0.0923**	**9.0770**	**67.6964**	**10.8347**	**98.3471**	**0.1102**	**0.0102**	**7.4580**
30	0.0573	9.4269	77.0766	17.4494	164.4940	0.1061	0.0061	8.1762
40	0.0221	9.7791	88.9525	45.2593	442.5926	0.1023	0.0023	9.0962
50	0.0085	9.9148	94.8889	117.3909	1,163.9085	0.1009	0.0009	9.5704
60	0.0033	9.9672	97.7010	304.4816	3,034.8164	0.1003	0.0003	9.8023
100	**0.0001**	**9.9993**	**99.9202**	**13,780.6123**	**137,796.1234**	**0.1000**		**9.9927**

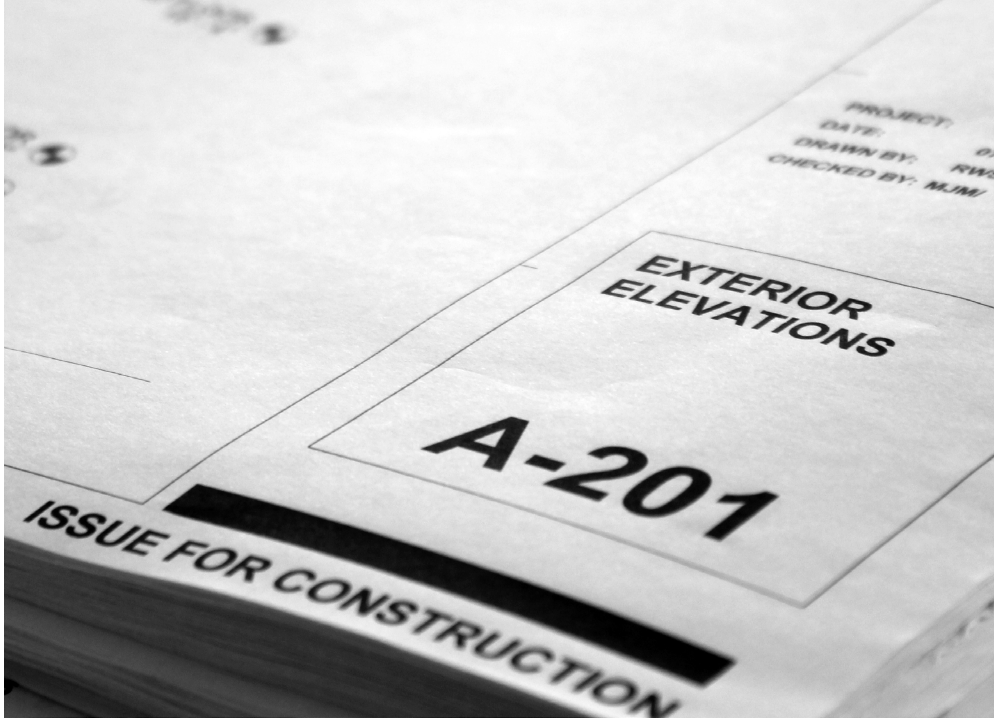
Interest Rate Tables

Factor Table: i = 12.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.8929	0.8929	0.0000	1.1200	1.0000	1.1200	1.0000	0.0000
2	0.7972	1.6901	0.7972	1.2544	2.1200	0.5917	0.4717	0.4717
3	0.7118	2.4018	2.2208	1.4049	3.3744	0.4163	0.2963	0.9246
4	0.6355	3.0373	4.1273	1.5735	4.7793	0.3292	0.2092	1.3589
5	**0.5674**	**3.6048**	**6.3970**	**1.7623**	**6.3528**	**0.2774**	**0.1574**	**1.7746**
6	0.5066	4.1114	8.9302	1.9738	8.1152	0.2432	0.1232	2.1720
7	0.4523	4.5638	11.6443	2.2107	10.0890	0.2191	0.0991	2.5515
8	0.4039	4.9676	14.4714	2.4760	12.2997	0.2013	0.0813	2.9131
9	0.3606	5.3282	17.3563	2.7731	14.7757	0.1877	0.0677	3.2574
10	**0.3220**	**5.6502**	**20.2541**	**3.1058**	**17.5487**	**0.1770**	**0.0570**	**3.5847**
11	0.2875	5.9377	23.1288	3.4785	20.6546	0.1684	0.0484	3.8953
12	0.2567	6.1944	25.9523	3.8960	24.1331	0.1614	0.0414	4.1897
13	0.2292	6.4235	28.7024	4.3635	28.0291	0.1557	0.0357	4.4683
14	0.2046	6.6282	31.3624	4.8871	32.3926	0.1509	0.0309	4.7317
15	**0.1827**	**6.8109**	**33.9202**	**5.4736**	**37.2797**	**0.1468**	**0.0268**	**4.9803**
16	0.1631	6.9740	36.3670	6.1304	42.7533	0.1434	0.0234	5.2147
17	0.1456	7.1196	38.6973	6.8660	48.8837	0.1405	0.0205	5.4353
18	0.1300	7.2497	40.9080	7.6900	55.7497	0.1379	0.0179	5.6427
19	0.1161	7.3658	42.9979	8.6128	63.4397	0.1358	0.0158	5.8375
20	**0.1037**	**7.4694**	**44.9676**	**9.6463**	**72.0524**	**0.1339**	**0.0139**	**6.0202**
21	0.0926	7.5620	46.8188	10.8038	81.6987	0.1322	0.0122	6.1913
22	0.0826	7.6446	48.5543	12.1003	92.5026	0.1308	0.0108	6.3514
23	0.0738	7.7184	50.1776	13.5523	104.6029	0.1296	0.0096	6.5010
24	0.0659	7.7843	51.6929	15.1786	118.1552	0.1285	0.0085	6.6406
25	**0.0588**	**7.8431**	**53.1046**	**17.0001**	**133.3339**	**0.1275**	**0.0075**	**6.7708**
30	0.0334	8.0552	58.7821	29.9599	241.3327	0.1241	0.0041	7.2974
40	0.0107	8.2438	65.1159	93.0510	767.0914	0.1213	0.0013	7.8988
50	0.0035	8.3045	67.7624	289.0022	2,400.0182	0.1204	0.0004	8.1597
60	0.0011	8.3240	68.8100	897.5969	7,471.6411	0.1201	0.0001	8.2664
100		**8.3332**	**69.4336**	**83,522.2657**	**696,010.5477**	**0.1200**		**8.3321**

Factor Table: i = 18.00%

n	*P/F*	*P/A*	*P/G*	*F/P*	*F/A*	*A/P*	*A/F*	*A/G*
1	0.8475	0.8475	0.0000	1.1800	1.0000	1.1800	1.0000	0.0000
2	0.7182	1.5656	0.7182	1.3924	2.1800	0.6387	0.4587	0.4587
3	0.6086	2.1743	1.9354	1.6430	3.5724	0.4599	0.2799	0.8902
4	0.5158	2.6901	3.4828	1.9388	5.2154	0.3717	0.1917	1.2947
5	**0.4371**	**3.1272**	**5.2312**	**2.2878**	**7.1542**	**0.3198**	**0.1398**	**1.6728**
6	0.3704	3.4976	7.0834	2.6996	9.4423	0.2859	0.1059	2.0252
7	0.3139	3.8115	8.9670	3.1855	12.1415	0.2624	0.0824	2.3526
8	0.2660	4.0776	10.8292	3.7589	15.3270	0.2452	0.0652	2.6558
9	0.2255	4.3030	12.6329	4.4355	19.0859	0.2324	0.0524	2.9358
10	**0.1911**	**4.4941**	**14.3525**	**5.2338**	**23.5213**	**0.2225**	**0.0425**	**3.1936**
11	0.1619	4.6560	15.9716	6.1759	28.7551	0.2148	0.0348	3.4303
12	0.1372	4.7932	17.4811	7.2876	34.9311	0.2086	0.0286	3.6470
13	0.1163	4.9095	18.8765	8.5994	42.2187	0.2037	0.0237	3.8449
14	0.0985	5.0081	20.1576	10.1472	50.8180	0.1997	0.0197	4.0250
15	**0.0835**	**5.0916**	**21.3269**	**11.9737**	**60.9653**	**0.1964**	**0.0164**	**4.1887**
16	0.0708	5.1624	22.3885	14.1290	72.9390	0.1937	0.0137	4.3369
17	0.0600	5.2223	23.3482	16.6722	87.0680	0.1915	0.0115	4.4708
18	0.0508	5.2732	24.2123	19.6731	103.7403	0.1896	0.0096	4.5916
19	0.0431	5.3162	24.9877	23.2144	123.4135	0.1881	0.0081	4.7003
20	**0.0365**	**5.3527**	**25.6813**	**27.3930**	**146.6280**	**0.1868**	**0.0068**	**4.7978**
21	0.0309	5.3837	26.3000	32.3238	174.0210	0.1857	0.0057	4.8851
22	0.0262	5.4099	26.8506	38.1421	206.3448	0.1848	0.0048	4.9632
23	0.0222	5.4321	27.3394	45.0076	244.4868	0.1841	0.0041	5.0329
24	0.0188	5.4509	27.7725	53.1090	289.4944	0.1835	0.0035	5.0950
25	**0.0159**	**5.4669**	**28.1555**	**62.6686**	**342.6035**	**0.1829**	**0.0029**	**5.1502**
30	0.0070	5.5168	29.4864	143.3706	790.9480	0.1813	0.0013	5.3448
40	0.0013	5.5482	30.5269	750.3783	4,163.2130	0.1802	0.0002	5.5022
50	0.0003	5.5541	30.7856	3,927.3569	21,813.0937	0.1800		5.5428
60	0.0001	5.5553	30.8465	20,555.1400	114,189.6665	0.1800		5.5526
100		**5.5556**	**30.8642**	**15,424,131.91**	**85,689,616.17**	**0.1800**		**5.5555**



2 CONSTRUCTION

2.1 Earthwork Construction and Layout

2.1.1 Excavation and Embankment

$$V_L = \left(1 + \frac{S_w}{100}\right) V_B \qquad \gamma_L = \frac{\gamma_B}{1 + \frac{S_w}{100}}$$

$$V_C = \left(1 - \frac{S_h}{100}\right) V_B \qquad \gamma_C = \frac{\gamma_B}{1 - \frac{S_h}{100}}$$

$$V_B = \left(\frac{\gamma_F}{\gamma_B} \times V_F\right) + \frac{W_L}{\gamma_B}$$

$$\text{Relative compaction (\%)} = RC = \frac{\gamma_{d,\text{field}}}{\gamma_{d,\max}} \times 100$$

$$\text{Shrinkage factor} = \frac{\text{Weight}/V_B}{\text{Weight}/V_C} = \frac{\text{bank unit weight}}{\text{compacted unit weight}}$$

$$\text{Shrinkage (\%)} = S_h = \frac{(\text{compacted unit weight}) - (\text{bank unit weight})}{\text{compacted unit weight}} \times 100$$

$$\text{Swell factor} = \frac{\text{bank unit weight}}{\text{loose unit weight}}$$

$$\text{Swell (\%)} = S_w = \left(\frac{\text{Weight}/V_B}{\text{Weight}/V_L} - 1\right) \times 100$$

$$\text{Load factor} = \frac{\text{loose unit weight}}{\text{bank unit weight}}$$

Note: Published definitions of swell factor and shrinkage factor vary. Unless otherwise noted in the question, the exam questions and solutions are consistent with the swell and shrinkage factor formulas shown here.

where

V_B = volume of undisturbed soil (bank measure)

V_F = volume of fill soil

V_L = volume of loose soil

V_C = volume of compacted soil

W_L = weight lost in stripping, waste, and transportation

$\gamma_{d,\text{field}}$ = dry unit weight of soil in the field

$\gamma_{d,\text{max}}$ = maximum dry density of soil measured in the laboratory

γ_B = unit weight of undistributed soil (bank measure)

γ_F = unit weight of fill soil

γ_L = unit weight of loose soil

γ_C = unit weight of compacted soil

Optimum soil moisture content for compaction:

Well-graded granular soils: 7 to 12%
Fine-grained soils: 12 to 25%

Amount of water to be added or removed from soil to achieve desired soil moisture content:

$$\text{Gallons of water} = \text{desired dry density, lb/ft}^3 \times \frac{(\text{desired \% of water content}) - (\text{\% of water content of borrow})}{100} \times \frac{\text{compacted volume of soil, ft}^3}{8.33\ \text{lb/gal}}$$

Available soil compaction techniques can be classified as:

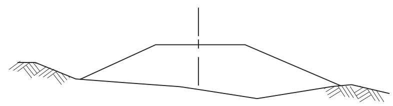
1. Static Pressure – A large stress is slowly applied to the soil and then released.
2. Impact – A stress is applied by dropping a large mass onto the surface of the soil.
3. Vibrating – A stress is applied repeatedly and rapidly via a mechanically driven plate or hammer.
4. Kneading – Shear is applied by alternating movement in adjacent positions.

Method of Compaction Categorized by Soil Type

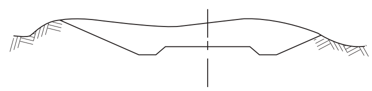
Soil Type	Impact	Pressure	Vibrating	Kneading
Gravel	Poor	No	Good	Very Good
Sand	Poor	No	Excellent	Good
Silt	Good	Good	Poor	Excellent
Clay	Excellent	Very Good	No	Good

2.1.2 Earthwork Volumes

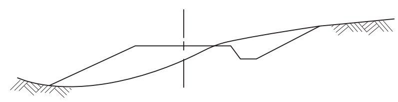
2.1.2.1 Cross-Section Methods



FILL



CUT



FILL AND CUT

Cross-Sectional End Areas

The **average end-area method** for earthwork calculates volume V between two consecutive cross sections as the average of their areas multiplied by the distance between them, where fill is positive and cut is negative:

$$V = L\left(\frac{A_1 + A_2}{2}\right)$$

where

V = volume

A_1, A_2 = end areas of cross sections 1 and 2

L = distance between cross sections

The **prismoidal formula** for earthwork calculates volume V between two consecutive cross sections, taking the area of the midsection into account:

$$V = L\left(\frac{A_1 + 4A_{\mathrm{m}} + A_2}{6}\right)$$

where

V = volume

A_1, A_2 = end areas of cross sections 1 and 2

A_m = area of midsection

L = distance between cross sections

2.1.2.2 Borrow Pit Grid Method

With the **grid formula**, the volume of material excavated from a borrow pit may be estimated by taking grade-rod readings at grid points before and after excavation. For differential elevations *a*, *b*, *c*, and *d* at the corners of a grid square:

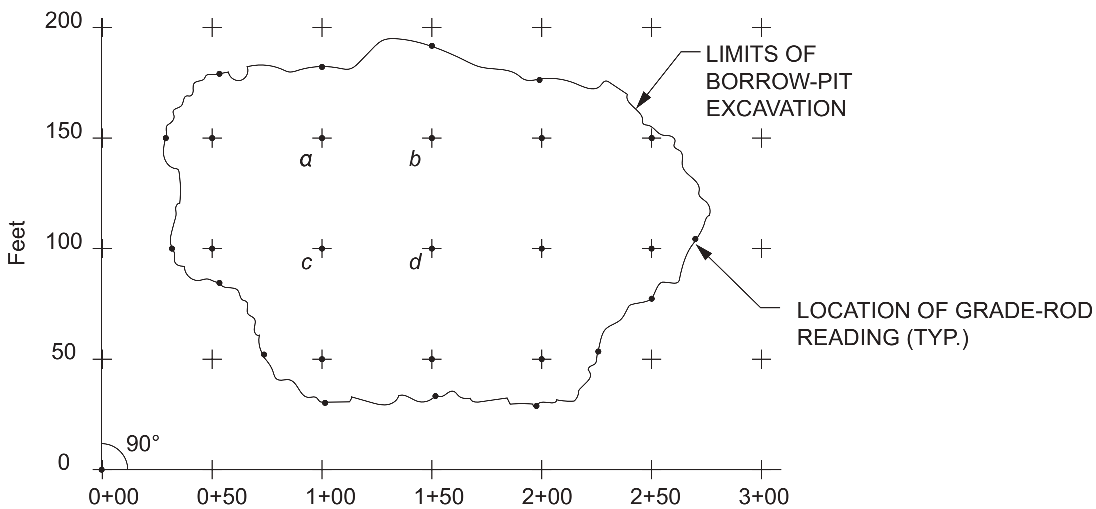
$$\text{Volume of material in one grid square} = \frac{1}{4}(a+b+c+d)\times(\text{area of grid square})$$

For partial grid squares at edges of an excavation, the amount of material *V* may be estimated using standard volume formulas for three-dimensional shapes, such as the following:

$$\text{Wedge (triangular prism): } V = \frac{1}{2}(b \times h \times l)$$

$$\text{Quarter of right circular cone: } V = \frac{1}{12}(\pi \times r^2 \times h)$$

Sample layout for use of the grid formula:



2.1.2.3 Earthwork Area Formulas

The **coordinate method** calculates area *A* as follows:

$$A = \frac{1}{2}\left[X_A(Y_B - Y_N) + X_B(Y_C - Y_A) + X_C(Y_D - Y_B) + \ldots + X_N(Y_A - Y_{N-1})\right]$$

The **trapezoid rule** calculates area *A* as follows:

$$A = w\left[\frac{1}{2}(h_1 + h_n) + h_2 + h_3 + h_4 + \ldots + h_{n-1}\right]$$

where w = length of the common interval

Simpson's Rule calculates area *A* for a section of earthwork as follows, given the elevation values of cut or fill at equal intervals (e.g., stations) along a baseline:

$$A = \frac{1}{3}\left[\text{first value} + \text{last value} + (2 \times \text{sum of odd-numbered values}) + (4 \times \text{sum of even-numbered values})\right] \times \text{length of interval}$$

To use Simpson's Rule, there must be an even number of intervals. The sum of odd-numbered values (e.g., 3rd, 5th, and 7th terms) and even-numbered values (e.g., 2nd, 4th, and 6th terms) does not include the first and last terms along the baseline.

2.1.2.4 Spoil Bank Volumes

Triangular Spoil Bank

Volume = section area × length

$$B = \left(\frac{4V}{L \times \tan R}\right)^{\frac{1}{2}}$$

$$H = \left(\frac{B \times \tan R}{2}\right)$$

where

B = base width

H = pile height

L = pile length

R = angle of repose

V = pile volume

Conical Spoil Pile

$$\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$D = \left(\frac{7.64V}{\tan R}\right)^{\frac{1}{3}}$$

$$H = \frac{D}{2} \times \tan R$$

where D = diameter of the pile base

Example Values of Angle of Repose of Excavated Soil

Material	Angle of Repose
Clay	35°
Common earth, dry	32°
Common earth, moist	37°
Gravel	35°
Sand, dry	25°
Sand, moist	37°

2.1.3 Site Layout and Control

Survey Leveling

Benchmark (BM) = permanent point of known elevation

Turning point (TP) = point temporarily used to transfer an elevation

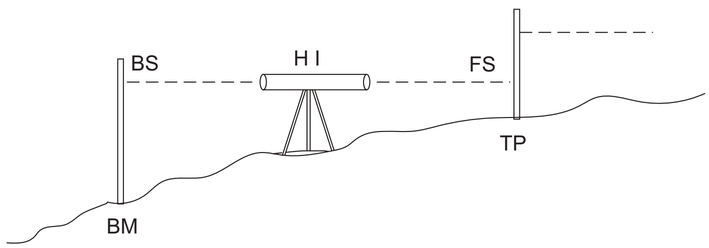
Backsight (BS) = rod reading taken on a point of known elevation to establish elevation of instrument's line of sight

Foresight (FS) = rod reading taken on a benchmark or turning point to determine its elevation

Height of instrument (HI) = elevation of line of sight through the level

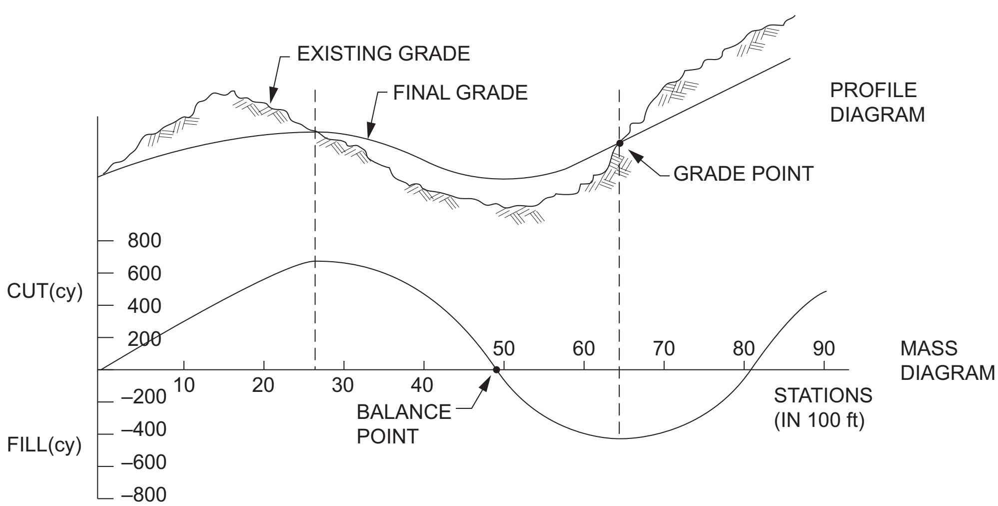
With reference to the diagram below:

- Elevation of BM + BS = HI
- Elevation of TP = HI – FS



2.1.4 Earthwork Balancing and Haul Distances

2.1.4.1 Mass Diagrams and Profile Diagrams



Profile diagram = plot of existing and final grades along route centerline for planned earthwork

Mass diagram = plot of cumulative earthwork volume moving up-station, with cut (excavation) plotted positive and fill (embankment) plotted negative

Grade points = locations on profile diagram where final grade matches existing grade, corresponding to maxima or minima on mass diagram

Balancing points = locations where cumulative volume of cut and fill is zero, i.e., where mass diagram crosses its baseline

- Any horizontal line between two balance points represents a distance between which cut-and-fill quantities are equal.
- The mass-diagram height at any point represents earthwork volume, which corresponds to an area on the profile diagram.

- The area under the mass diagram at any point represents earthwork volume multiplied by a distance.
- Dividing any area on the mass diagram by its height yields the average haul distance for that earthwork.

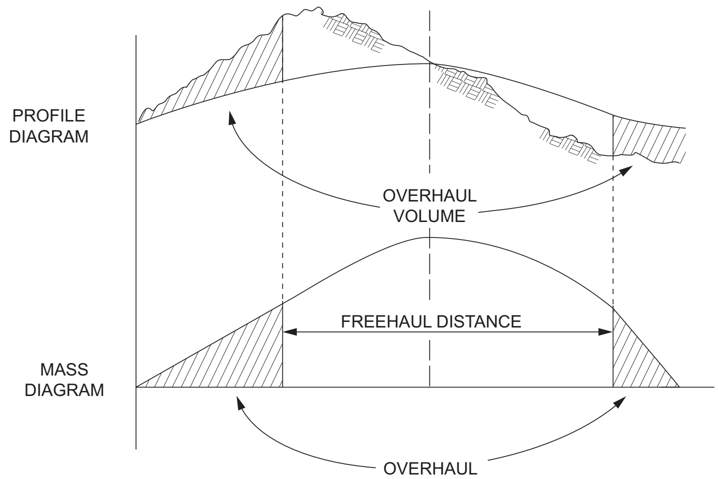
2.1.4.2 Freehaul and Overhaul

Freehaul = distance below which all earthmoving is considered part of the contract base price

Overhaul = any volume of material moved beyond the freehaul distance

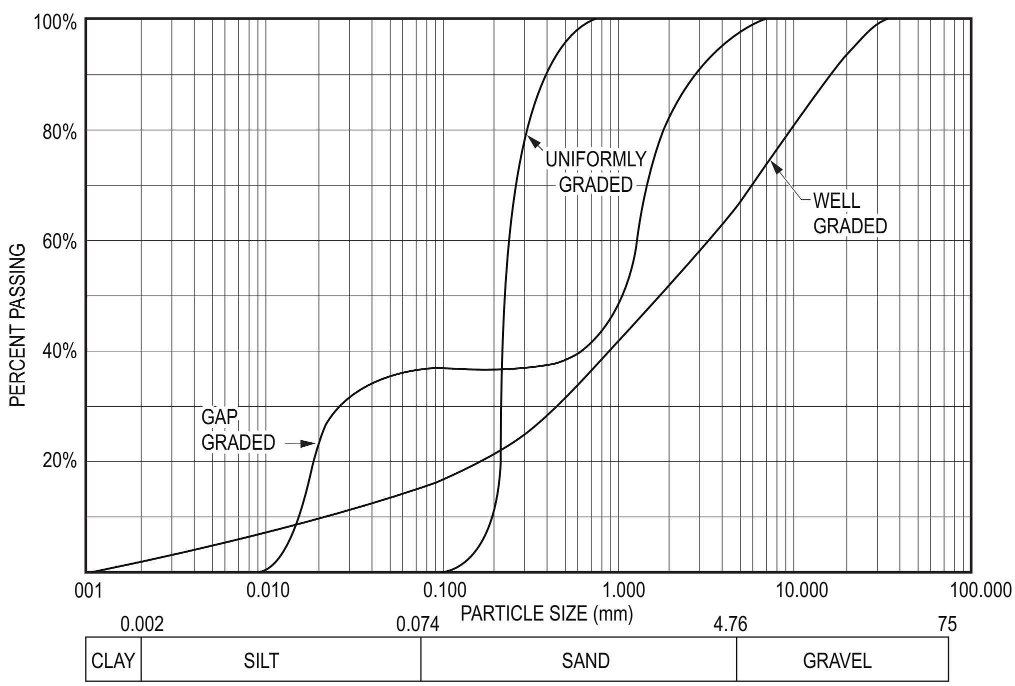
Overhaul distance = distance determined by deducting the freehaul distance from the distance between the centers of gravity of the remaining mass of excavation and the remaining mass of embankment

Overhaul quantity = overhaul volume multiplied by overhaul distance



2.1.5 Site and Subsurface Investigations

Refer to "Subsurface Exploration and Planning" and "Compaction: Laboratory and Field Compaction" in the Geotechnical chapter.



Particle Size Distribution Curves

2.2 Estimating Quantities and Costs

2.2.1 Quantity Takeoff Methods

Refer to "Earthwork Volumes" in this chapter for the various methods and formulas used to calculate earthwork volumes. See Chapter 1 for general area and volume formulas.

2.2.2 Cost Estimating

Cost Estimate Classification Matrix for Building and General Construction Industries

	Primary Characteristic	Secondary Characteristics		
Estimate Class	**Maturity Level of Project Definition Deliverables (expressed as % of complete definition)**	**End Usage (typical purpose of estimate)**	**Methodology (typical estimating method)**	**Expected Accuracy Range (typical variation in low and high ranges at an 80% confidence interval)**
Class 5	0% to 2%	Functional area or concept screening	ft^2 or m^2 factoring, parametric models, judgement, or analogy	L: –20% to –30% H: +30% to +50%
Class 4	1% to 15%	Schematic design or concept study	Parametric models, assembly-driven models	L: –10% to –20% H: +20% to +30%
Class 3	10% to 40%	Design development, budget authorization, feasibility	Semidetailed unit costs with assembly-level line items	L: –5% to –15% H: +10% to +20%
Class 2	30% to 75%	Control or bid/tender, semidetailed	Detailed unit cost with forced-detailed take-off	L: –5% to –10% H: +5% to +15%
Class 1	65% to 100%	Check estimate or prebid/ tender, change order	Detailed unit cost with detailed take-off	L: –3% to –5% H: +3% to +10%

2.2.3 Cost Indexes

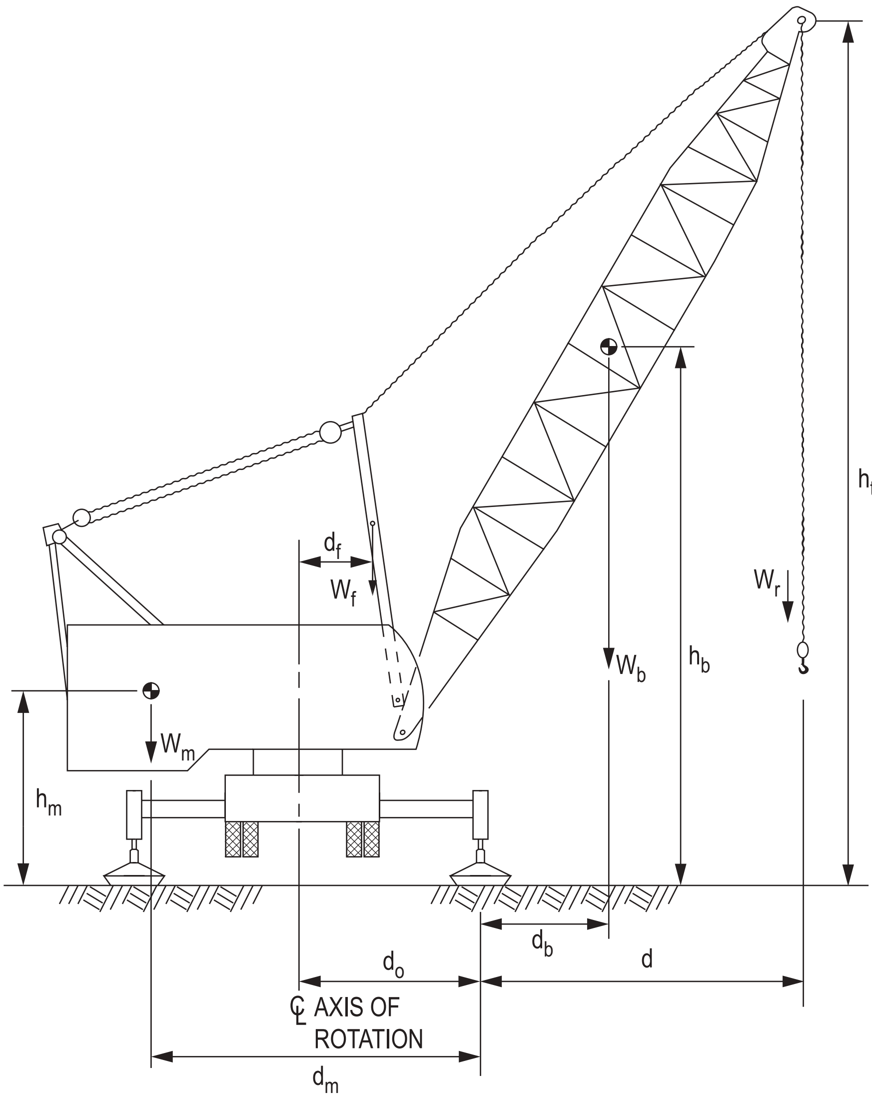
Cost indexes are used to update historical cost data to the present. If a purchase cost is available for an item of equipment in year M, the equivalent current cost would be found by:

$$\text{Current \$} = (\text{Cost in year } M)\left(\frac{\text{Current Index}}{\text{Index in year } M}\right)$$

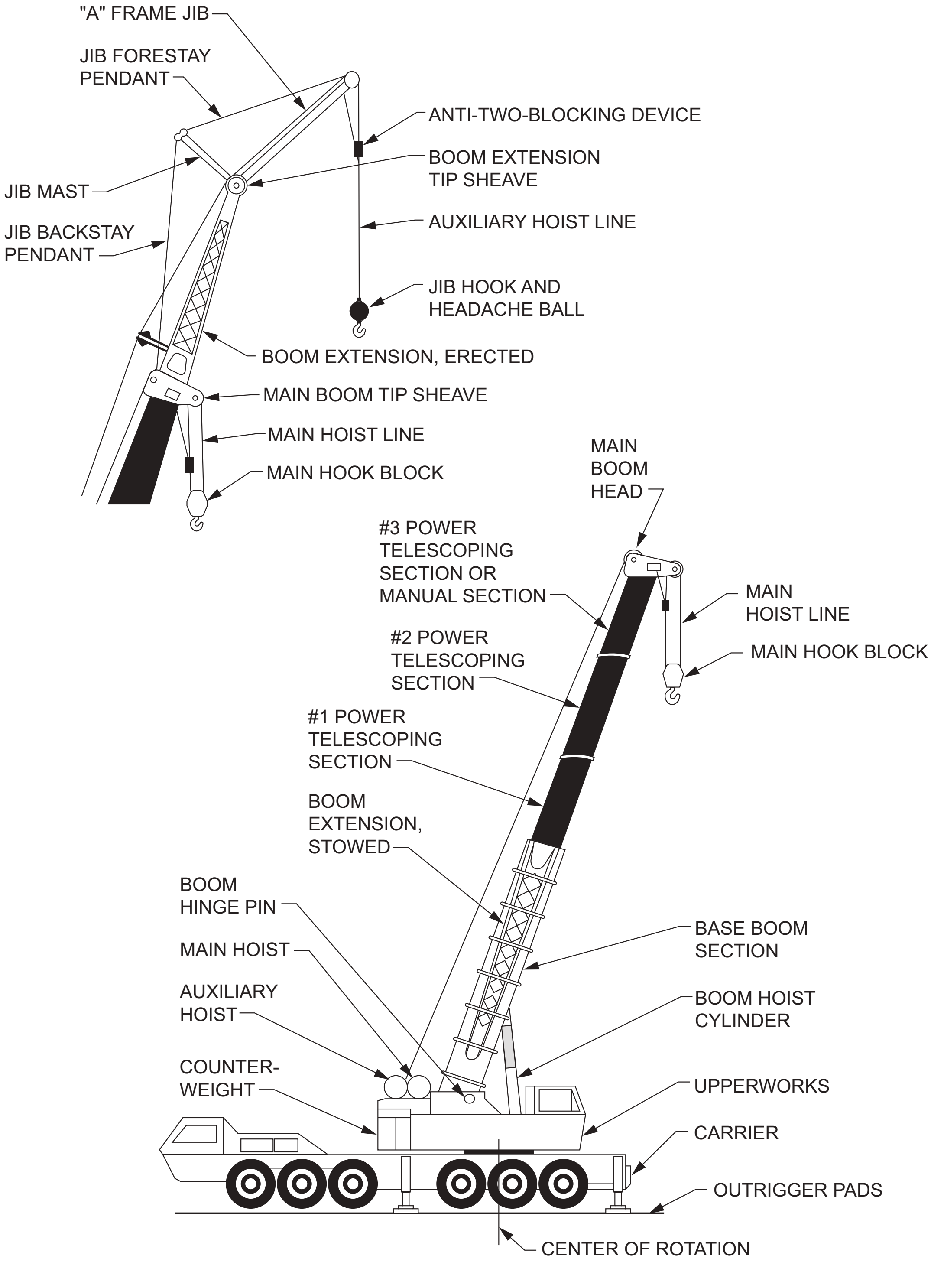
2.3 Construction Operations and Methods

2.3.1 Crane Stability

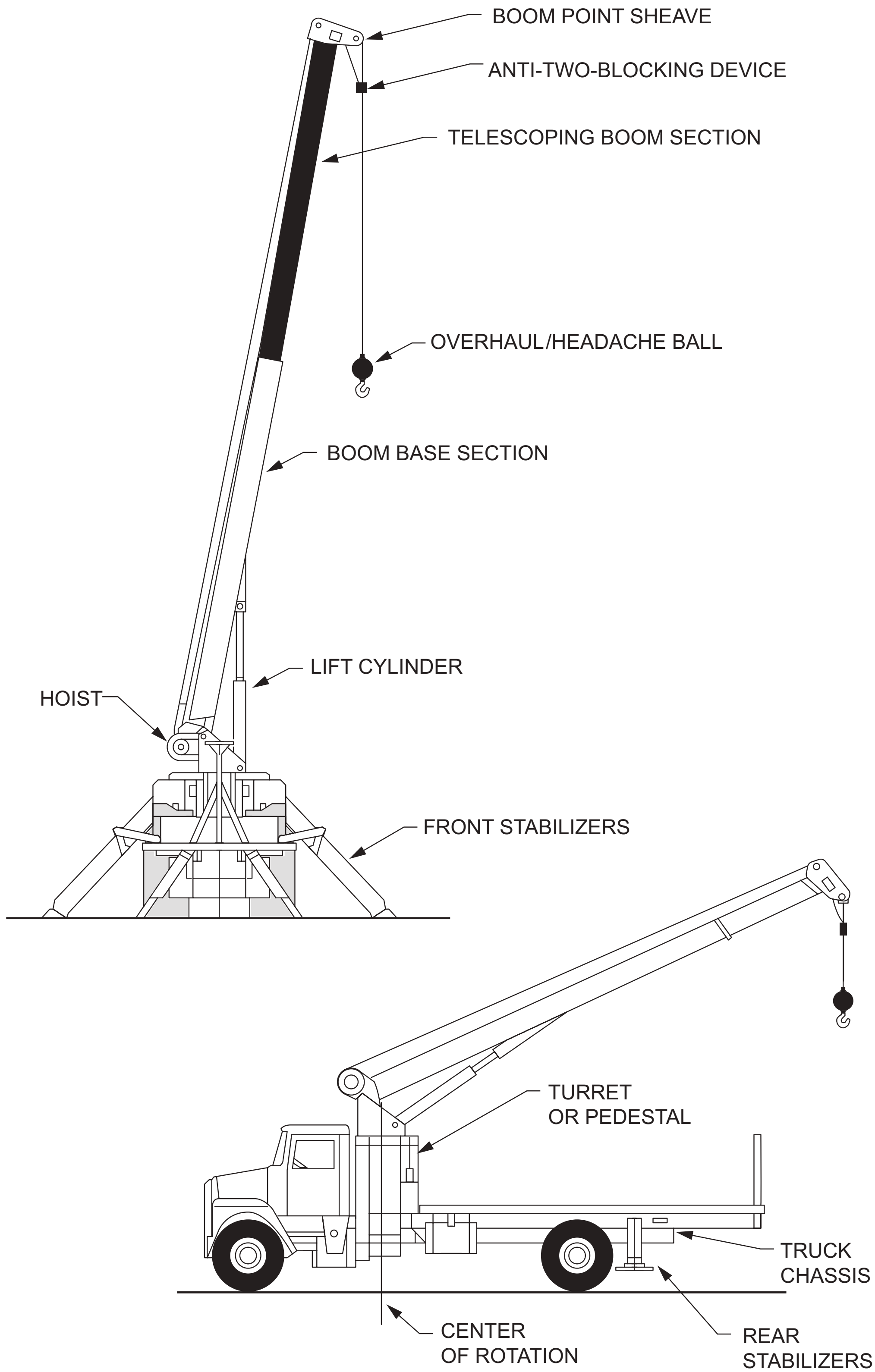
2.3.1.1 Diagram for Stability Calculations



2.3.1.2 Telescoping Boom Crane Components

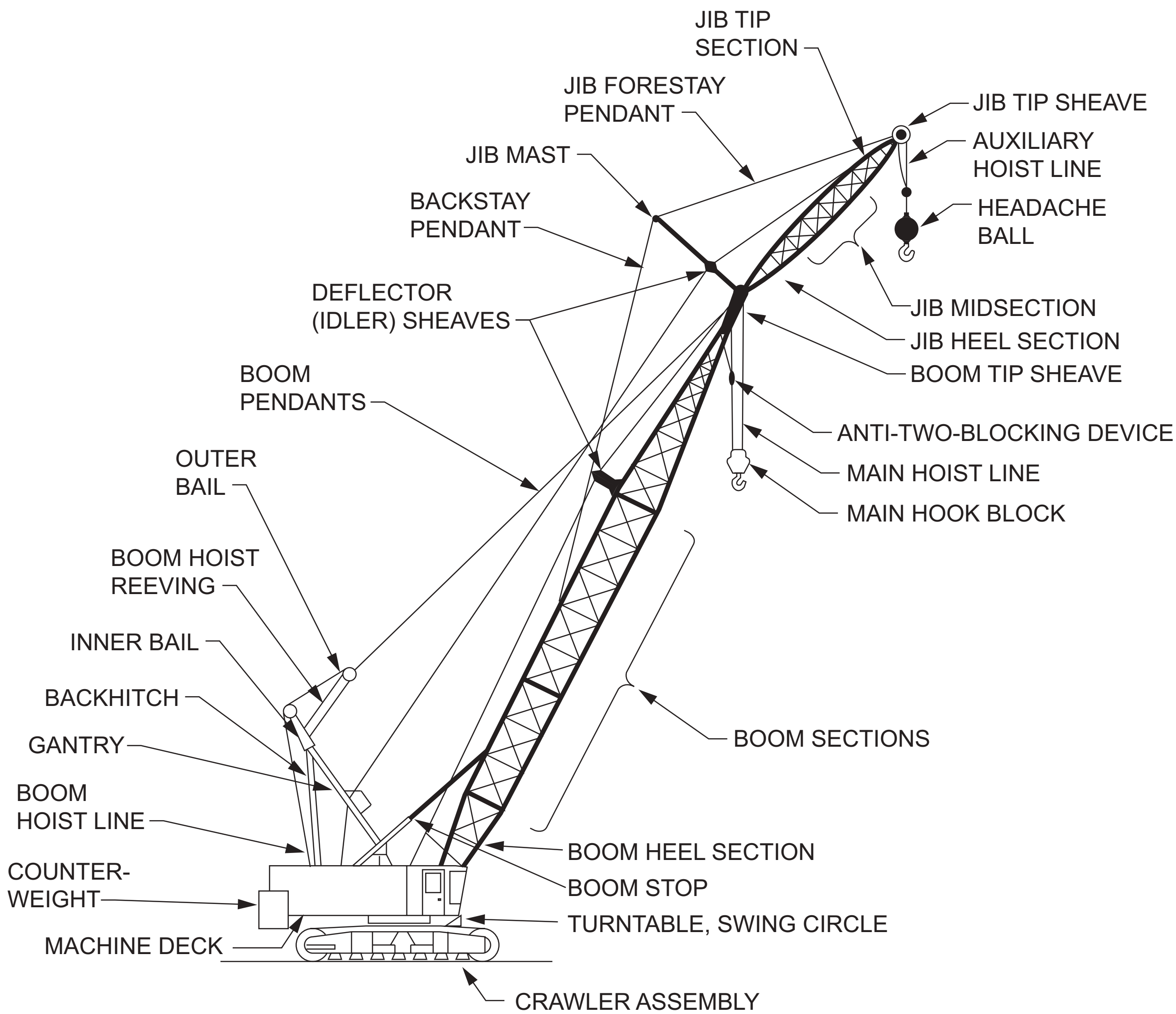


Source: Headley, James. *Mobile Cranes*. 4th ed. Sanford, FL: Crane Institute of America, 2002. Used by permission of Crane Institute of America.

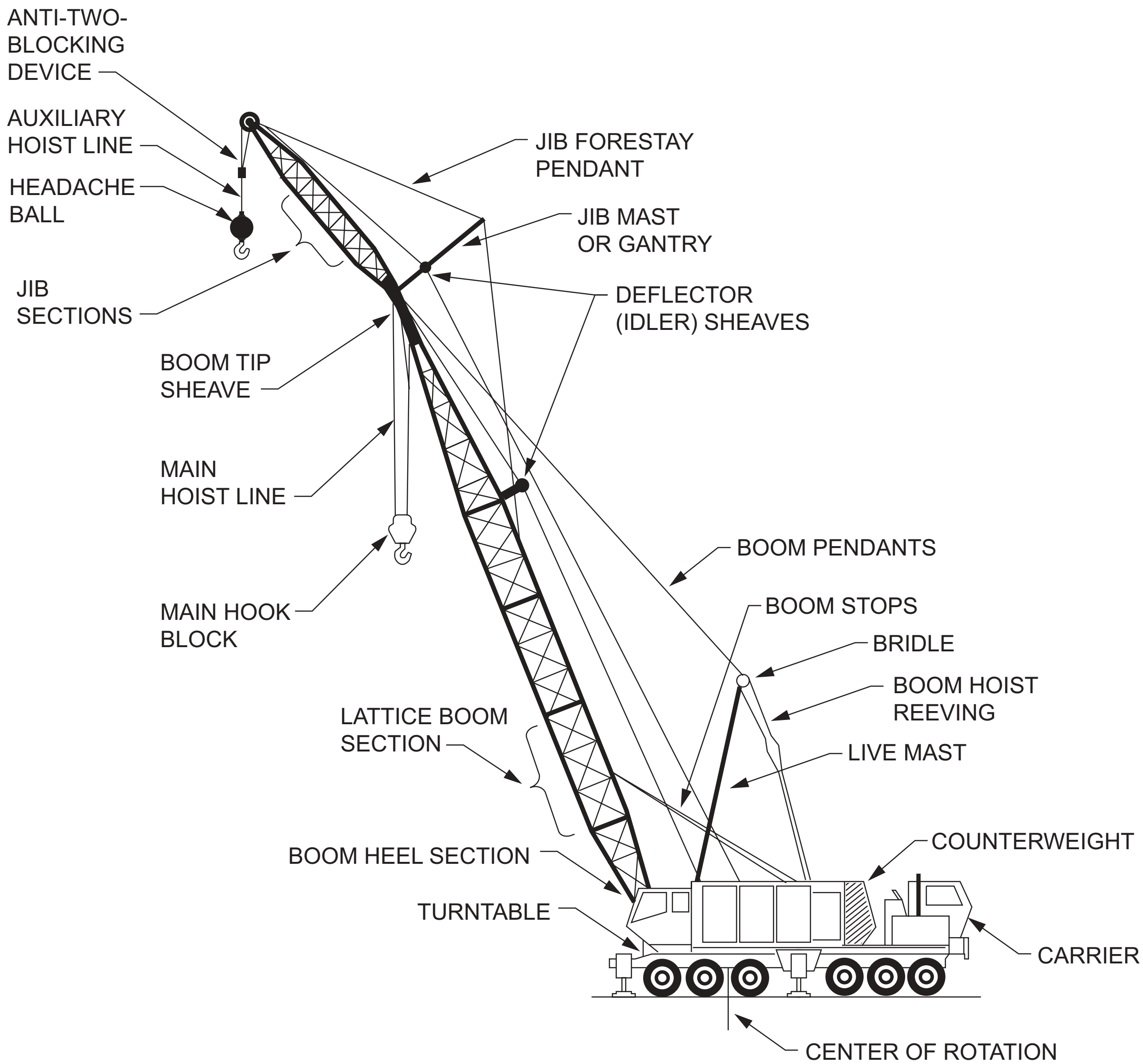


Source: Headley, James. *Mobile Cranes*. 4th ed. Sanford, FL: Crane Institute of America, 2002. Used by permission of Crane Institute of America.

2.3.1.3 Lattice Boom Crane Components



Source: Headley, James. *Mobile Cranes*. 4th ed. Sanford, FL: Crane Institute of America, 2002. Used by permission of Crane Institute of America.



Source: Headley, James. *Mobile Cranes*. 4th ed. Sanford, FL: Crane Institute of America, 2002. Used by permission of Crane Institute of America.

2.3.2 Dewatering and Pumping

See Chapter 3: Geotechnical and Chapter 6: Water Resources and Environmental

2.3.3 Equipment Operations

2.3.3.1 Production Rate for Soil Compaction

$$\text{Compacted cubic yards per hour} = \frac{1}{n}\left(16.3 \times W \times S \times L \times \text{efficiency}\right)$$

where

W = compacted width per roller pass (ft)

S = average roller speed (mph)

L = compacted lift thickness (in.)

n = number of roller passes required to achieve required density

To convert the production rate from compacted cubic yards to bank cubic yards, apply a shrinkage factor.

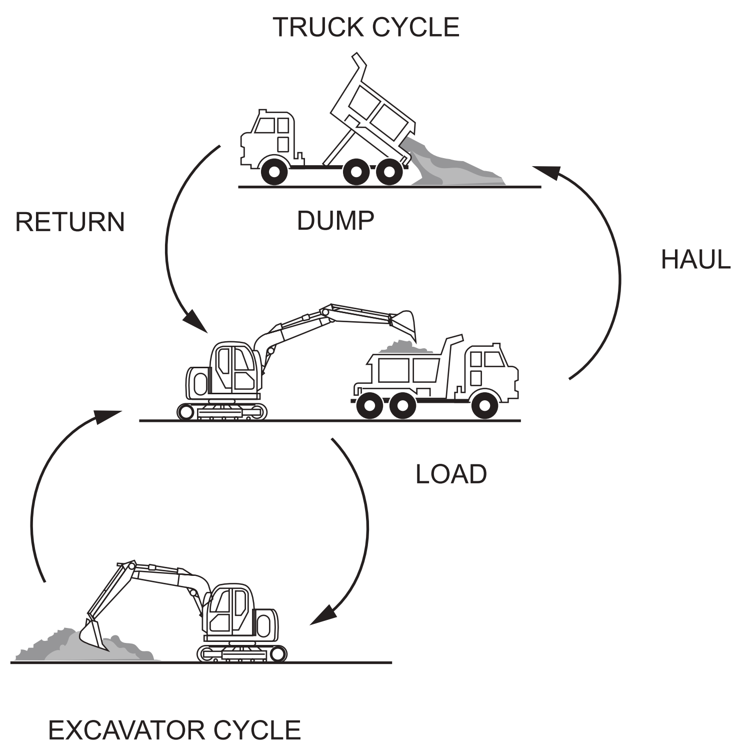
2.3.3.2 Production Rate for Loading and Hauling Earthwork

$$\text{Number of excavator bucket loads per truck} = \frac{\text{truck capacity}}{\text{bucket capacity}}$$

Load time = number of bucket swings × bucket cycle time

$$\text{Haul time, in min} = \frac{\text{Haul distance, ft}}{\text{Haul speed, mph}} \times \frac{60\text{ min}}{\text{hr}} \times \frac{\text{mi}}{5{,}280\text{ ft}}$$

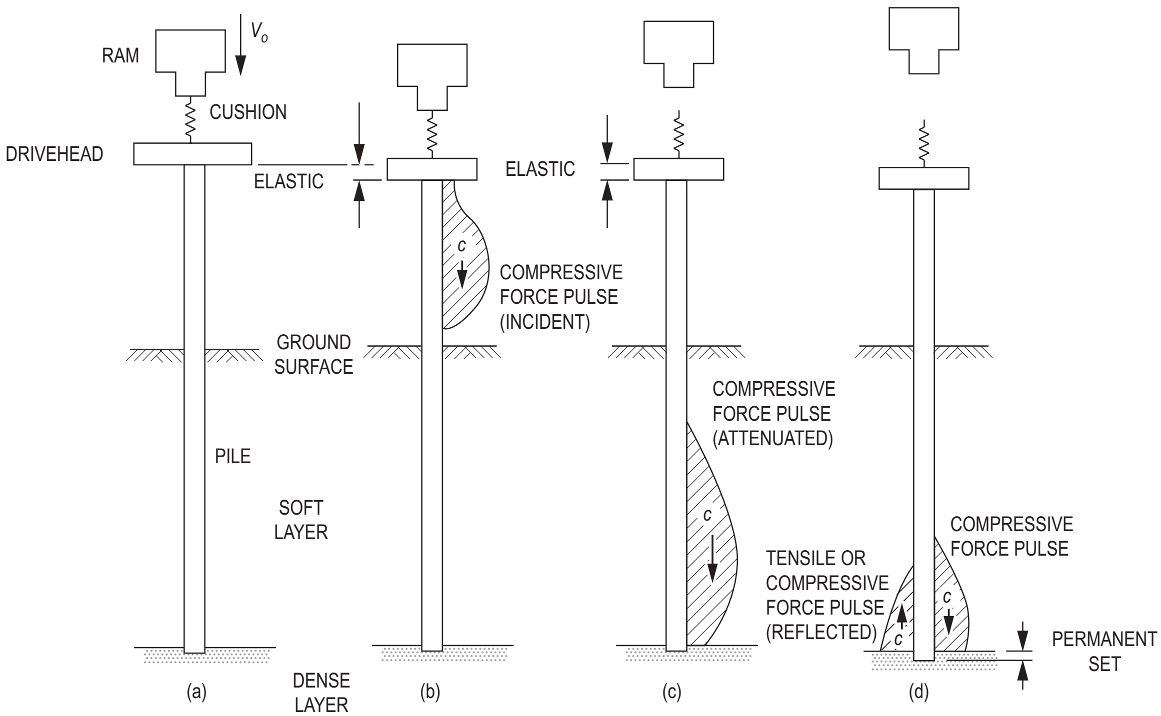
All capacity measurements are in loose cubic yards.



Loading and Hauling Earthwork Cycles

2.3.4 Pile Dynamics

2.3.4.1 Hammer-Pile Driving System



Hammer-Pile-Soil System

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI–06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 9-40, p. 9-100. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

2.3.4.2 Pile Driving Formulas

***Modified Engineering-News* Formula**

$$P_{\text{allow}} = \frac{R}{6} = \frac{2E_n}{S+k}$$

where

P_{allow} = allowable pile load (lb)

R = total soil resistance (lb)

E_n = driving energy (ft-lb) = weight of hammer × drop height

S = pile penetration per blow (set) (in.)

k = constant based on hammer type: 0.1 for single-acting air hammer, 1.0 for drop hammer

Modified Gates Formula with Specialized Units

$$R_{ndr} = 1.75\sqrt{E_d}\log_{10}(10N_b) - 100$$

where

R_{ndr} = nominal driving resistance (kips)

E_d = developed hammer energy, $(W)(h)$, during the observed set (ft-lb)

W = ram weight (lb)

h = average hammer stroke during set observation (ft)

N_b = number of hammer blows per inch (blows/in.)

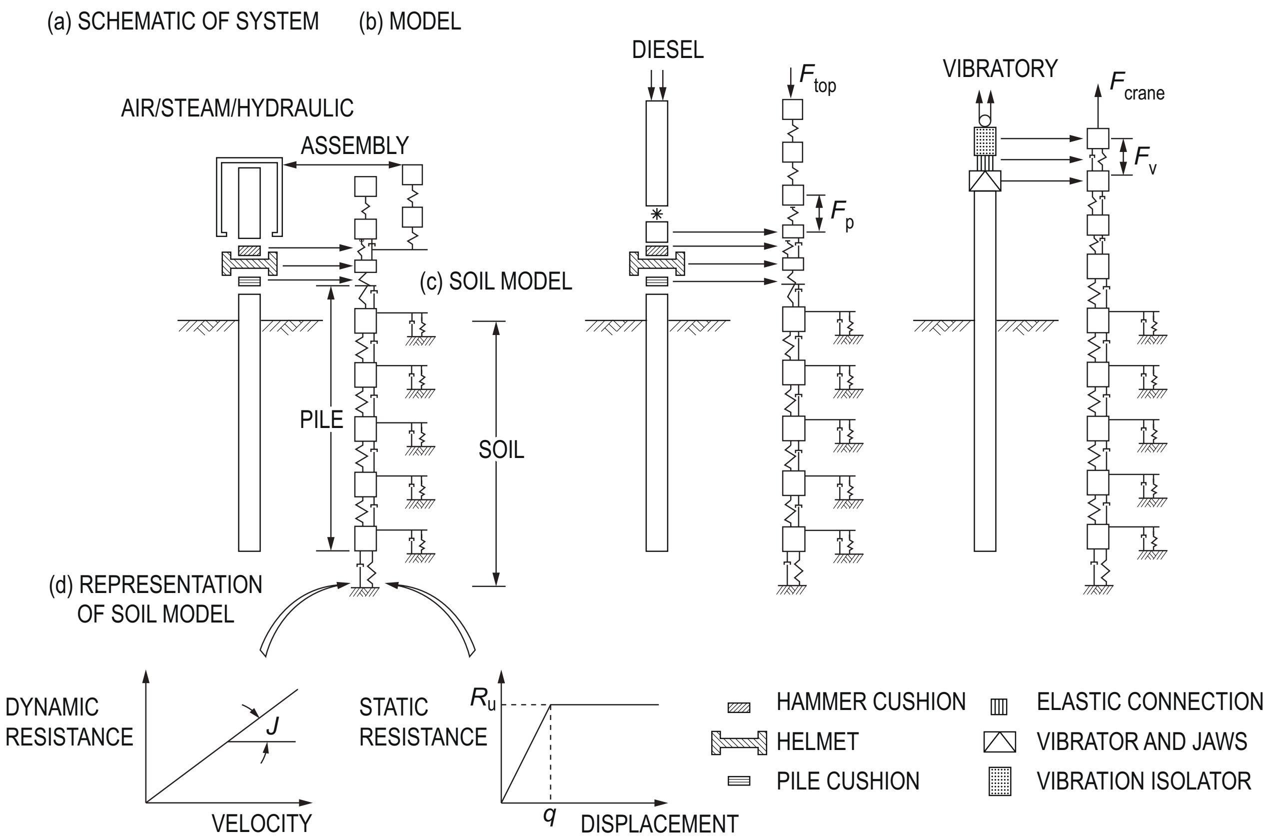
The number of hammer blows per foot of pile penetration required to obtain the nominal resistance is calculated as follows:

$$N_{ft} = 12(10^x)$$

in which

$$x = \left[\left(R_{ndr} + 100\right)/\left(1.75\sqrt{E_d}\right)\right] - 1$$

2.3.4.3 Wave Equation Analysis

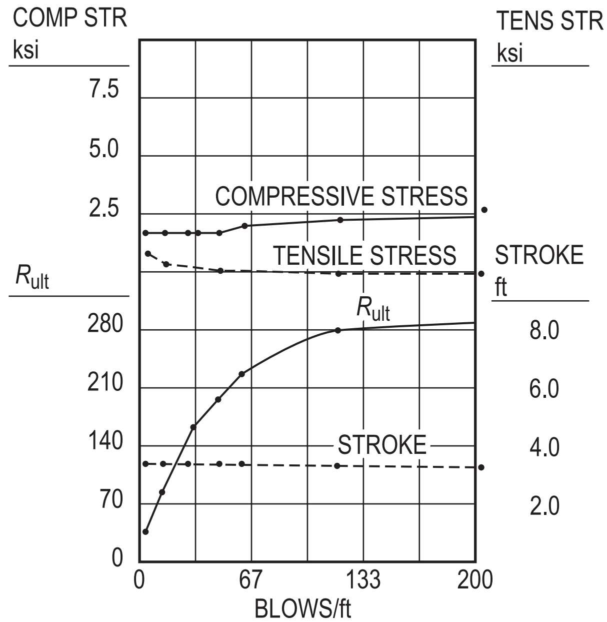


Typical Wave Equation Models

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI–06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 9-41, p. 9-104. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

2.3.4.4 Summary Output for Wave Equation Analysis

Summary of stroke, compressive stress, tensile stress, and driving capacity versus blow count (blows/ft) for air-steam hammer.



Wave Equation Bearing Graph

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI–06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 9-42, p. 9-107. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

Maximum Allowable Stresses in Pile for Top-Driven Piles (after AASHTO, 2002; FHWA, 2006a)

Pile Type	Maximum Allowable Stresses (f_y = yield stress of steel; f'_c = 28-day compressive strength of concrete; f_{pe} = pile prestress)
Steel H-Piles	**Design Stress** $0.25 f_y$ $0.33 f_y$ If damage is unlikely, and confirming static and /or dynamic load tests are performed and evaluated by engineer **Driving Stress** $0.9 f_y$
	32.4 ksi (223 MPa) for ASTM A-36 (f_y = 36 ksi; 248 MPa) 45.0 ksi (310 MPa) for ASTM A-572 or A-690 (f_y = 50 ksi; 345 MPa)
Unfilled Steel Pipe Piles	**Design Stress** $0.25 f_y$ $0.33 f_y$ If damage is unlikely, and confirming static and /or dynamic load tests are performed and evaluated by engineer **Driving Stress** $0.9 f_y$
	27.0 ksi (186 MPa) for ASTM A-252 Grade 1 (f_y = 30 ksi; 207 MPa) 31.5 ksi (217 MPa) for ASTM A-252 Grade 2 (f_y = 35 ksi; 241 MPa) 40.5 ksi (279 MPa) for ASTM A-252 Grade 3 (f_y = 45 ksi; 310 MPa)
Concrete-Filled Steel Pipe Piles	**Design Stress** $0.25 f_y$ (on steel area) ***plus*** $0.40\ f'_c$ (on concrete area) **Driving Stress** $0.9 f_y$
	27.0 ksi (186 MPa) for ASTM A-252 Grade 1 (f_y = 30 ksi; 207 MPa) 31.5 ksi (217 MPa) for ASTM A-252 Grade 2 (f_y = 35 ksi; 241 MPa) 40.5 ksi (279 MPa) for ASTM A-252 Grade 3 (f_y = 45 ksi; 310 MPa)
Precast Prestressed Concrete Piles	**Design Stress** $0.33\ f'_c - 0.27 f_{pe}$ (on gross concrete area); f'_c minimum of 5.0 ksi (34.5 MPa) f_{pe} generally > 0.7 ksi (5 MPa)
	Driving Stress Compression Limit $< 0.85\ f'_c - f_{pe}$ (on gross concrete area) Tension Limit (1) $< 3\ (f'_c)^{1/2} + f_{pe}$ (on gross concrete area) **US Units*** $< 0.25\ (f'_c)^{1/2} + f_{pe}$ (on gross concrete area) **SI Units*** Tension Limit (2) $< f_{pe}$ (on gross concrete area) (1) Normal environments (2) Severe corrosive environments * Note: f'_c and f_{pe} must be in psi and MPa for US and SI equations, respectively.
Conventionally Reinforced Concrete Piles	**Design Stress** $0.33\ f'_c$ (on gross concrete area); f'_c minimum of 5.0 ksi (34.5 MPa)
	Driving Stress Compression Limit $< 0.85\ f'_c$; Tension Limit $< 0.70 f_y$ (of steel reinforcement)

Maximum Allowable Stresses in Pile for Top-Driven Piles (after AASHTO, 2002; FHWA, 2006a) (cont'd)

Pile Type	Maximum Allowable Stresses (f_y = yield stress of steel; f'_c = 28-day compressive strength of concrete; f_{pe} = pile prestress)
Timber Pile	**Design Stress** 0.8 to 1.2 ksi (5.5 to 8.3 MPa) for pile toe area depending upon species
	Driving Stress Compression Limit < 3 s_a Tension Limit < 3 s_a σ_a = AASHTO allowable working stress

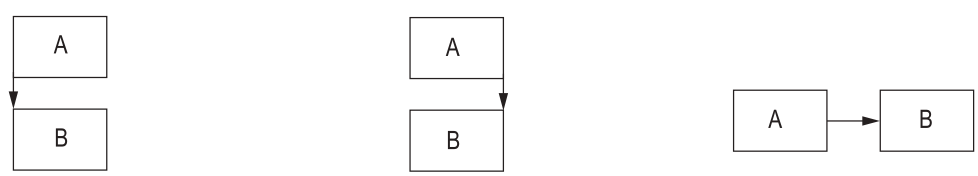
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI–06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 9-11, p. 9-110. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

2.4 Scheduling

2.4.1 Critical Path Method (CPM) Network Analysis

2.4.1.1 CPM Precedence Relationships

ACTIVITY-ON-NODE

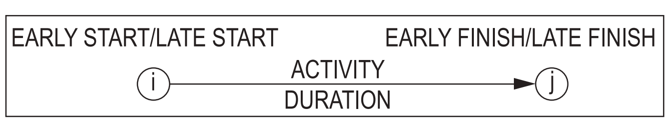


START-TO-START: START OF B DEPENDS ON THE START OF A

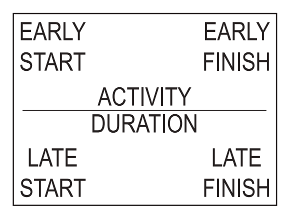
FINISH-TO-FINISH: FINISH OF B DEPENDS ON THE FINISH OF A

FINISH-TO-START: START OF B DEPENDS ON THE FINISH OF A

ACTIVITY-ON-ARROW ANNOTATION



ACTIVITY-ON-NODE ANNOTATION



Nomenclature

ES = early start = latest EF of predecessors

EF = early finish = ES + duration

LS = late start = LF – duration

LF = late finish = earliest LS of successors

D = duration

Total Float = LS – ES or LF – EF

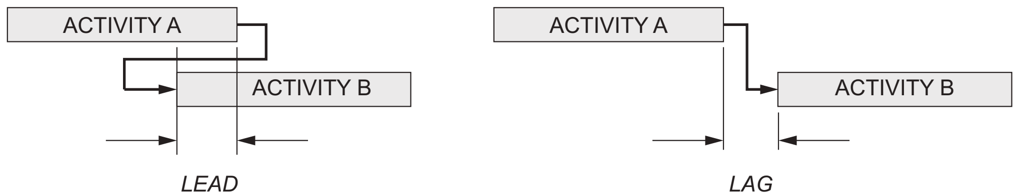
Free Float = earliest ES of successors – EF

Critical Path = longest continuous chain of activities through the network schedule that establishes the minimum overall project duration

Critical Activity = activity on the critical path (i.e., an activity with zero total float)

2.4.1.2 Lead and Lag Relationships

Activities A and B have a finish-to-start relationship with lead or lag time, as shown:



2.4.1.3 Earned-Value Analysis

$BCWS$ = Budgeted cost of work scheduled (Planned)

$ACWP$ = Actual cost of work performed (Actual)

$BCWP$ = Budgeted cost of work performed (Earned)

Variances

$CV = BCWP - ACWP$ (Cost Variance = Earned – Actual)

$SV = BCWP - BCWS$ (Schedule Variance = Earned – Planned)

Indices

$$CPI = \frac{BCWP}{ACWP} \quad \left(\text{Cost Performance Index} = \frac{\text{Earned}}{\text{Actual}}\right)$$

$$SPI = \frac{BCWP}{BCWS} \quad \left(\text{Schedule Performance Index} = \frac{\text{Earned}}{\text{Planned}}\right)$$

Forecasting

BAC = Original project estimate (Budget at completion)

$$ETC = \frac{BAC - BCWP}{CPI} \quad (\text{Estimate to complete})$$

$$EAC = (ACWP + ETC) \quad (\text{Estimate at completion})$$

$$\text{Project completion (\%)} = \frac{\text{Actual units complete}}{\text{Total units budgeted}} \times 100$$

where units may be cost, time, or other resources

2.4.2 Resource Scheduling and Leveling

Resource scheduling is tabulating project resource demands over time, often based on an early start schedule.

Resource leveling is minimizing resource conflicts either by adjusting the start times of activities to reduce peak demand for resources, sometimes within float, or by extending activity durations to reduce peak demand within a constrained resource limit.

2.4.3 Time-Cost Trade-Off

A time-cost trade-off is a strategy for optimizing project time and cost either by identifying least-cost solutions for adding resources to shorten the duration of critical-path activities or by identifying least-time solutions for reallocating resources over an extended duration of critical-path activities to reduce total project cost.

2.5 Material Quality Control and Production

2.5.1 Material Properties and Testing

Concrete Testing

- Samples for strength tests of each class of concrete placed each day shall be taken not less than once a day, nor less than once for each 150 yd^3 of concrete, nor less than once for each 5,000 ft^2 of surface area for slabs or walls.
- When the total quantity of a given class of concrete is less than 50 yd^3, strength tests are not required when evidence of satisfactory strength is submitted to and approved by the building official.
- A strength test shall be the average of the strengths of at least two 6 in. by 12 in. cylinders or at least three 4 in. by 8 in. cylinders made from the same sample of concrete and tested at 28 days (or at the test age designated for determination of concrete compressive strength).
    - f'_c = specified compressive strength of concrete (psi)
- The strength level of an individual class of concrete shall be considered satisfactory if both of the following requirements are met:
    - Every arithmetic average of any three consecutive strength tests equals or exceeds f'_c.
    - No strength test falls below f'_c by more than 500 psi when f'_c is 5,000 psi or less; or by more than 0.10 f'_c when f'_c is more than 5,000 psi.
- For field-cured concrete test cylinders, procedures for protecting and curing concrete shall be improved when the strength of field-cured cylinders (at the test age designated for f'_c) is less than 85% of that of companion laboratory-cured cylinders. The 85% limitation shall not apply if the field-cured strength exceeds f'_c by more than 500 psi.
- Concrete test cores drilled from existing structures shall be tested no later than 7 days after drilled from structure. The concrete in an area represented by core tests shall be considered structurally adequate if the average of three cores is equal to at least 85% of f'_c and if no single core is less than 75% of f'_c.

Source: Adapted from American Concrete Institute. Authorized reprint from ACI 318-19: *Building Code Requirements for Structural Concrete* and ACI 301-20: Specifications for Concrete Construction. American Concrete Institute.

2.5.2 Concrete Proportioning and Placement

2.5.2.1 Mixture Proportioning

$$\text{Yield} = \text{volume of fresh concrete produced in a batch}$$
$$= \frac{\text{total mass of batched materials}}{\text{density of freshly mixed concrete}}$$
$$= \text{sum of absolute volumes of concrete ingredients}$$

$$\text{Absolute volume} = \frac{\text{mass of loose material}}{\text{relative density (or specific gravity) of material} \times \text{density of water}}$$

Relative densities of aggregates in mix-design calculations are based on saturated surface-dry (SSD) conditions.

Aggregate moisture conditions:

$$\text{Total moisture (\%)} = \frac{\text{wet weight} - \text{oven dry weight}}{\text{oven dry weight}} \times 100$$

$$\text{Absorbed moisture (\%)} = \frac{\text{SSD weight} - \text{oven dry weight}}{\text{oven dry weight}} \times 100$$

Free moisture (%) = total moisture (%) – absorbed moisture (%)

$$\text{Water-cementitious material ratio} = \frac{w}{cm} = \frac{\text{mass of water}}{\text{mass of cementitious materials}}$$

where cementitious materials include portland cement, blended cement, fly ash, slag cement, silica fume, and natural pozzolans.

2.5.2.2 Concrete Exposure Categories and Classes

Category	Class	Condition	
Freezing and Thawing (F)	F0	Concrete not exposed to freezing-and-thawing cycles	
	F1	Concrete exposed to freezing-and-thawing cycles with limited exposure to water	
	F2	Concrete exposed to freezing-and-thawing cycles with frequent exposure to water	
	F3	Concrete exposed to freezing-and-thawing cycles with frequent exposure to water and exposure to deicing chemicals	
Sulfate (S)		**Water-soluble sulfate (SO_4^{2-}) in soil (percent by mass)**	**Dissolved sulfate (SO_4^{2-}) in water (ppm)**
	S0	$SO_4^{2-} < 0.10$	$SO_4^{2-} < 150$
	S1	$0.10 \le SO_4^{2-} < 0.20$	$150 \le SO_4^{2-} < 1{,}500$ or seawater
	S2	$0.20 \le SO_4^{2-} \le 2.00$	$1{,}500 \le SO_4^{2-} \le 10{,}000$
	S3	$SO_4^{2-} > 2.00$	$SO_4^{2-} > 10{,}000$
In Contact with Water (W)	W0	Concrete dry in service	
	W1	Concrete in contact with water where low permeability is not required	
	W2	Concrete in contact with water where low permeability is required	
Corrosion Protection of Reinforcement (C)	C0	Concrete dry or protected from moisture	
	C1	Concrete exposed to moisture, but not to an external source of chlorides	
	C2	Concrete exposed to moisture and an external source of chlorides from deicing chemicals, salt, brackish water, seawater, or spray from these sources	

Source: Adapted from American Concrete Institute. Authorized reprint from ACI 318-19: *Building Code Requirements for Structural Concrete*. American Concrete Institute, 2019, p. 358.

2.5.2.3 Concrete Cover Requirements for Reinforcement

Specified Concrete Cover for Cast-in-place Nonprestressed Concrete Members

Concrete Exposure	Member	Reinforcement	Specified Cover (in.)
Cast against and permanently in contact with ground	All	All	3
Exposed to weather or in contact with ground	All	No. 6 through No. 18 bars	2
		No. 5 bar, W31 or D31 wire, and smaller	1 1/2
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 14 and No. 18 bars	1 1/2
		No. 11 bar and smaller	3/4
	Beams, columns, pedestals, and tensions ties	Primary reinforcement, stirrups, ties, spirals, and hoops	1 1/2

Specified Concrete Cover for Cast-in-place Prestressed Concrete Members

Concrete Exposure	Member	Reinforcement	Specified Cover (in.)
Cast against and permanently in contact with ground	All	All	3
Exposed to weather or in contact with ground	Slabs, joists, and walls	All	1
	All other	All	1 1/2
Not exposed to weather or in contact with ground	Slabs, joists, and walls	All	3/4
	Beams, columns, and tensions ties	Primary reinforcement	1 1/2
		Stirrups, ties, spirals, and hoops	1

Source: American Concrete Institute. Authorized reprint from ACI 318-19: *Building Code Requirements for Structural Concrete*. American Concrete Institute, 2019, p. 382–383.

2.5.3 Concrete Maturity and Early Strength Evaluation

2.5.3.1 Concrete Maturity

Concrete maturity can be estimated using the Time-Temperature Factor method:

$$M = \sum_0^t \left(T - T_0\right)\Delta t$$

where

M = maturity index (°F-hours)

T = average concrete temperature (°F) during time interval Δt

T_0 = datum temperature (°F), usually 32°F unless otherwise specified

t = elapsed time (hr)

Δt = time intervals (hr)

2.5.3.2 Nondestructive Test Methods

A nondestructive testing (NDT) program may be undertaken for a variety of purposes regarding the strength or condition of hardening concrete, including:

- Determination of in-place concrete strength
- Monitoring rate of concrete strength gain
- Location of defects, such as voids or honeycombing in concrete
- Determination of relative strength of comparable members
- Evaluation of concrete cracking and delamination
- Evaluation of damage from mechanical or chemical actions
- Steel reinforcement location, size, and corrosion activity
- Member dimensions

Nondestructive Test Methods for Concrete

Property or Condition	Primary NDT Method	Secondary NDT Method
In-place Strength	Pullout test Pull-off test (for bond strength) Probe penetration Maturity method (new construction)	Rebound hammer
General Quality and Uniformity	Visual inspection Rebound hammer Pulse velocity	Probe penetration
Thickness	Impact-echo Ground-penetrating radar Ultrasonic echo	Eddy-current thickness gauge
Dynamic Modulus of Elasticity	Resonate frequency (on small specimens)	Pulse velocity Impact-echo Spectral analysis of surface waves
Density	Gamma radiometry (nuclear gauge)	
Reinforcement Location	Covermeter Ground-penetrating radar	Radiography Ultrasonic echo
Reinforcement Bar Size	Covermeter	Radiography
Corrosion State of Steel Reinforcement	Half-cell potential Polarization resistance	Radiography
Presence of Near-Surface Defects	Sounding Infrared thermography	Ground-penetrating radar Radiography
Presence of Internal Defects	Impact-echo Ultrasonic echo Impulse response	Pulse velocity Spectral analysis of surface waves Radiography

Source: Wilson, Michelle L., and Paul D. Tennis. 2021. *Design and Control of Concrete Mixtures*. 17th edition. Engineering Bulletin, EB001.17. Washington, DC: Portland Cement Association. Courtesy of Portland Cement Association (PCA).

2.5.4 Soil Stabilization Methods

Soil Stabilization Methods

Method	Soil Type	Plasticity Index	Treatment Notes
Portland cement	Primarily granular soils	Less than 10	Cement stabilization is used to strengthen granular soils by mixing in portland cement, typically 3% to 7% of soil dry weight.
Lime	Clay-containing soils	10 to 50 or more	Lime stabilization is used to treat subbase or base materials, which may include predominantly fine-grained or clay-gravel soils. Lime modification is used to improve fine-grained soils with small amounts of lime, typically 0.5% to 3% of soil dry weight.

2.6 Health and Safety

2.6.1 Safety Management and Statistics

2.6.1.1 Safety Incidence Rate

$$IR = N \times 200{,}000 / T$$

where

IR = total injury/illness incidence rate

N = number of injuries and illnesses

T = total hours worked by all employees during the period in question

2.6.1.2 Experience Modification Rate

The experience modification rate (EMR) is an annual adjustment factor for workers' compensation insurance premiums based on actual loss experience for the previous three full years. If an employer's past insurance claim history is better or worse than the average for similar types of businesses, that employer's premium is adjusted respectively downward or upward proportional to the EMR.

Worker's compensation costs fall into three categories:

Paid losses = money spent on a claim

Reserved losses = money set aside (outstanding) for future payments

Incurred losses = combined total paid plus reserved amounts

$$EMR = \frac{\text{Actual Losses}}{\text{Expected Losses}} = \frac{B + H + (E \times W) + (1 - W) \times F}{D + H + (F \times W) + (1 - W) \times F}$$

where

A = total of all actual incurred losses

B = total of all actual primary losses

C = total of all expected losses

D = total of all expected primary losses

E = actual excess losses = $A - B$

F = expected excess losses = $C - D$

W = weighting value (tabular value based on expected losses, C)

H = ballast value (tabular value based on expected losses, C)

2.6.2 Work Zone and Public Safety

Permissible Noise Exposure (OSHA)

$$D = 100 \times \sum \frac{C_i}{T_i}$$

where

D = noise dose (%)

C_i = time spent at specified sound pressure level (SPL) (hr)

T_i = time permitted at SPL (hr)

$\sum C_i$ = 8 hours

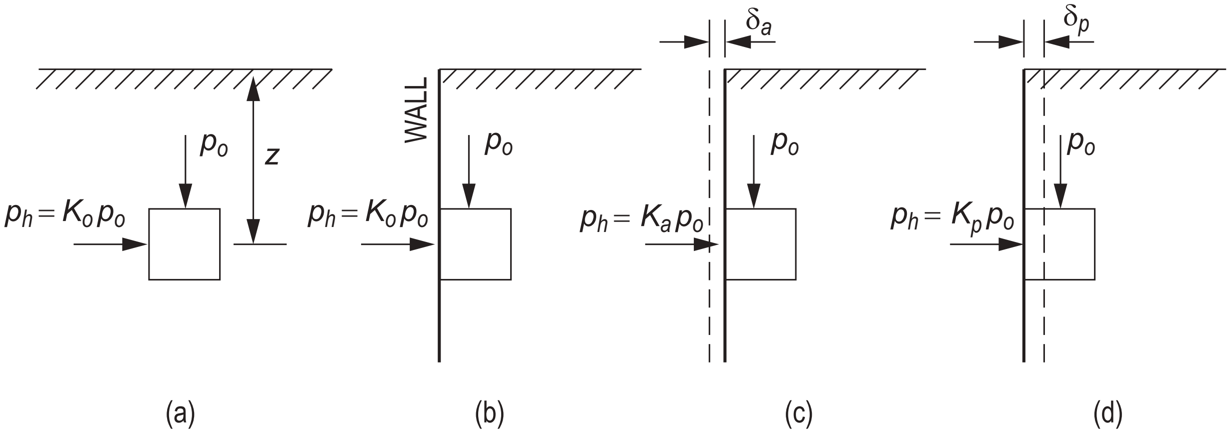
Noise Level (dBA)	Permissible Time (hr)
80	32
85	16
90	8
95	4
100	2
105	1
110	0.5
115	0.25
120	0.125
125	0.063
130	0.031

If D > 100%, noise abatement is required.

Exposure to impulsive or impact noise should not exceed 140 dB sound pressure level (SPL).

3 GEOTECHNICAL

3.1 Lateral Earth Pressures



Stress States on a Soil Element Subjected Only to Body Stresses:
(a) In situ geostatic effective vertical and horizontal stresses
(b) Insertion of hypothetical infinitely rigid, infinitely thin frictionless wall and removal of soil to left of wall
(c) Active condition of wall movement away from retained soil
(d) Passive condition of wall movement into retained soil

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 2-19, p. 2-43. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.1.1 At-Rest Coefficients

Normally Consolidated Soils

$$K_o = 1 - \sin \phi'$$

Overconsolidated Soils

$$K_o = (1 - \sin \phi') OCR^{\Omega}$$

$$\Omega = \sin \phi'$$

where

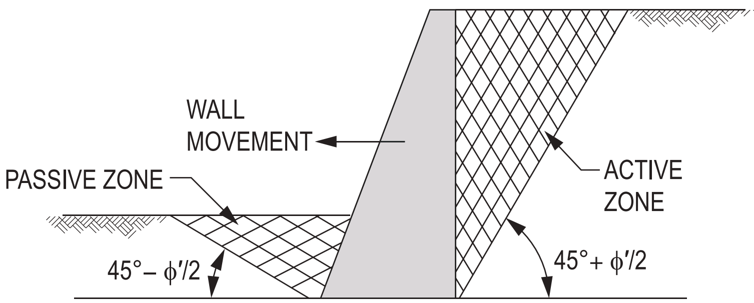
K_o = at-rest earth coefficient

ϕ' = effective friction angle

OCR = overconsolidation ratio

Ω = OCR factor

3.1.2 Rankine Earth Coefficients



Development of Rankine Active and Passive Failure Zones for a Smooth Retaining Wall
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 2-20, p. 2-45. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Rankine Active and Passive Coefficients (Friction Only)

$$K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'} = \tan^2\left(45 - \frac{\phi'}{2}\right)$$

$$K_p = \frac{1 + \sin\phi'}{1 - \sin\phi'} = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$p_a = K_a p_o$$

$$p_p = K_p p_o$$

where

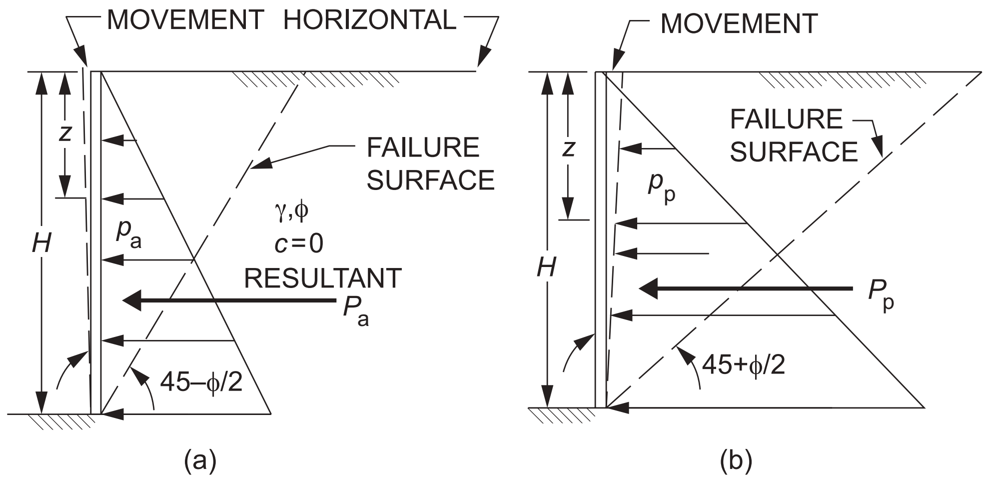
K_a = coefficient of active earth pressure

K_p = coefficient of passive earth pressure

p_o = overburden pressure

p_a = active pressure

p_p = passive pressure

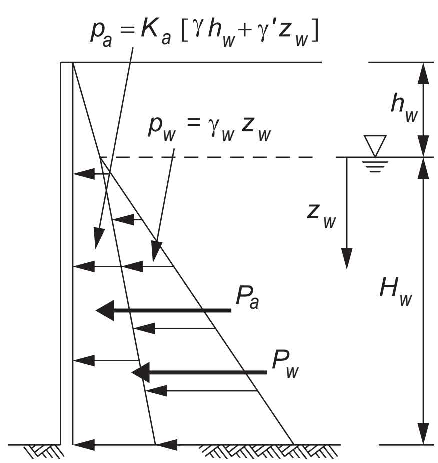


Active pressure at depth z: $p_a = K_a \gamma z$ Passive pressure at depth z: $p_p = K_p \gamma z$

Active force within depth z: $P_a = \dfrac{K_a \gamma z^2}{2}$ Passive force within depth z: $P_p = \dfrac{K_p \gamma z^2}{2}$

Failure Surfaces, Pressure Distribution and Forces: (a) Active case, (b) Passive case

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 2-21, p. 2-46. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



General Distribution of Combined Active Earth Pressure and Water Pressure

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, fig. 2-22, p. 2-46. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

where

h_w = depth to water level from ground surface

z_w = depth below groundwater level

H_w = height of water below base of wall

γ = unit weight of soil

γ' = effective unit weight of soil

p_w = water pressure

P_a = lateral force from active pressure

P_p = lateral force from passive pressure

u = pore water pressure

Rankine Active and Passive Coefficients (Friction and Cohesion)

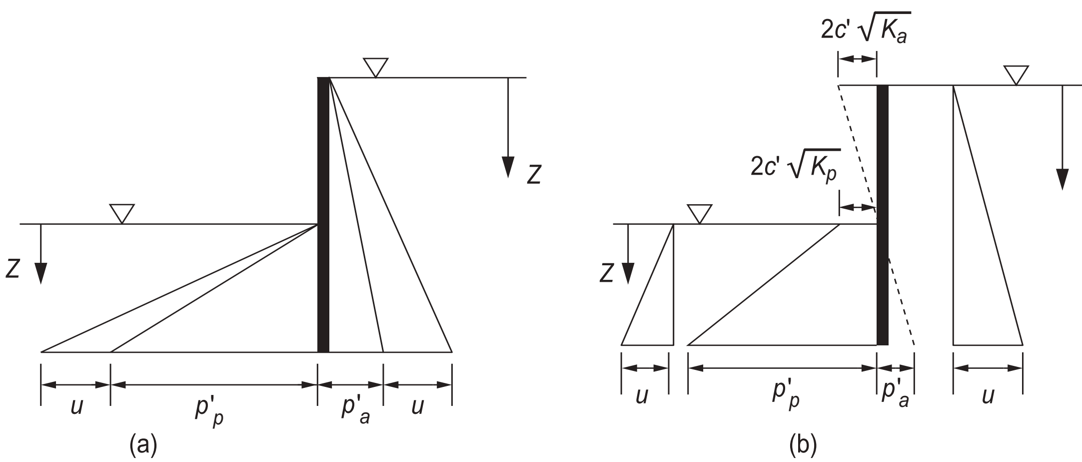
$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) - \frac{2c'}{p'_o}\tan\left(45 - \frac{\phi'}{2}\right)$$

$$K_p = \tan^2\left(45 + \frac{\phi'}{2}\right) + \frac{2c'}{p'_o}\tan\left(45 + \frac{\phi'}{2}\right)$$

$$p'_a = K_a\left(\gamma z - u\right) - 2c'\sqrt{K_a}$$

$$p'_p = K_p\left(\gamma z - u\right) + 2c'\sqrt{K_p}$$

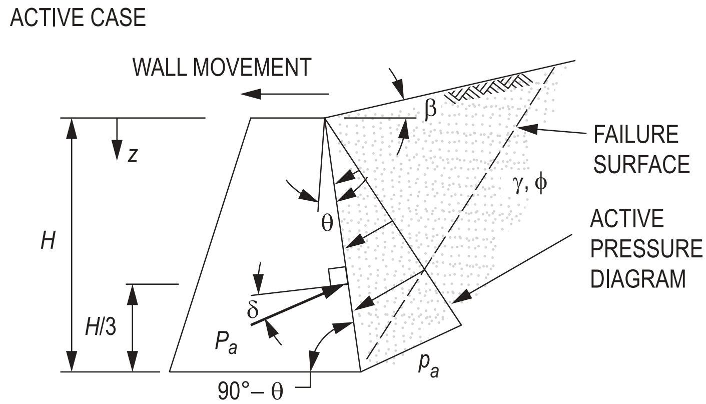
Source: NAVFAC DM 7.2 *Foundations and Earth Structures Design Manual*, Department of the Navy—Naval Facilities Engineering Command, Alexandria, VA, September 1986, Figure 2, p. 7.2-62.



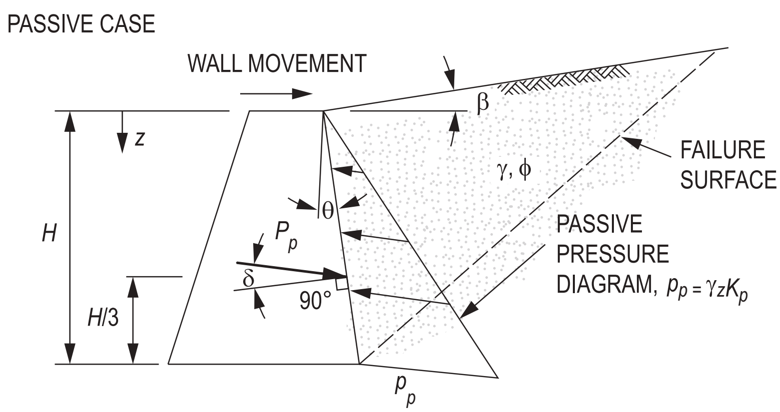
(a) Wall Pressures for a Cohesionless Soil and (b) Wall Pressures for Soil with a Cohesion Intercept, with groundwater in both cases (after Padfield and Mair, 1984)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 10-6, p. 10-15.www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf

3.1.3 Coulomb Earth Pressures



$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta\cos(\theta + \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\cos(\theta + \delta)\cos(\theta - \beta)}}\right]^2}$$



$$K_p = \frac{\cos^2(\theta + \phi)}{\cos^2\theta\cos(\theta - \delta)\left[1 - \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi + \beta)}{\cos(\theta - \delta)\cos(\theta - \beta)}}\right]^2}$$

Coulomb Coefficients K_a and K_p for Sloping Wall with Wall Friction and Sloping Cohesionless Backfill (after NAVFAC, 1986b)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 10-5, p. 10-13. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf

where

ϕ = friction angle of soil

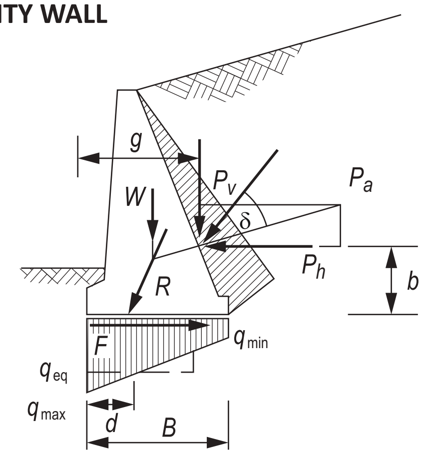
δ = friction between wall and soil

β = angle of backfill slope (β = 0° for horizontal surface)

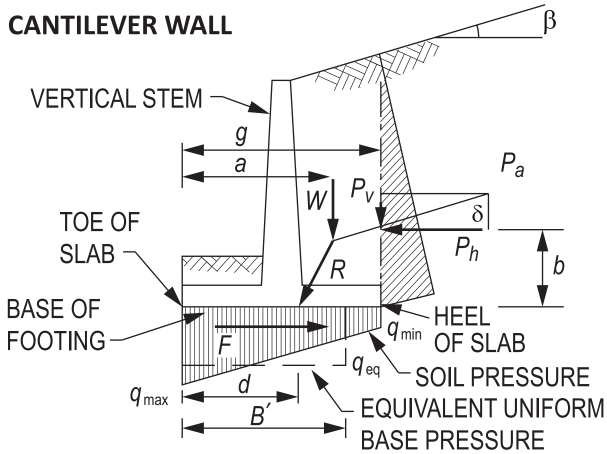
θ = angle of wall face (θ = 0° for vertical wall)

Design Criteria for Cast-in-Place (CIP) Concrete Retaining Walls (after NAVFAC)

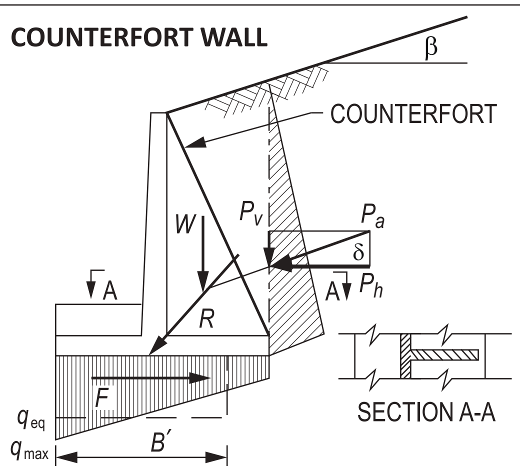
GRAVITY WALL



CANTILEVER WALL



COUNTERFORT WALL



Definitions

B = width of the base of the footing

$\tan \delta_b$ = friction factor between soil and base (see table below)

W = weight at the base of wall; includes weight of wall for gravity walls; includes weight of soil above footing for cantilever and counterfort walls

c = cohesion of the foundation soil

c_a = adhesion between concrete and soil

δ = angle of wall friction

P_p = passive resistance

Location of Resultant, *R*

Based on moments about toe (assuming $P_p = 0$)

$$d = \frac{Wa + P_v g - P_h b}{W + P_v}$$

Criteria for Eccentricity, *e*

$$e = d - \frac{B}{2}$$

$e \le B/6$ for soils

$e \le B/4$ for rocks

Factor of Safety against Sliding

$$FS_s = \frac{(W + P_v)\tan \delta_b + c_a B}{P_h} \ge 1.5(\text{min})$$

Applied Stress at Base (q_{max}, q_{min}, q_{eq})

$$q_{max} = \frac{(W + P_v)}{B}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{(W + P_v)}{B}\left(1 - \frac{6e}{B}\right)$$

Equivalent uniform (Meyerhof) applied stress, q_{eq}, is given as follows:

$$q_{eq} = \frac{(W + P_v)}{B'}$$

where $B' = B - 2e$

Use uniform stress, q_{eq}, for soils and settlement analysis; use trapezoidal distribution with q_{max} and q_{min} for rocks and structural analysis.

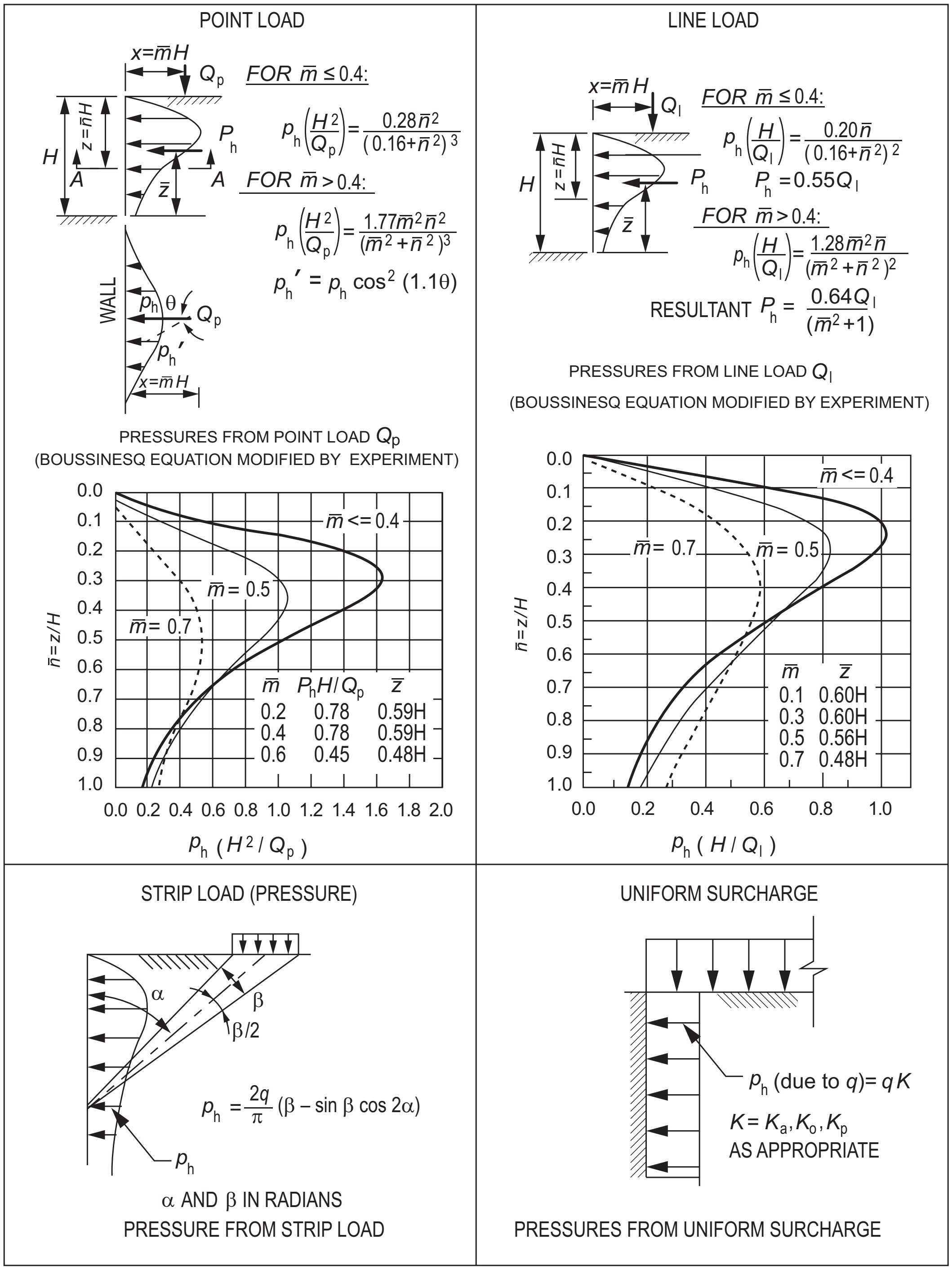
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 10-17, p. 10-39. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf

Wall Friction and Adhesion for Dissimilar Materials (after NAVFAC, 1986b)

Interface Materials	Friction Factor, tan δ	Friction Angle, δ (degrees)
Mass concrete on the following foundation materials:		
Clean sound rock	0.70	35
Clean gravel, gravel sand mixtures, coarse sand	0.55 to 0.60	29 to 31
Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel	0.45 to 0.55	24 to 29
Clean fine sand, silty or clayey fine to medium sand	0.35 to 0.45	19 to 24
Fine sandy silt, nonplastic silt	0.30 to 0.35	17 to 19
Very stiff and hard residual or preconsolidated clay	0.40 to 0.50	22 to 26
Medium stiff and stiff clay and silty clay (masonry on foundation materials has same friction factor)	0.30 to 0.35	17 to 19
Steel sheet piles against the following soils:		
Clean gravel, gravel-sand mixtures, well-graded rock fill with spalls	0.40	22
Clean sand, silty sand-gravel mixtures, single size hard rock fill	0.30	17
Silty sand, gravel or sand mixed with silt or clay	0.25	14
Fine sandy silt, nonplastic silt	0.20	11
Formed concrete or concrete sheet piling against the following soils:		
Clean gravel, gravel-sand mixture, well-graded rock fill with spalls	0.40 to 0.50	22 to 26
Clean sand, silty sand-gravel mixture, single size hard rock fill	0.30 to 0.40	17 to 22
Silty sand, gravel or sand mixed with silt or clay	0.30	17
Fine sandy silt, nonplastic silt	0.25	14
Various structural materials:		
Masonry on masonry, igneous and metamorphic rocks:		
Dressed soft rock on dressed soft rock	0.70	35
Dressed hard rock on dressed soft rock	0.65	33
Dressed hard rock on dressed hard rock	0.55	29
Masonry on wood (cross grain)	0.50	26
Steel on steel at sheet pile interlocks	0.30	17
Interface Materials (Cohesion)	**Adhesion c_a**	
	(kPa)	**(psf)**
Very soft cohesive soil (0–12 kPa)	0–12	0–250
Soft cohesive soil (12–24 kPa)	12–24	250–500
Medium stiff cohesive soil (24–48 kPa)	24–36	500–750
Stiff cohesive soil (48–96 kPa)	36–45	750–950
Very stiff cohesive soil (96–192 kPa)	45–62	950–1,300

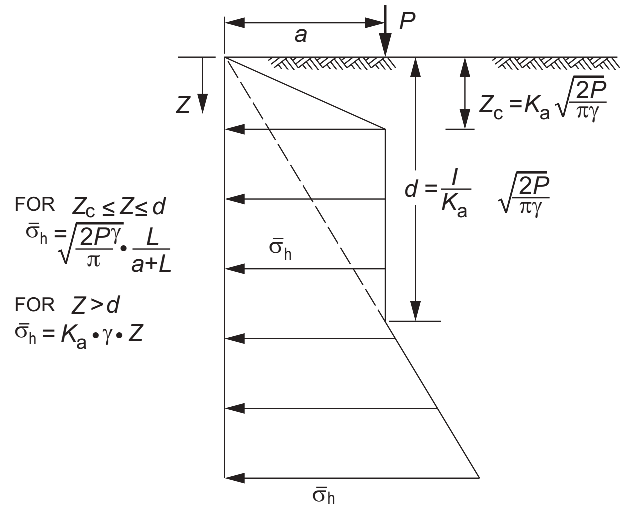
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 10-1, p. 10-18. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf

3.1.4 Load Distribution from Surcharge



Lateral Pressure Due to Surcharge Loadings (after USS Steel, 1975):
Solutions for point, line, and strip loading are semi-empirical and based on an assumption of unyielding walls.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 10-14, p. 10-32. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.



P (ROLLER LOAD) = $\frac{\text{DEAD WT. OF ROLLER + CENTRIFUGAL FORCE}}{\text{WIDTH OF ROLLER}}$

a = DISTANCE OF ROLLER FROM WALL
L = LENGTH OF ROLLER
K_a = COEFFICIENT OF ACTIVE EARTH PRESSURE

Horizontal Pressure on Walls from Compaction Effort

Source: Naval Facilities Engineering Command. *Foundations and Earth Structures*. NAVFAC DM 7.2. Washington, DC: U.S. Department of the Navy, September 1986, Fig. 13, p. 7.2-77. https://web.mst.edu/~rogersda/umrcourses/ge441/DM7_02.pdf

3.1.5 Pseudostatic Analysis and Earthquake Loads

Mononobe-Okabe Theory

$$P_{AE} = \frac{1}{2}K_{AE}\gamma H^2(1-k_v)$$

$$P_{PE} = \frac{1}{2}K_{PE}\gamma H^2(1-k_v)$$

$$K_{AE} = \frac{\cos^2(\phi-\theta-\beta)}{\cos\theta(\cos^2\beta)\cos(\beta+\delta+\theta)D}$$

$$D = \left[1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\theta-i)}{\cos(\delta+\beta+\theta)\cos(i-\beta)}}\right]^2$$

$$K_{PE} = \frac{\cos^2(\phi-\theta+\beta)}{\cos\theta(\cos^2\beta)\cos(\delta-\beta+\theta)D'}$$

$$D' = \left[1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi+i-\theta)}{\cos(\delta-\beta+\theta)\cos(i-\beta)}}\right]^2$$

$$\theta = \tan^{-1}\left[\frac{k_h}{1-k_v}\right]$$

where

P_{AE} = dynamic earth pressures in the active state

P_{PE} = dynamic earth pressures in the passive state

γ = effective unit height of the backfill

H = height of the wall

ϕ = angle of internal friction of the backfill

δ = angle of friction of the wall/backfill interface

ξ = angle of failure plane

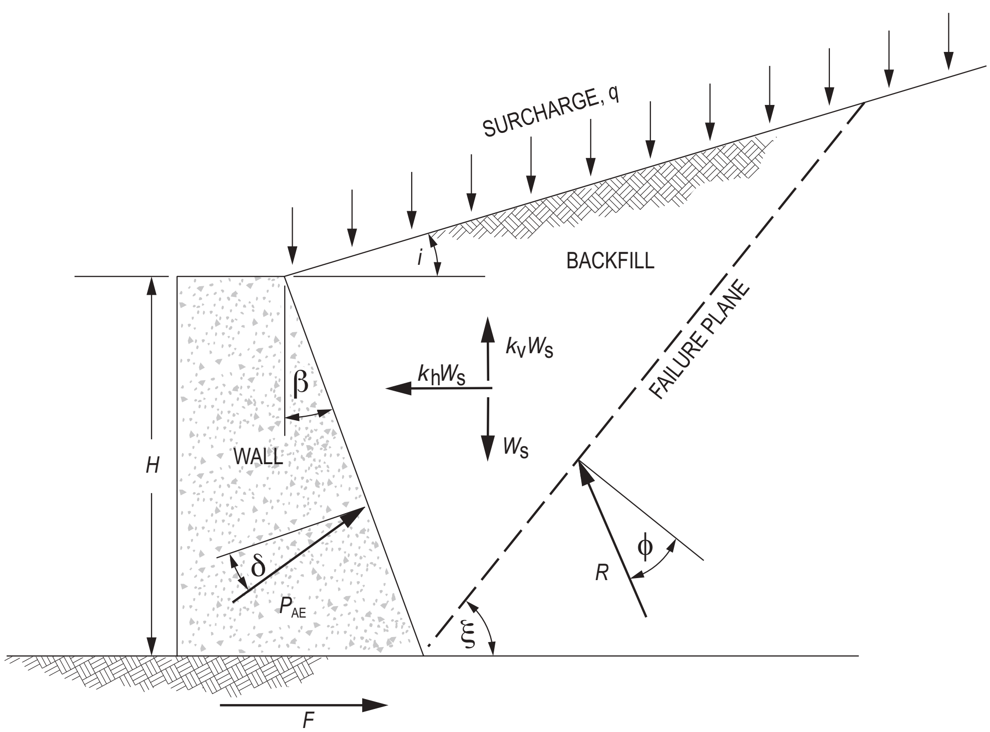
i = slope of the surface of the backfill

β = slope of the back of the wall

k_h = horizontal seismic coefficient, expressed as a fraction of g

k_v = vertical seismic coefficient, expressed as a fraction of g

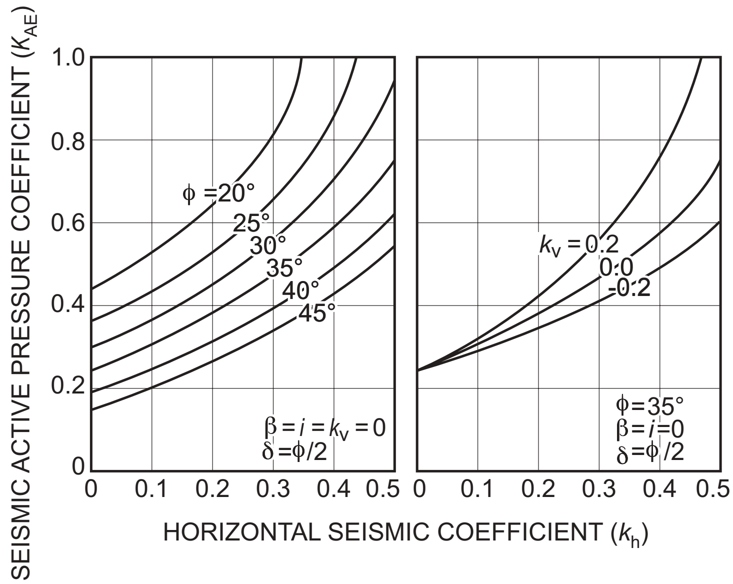
g = acceleration due to gravity



Forces Behind a Gravity Wall

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4, Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, June 1999, Fig. 55, p. 115. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf

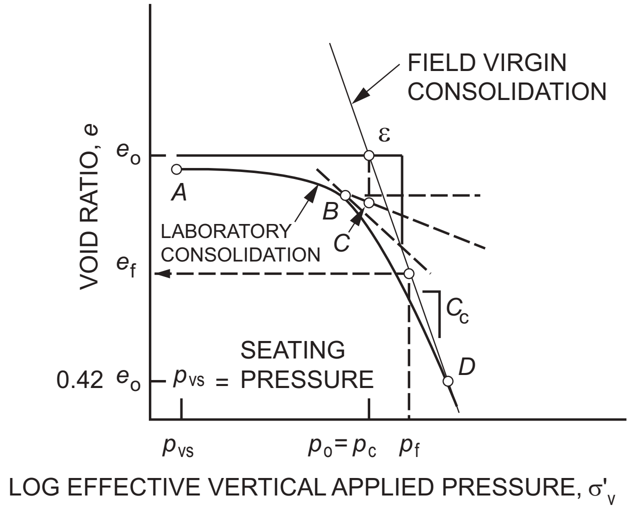
where W_s = weight of the sliding wedge



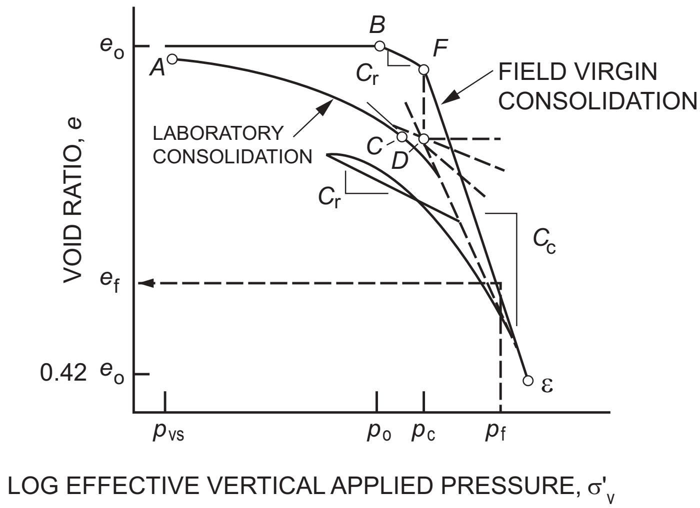
Effects of Seismic Coefficients and Friction Angle on Seismic Active Pressure Coefficient (after Lam and Martin, 1986)

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4, Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, June 1999, Fig. 56, p. 116. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

3.2 Consolidation



(a) NORMALLY CONSOLIDATED SOIL

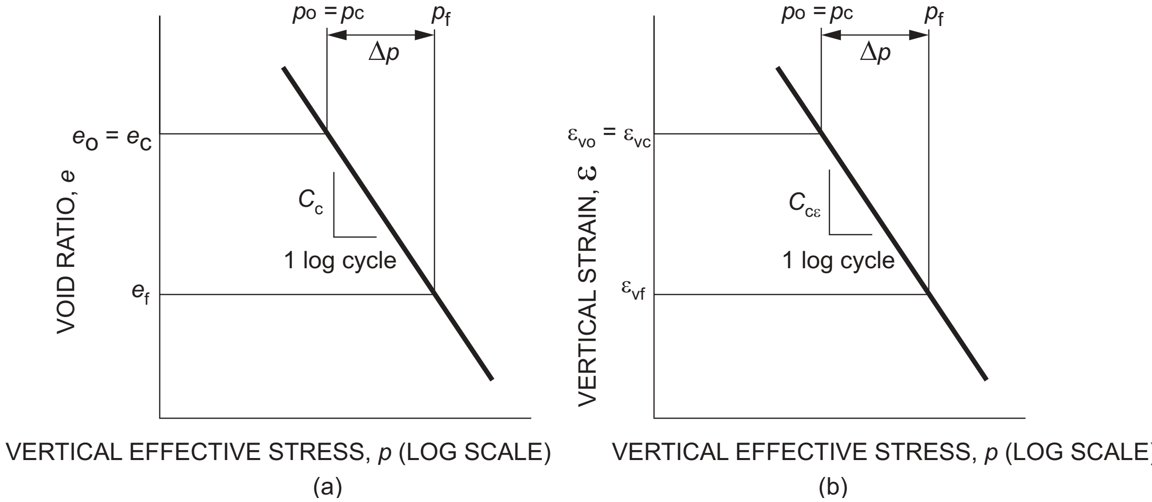


(b) OVERCONSOLIDATED SOIL

Construction of Field Virgin Consolidation Relationships (adapted from USACE, 1994)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 7-9, p. 7-22. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.2.1 Normally Consolidated Soils



Typical Consolidation Curve for Normally Consolidated Soil:
(a) Void ratio versus vertical effective stress and (b) Vertical strain versus vertical effective stress
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 7-10, p. 7-25. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

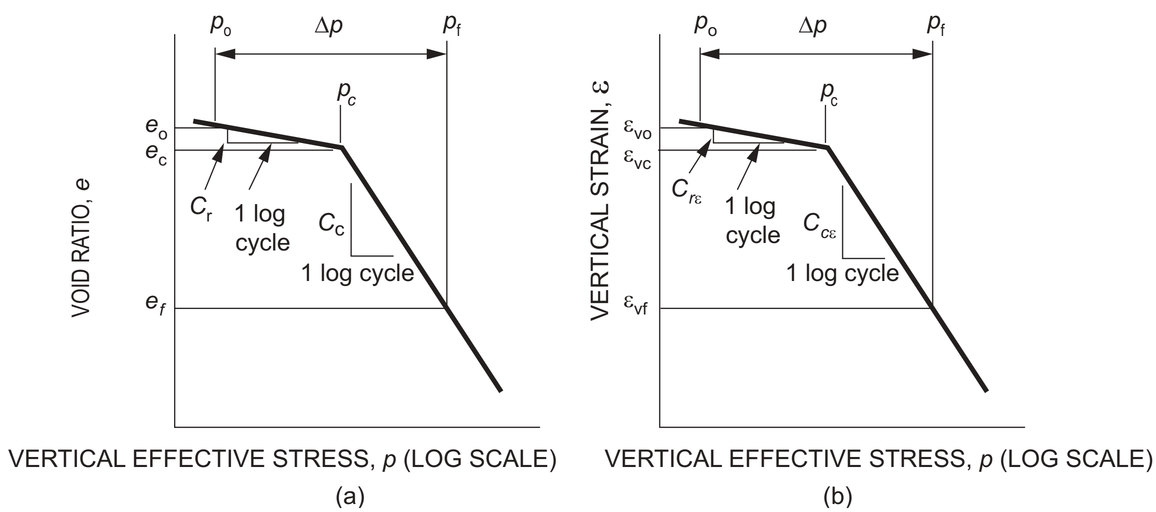
$$S_c = \sum_{1}^{n} \frac{C_c}{1 + e_o} H_o \log_{10}\left(\frac{p_f}{p_o}\right)$$

$$S_c = \sum_{1}^{n} C_{c\varepsilon} H_o \log_{10}\left(\frac{p_f}{p_o}\right)$$

$$C_{c\varepsilon} = \frac{C_c}{1 + e_o}$$

Equation variables are defined in the following section.

3.2.2 Overconsolidated Soils



Typical Consolidation Curve for Overconsolidated Soil:
(a) Void ratio versus vertical effective stress and (b) Vertical strain versus vertical effective stress
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 7-11, p. 7-26. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

$$S_c = \sum_1^n \frac{H_o}{1+e_o}\left(C_r \log_{10}\frac{p_c}{p_o} + C_c \log_{10}\frac{p_f}{p_c}\right)$$

$$S_c = \sum_1^n H_o\left(C_{r\varepsilon} \log_{10}\frac{p_c}{p_o} + C_{c\varepsilon} \log_{10}\frac{p_f}{p_c}\right)$$

where

C_c = compression index

$C_{c\varepsilon}$ = modified compression index

C_r = recompression index

$C_{r\varepsilon}$ = modified recompression index

e_o = initial void ratio

H_o = thickness of compressible layer

p_o = initial overburden pressure (= p_c for normally consolidated soils)

p_c = preconsolidation pressure

p_f = final overburden pressure = $p_o + \Delta p$

S_c = consolidation settlement

Δp = increase in effective stress

3.2.3 Time Rate of Settlement

C_1 = excess pore water pressure profile at time 1

C_2 = excess pore water pressure profile at time 2

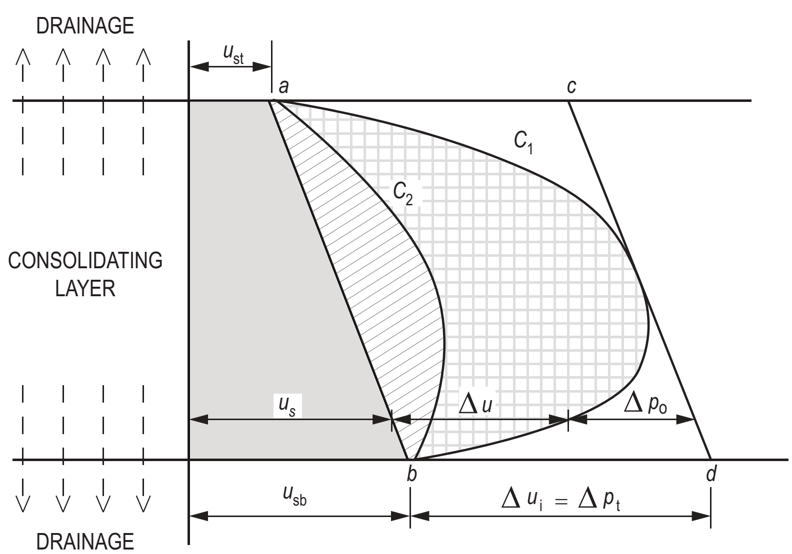


Diagram Illustrating Consolidation of a Layer of Clay Between Two Pervious Layers (modified, after Terzaghi *et alia*, 1996)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 7-13, p. 7-28. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

where

u_{st} = hydrostatic pore water pressure at top of layer

u_{sb} = hydrostatic pore water pressure at bottom of layer

u_s = hydrostatic pore water pressure at any depth

Δp_0 = effective vertical stress increment

Δp_t = total vertical stress increment

$\Delta p_t = \Delta u + \Delta p_0$

Δu_i = initial excess pore water pressure

Δu = excess pore water pressure at any depth after time t

$u_t = u_s + \Delta u$ = total pore water pressure at any depth after time t

$$T_v = \frac{c_v t}{\left(H_d\right)^2}$$

$$t = \frac{T_v \left(H_d\right)^2}{c_v}$$

where

c_v = coefficient of consolidation (ft^2/day or m^2/day)

H_d = longest distance to drainage boundary (ft or m)

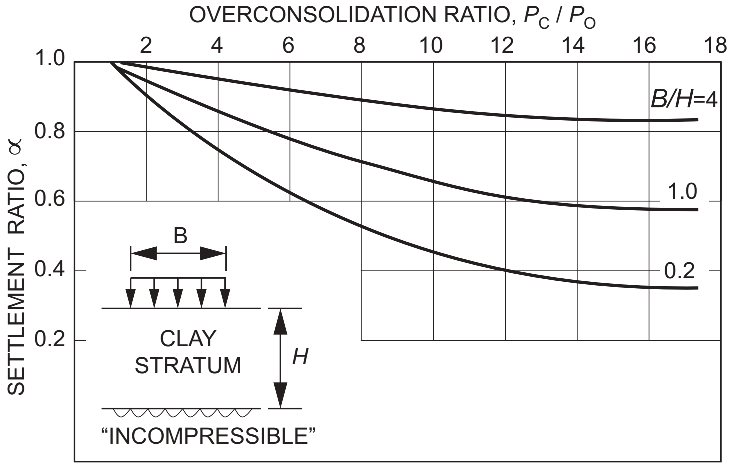
t = time (day)

Average Degree of Consolidation, *U*, Versus Time Factor, T_v, for Uniform Initial Increase in Pore Water Pressure

U%	T_v
0	0.000
10	0.008
20	0.031
30	0.071
40	0.126
50	0.197
60	0.287
70	0.403
80	0.567
90	0.848
93.1	1.000
95.0	1.163
98.0	1.500
99.4	2.000
100.0	Infinity

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 7-4, p. 7-32. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

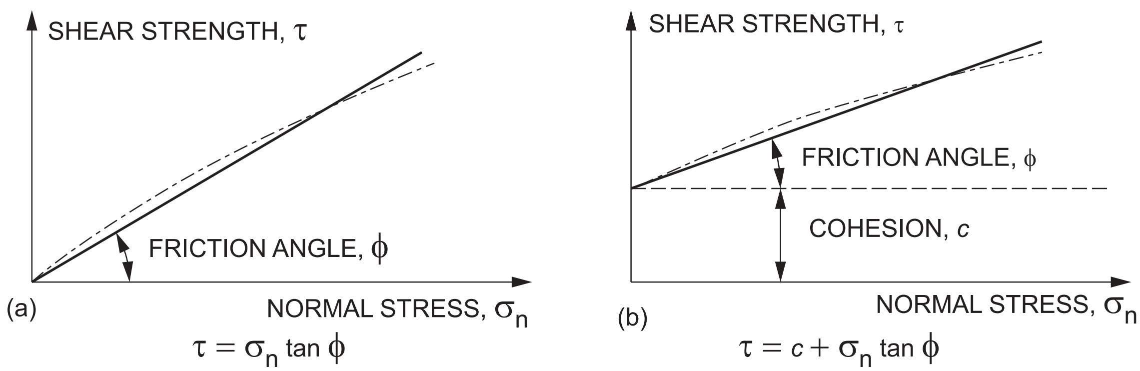
3.2.4 Settlement Ratio for Overconsolidation



Relation Between Settlement Ratio and Overconsolidation Ratio
Source: Unified Facilities Criteria (UFC). *Soil Mechanics*. UFC 3-220-10N. Washington, DC: U.S. Department of Defense, June 2005, Fig. 8, p. 225.
www.wbdg.org/FFC/DOD/UFC/ufc_3_220_10n_2005.pdf.

3.3 Effective and Total Stresses

3.3.1 Shear Strength—Total Stress



Shear Strength of (a) Cohesionless Soils and (b) Cohesive Soils
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 2-18, p. 2-41.
www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

where

τ = shear strength

c = total cohesion

σ_n = normal stress

ϕ = friction angle

3.3.2 Shear Strength Effective Stress

$$\tau' = c' + (\sigma_n - u)\tan\phi' = c' + \sigma'_n \tan\phi'$$

where

τ' = effective shear strength

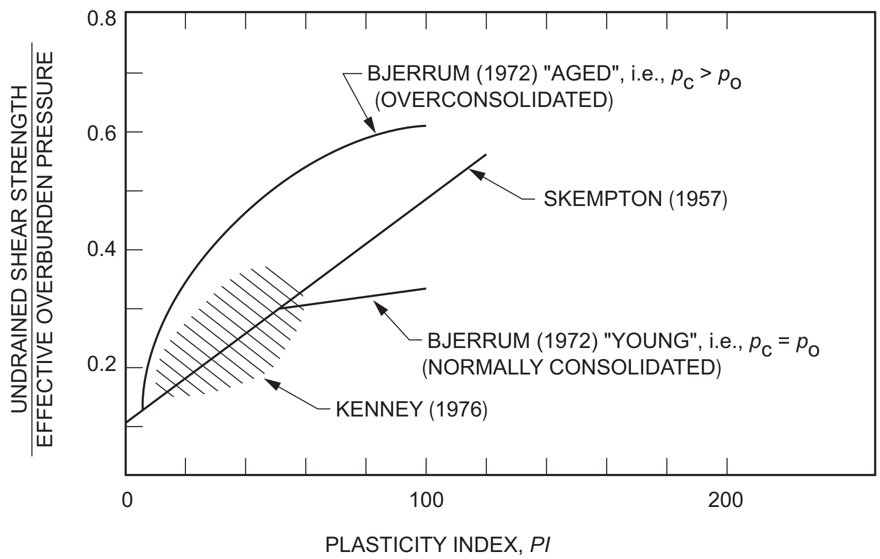
u = pore water pressure

c' = effective cohesion

σ'_n = effective normal stress

ϕ' = effective friction angle

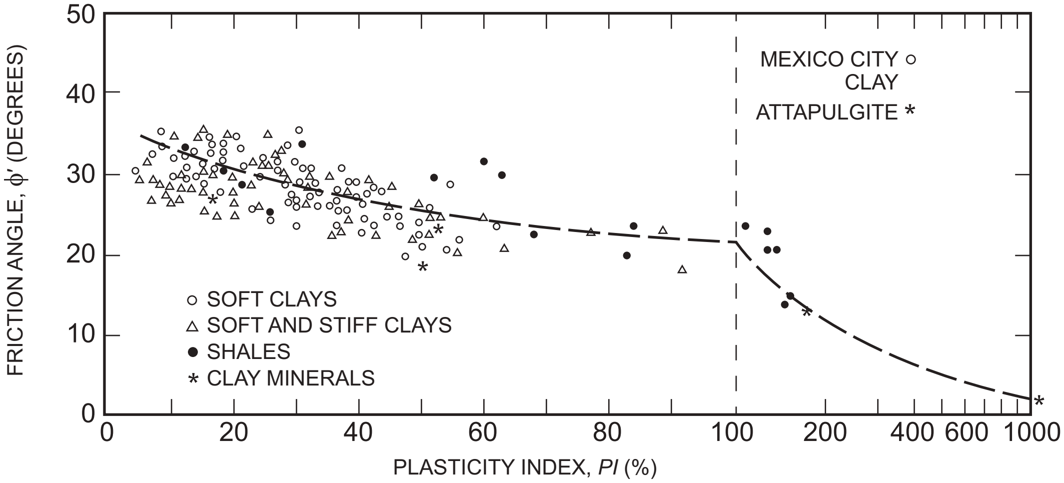
3.3.3 Undrained Shear Strength of Clays



Relationship Between the Ratio of Undrained Shear Strength to Effective Overburden Pressure and Plasticity Index for Normally Consolidated and Overconsolidated Clays (after Holtz and Kovacs, 1981)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-20, p. 5-54. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.3.4 Drained Shear Strength of Clays

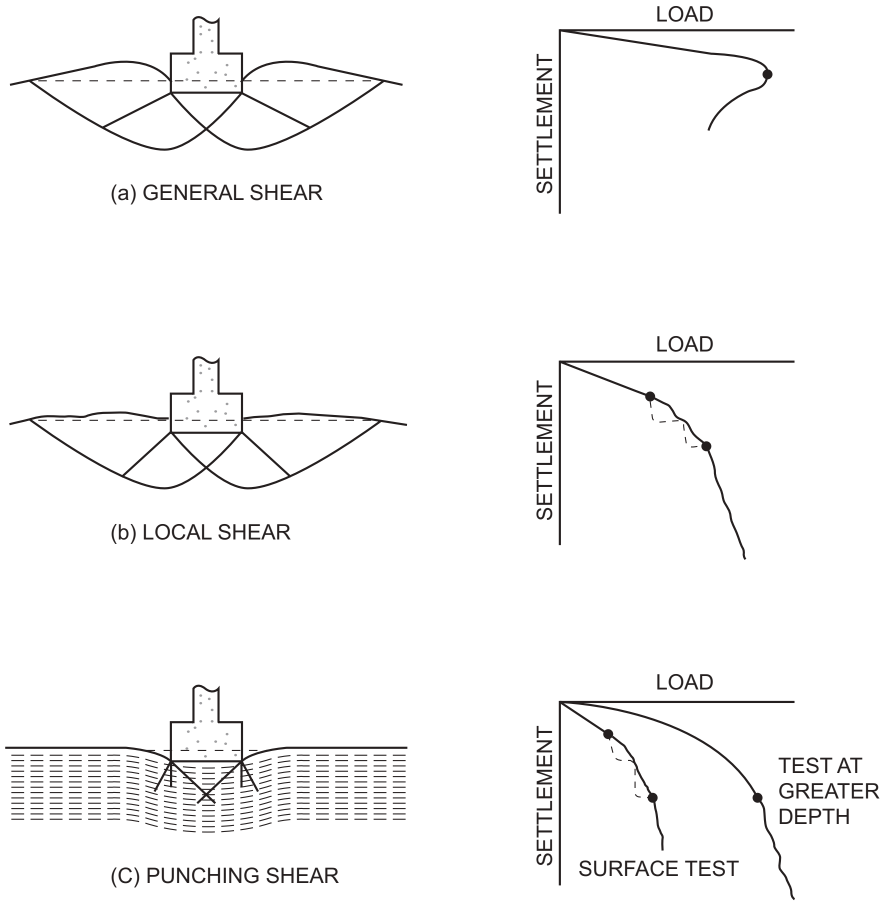


Relationship Between ϕ' and *PI* (after Terzaghi *et alia*, 1996)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-21, p. 5-56. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.4 Bearing Capacity

3.4.1 Bearing Capacity Theory



Modes of Bearing Capacity Failure (after Vesic, 1975): (a) General shear (b) Local shear (c) Punching shear

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 8-14, p. 8-17. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

3.4.2 Bearing Capacity Equation for Concentrically Loaded Strip Footings

$q_{ult} = c(N_c) + q(N_q) + 0.5(\gamma)(B_f)(N_\gamma)$

$q = q_{appl} + \gamma_a D_f$

N_q, N_c, N_γ: Refer to Bearing Capacity Factors table below.

where

q_{ult} = ultimate bearing pressure

c = cohesion of the soil

q = total surcharge at base of the footing

q_{appl} = applied surcharge at surface *

γ_a = unit weight of soil above base of footing

D_f = depth of footing

γ = unit weight of soil

B_f = width of footing

L_f = length of footing

N_q = bearing capacity factor for surcharge

N_c = bearing capacity factor for cohesion

N_γ = bearing capacity factor for soil weight

* Surface surcharge may be a temporary condition. Check all loading conditions that may apply.

3.4.2.1 Bearing Capacity Equation for Concentrically Loaded Square or Rectangular Footings

$$q_{ult} = c(N_c)s_c + q(N_q)s_q + 0.5\gamma(B_f)(N_\gamma)s_\gamma$$

Shape Correction Factors (AASHTO, 2004 with 2006 Interims)

Factor	Friction Angle	Cohesion Term (s_c)	Unit Weight Term (s_γ)	Surcharge Term (s_q)
Shape Factors, s_c, s_γ, s_q	$\phi = 0$	$1 + \left(\frac{B_f}{5L_f}\right)$	1.0	1.0
	$\phi > 0$	$1 + \left(\frac{B_f}{L_f}\right)\left(\frac{N_q}{N_c}\right)$	$1 - 0.4\left(\frac{B_f}{L_f}\right)$	$1 + \left(\frac{B_f}{L_f}\tan\phi\right)$

Source: Based on information from *LRFD Bridge Design Specifications*. 3rd ed. Washington, DC: American Association of State Highway and Transportation Officials, 2004/2006 with Interims.

Bearing Capacity Factors

ϕ	N_c	N_q	N_γ
0	5.14	1.0	0.0
1	5.4	1.1	0.1
2	5.6	1.2	0.2
3	5.9	1.3	0.2
4	6.2	1.4	0.3
5	6.5	1.6	0.5
6	6.8	1.7	0.6
7	7.2	1.9	0.7
8	7.5	2.1	0.9
9	7.9	2.3	1.0
10	8.4	2.5	1.2
11	8.8	2.7	1.4
12	9.3	3.0	1.7
13	9.8	3.3	2.0
14	10.4	3.6	2.3
15	11.0	3.9	2.7
16	11.6	4.3	3.1
17	12.3	4.8	3.5
18	13.1	5.3	4.1
19	13.9	5.8	4.7
20	14.8	6.4	5.4
21	15.8	7.1	6.2
22	16.9	7.8	7.1

ϕ	N_c	N_q	N_γ
23	18.1	8.7	8.2
24	19.3	9.6	9.4
25	20.7	10.7	10.9
26	22.3	11.9	12.5
27	23.9	13.2	14.5
28	25.8	14.7	16.7
29	27.9	16.4	19.3
30	30.1	18.4	22.4
31	32.7	20.6	26.0
32	35.5	23.2	30.2
33	38.6	26.1	35.2
34	42.2	29.4	41.1
35	46.1	33.3	48.0
36	50.6	37.8	56.3
37	55.6	42.9	66.2
38	61.4	48.9	78.0
39	67.9	56.0	92.3
40	75.3	64.2	109.4
41	83.9	73.9	130.2
42	93.7	85.4	155.6
43	105.1	99.0	186.5
44	118.4	115.3	224.6
45	133.9	134.9	271.8

Source: From *LRFD Bridge Design Specifications*, 3rd ed., 2004/2006 with Interims, by the American Association of State Highway and Transportation Officials, Washington, DC Used with permission.

Correction Factor for Location of Groundwater Table (AASHTO, 2004 with 2006 Interims)

D_w	$C_{w\gamma}$	C_{wq}
0	0.5	0.5
D_f	0.5	1.0
$> 1.5B_f + D_f$	1.0	1.0

Note: For intermediate positions of the groundwater table, interpolate between the values shown above.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual.* Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Table 8-5, p. 8-27. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

3.4.2.2 Eccentric and Inclined Loaded Footings

$B'_f = B_f - 2e_B$

$L'_f = L_f - 2e_L$

$A' = B'_f \times L'_f$

where

A' = effective footing area

B'_f = effective footing width

L_f = footing length

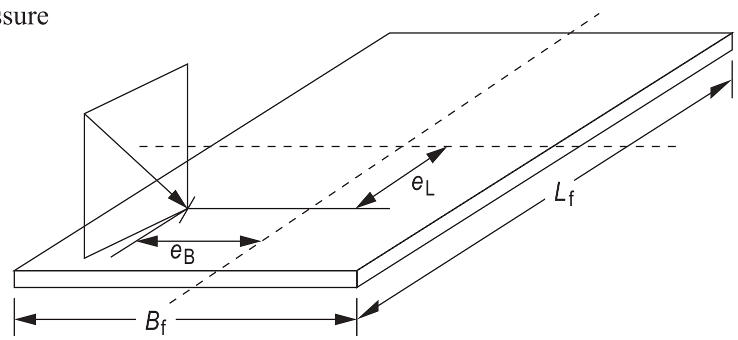
L'_f = effective footing length

e_B = eccentricity in the B_f direction

e_L = eccentricity in the L_f direction

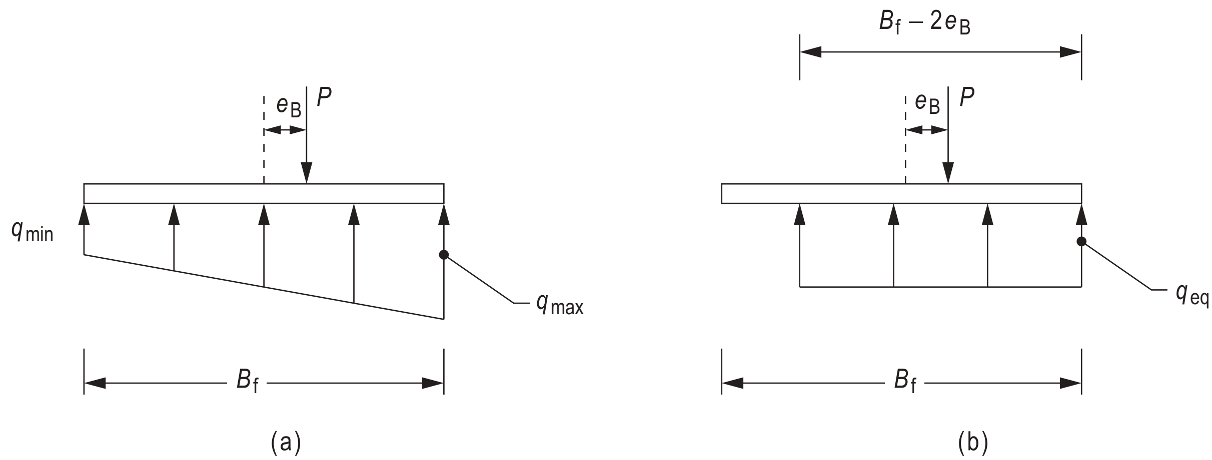
P = total vertical load

q = contact pressure



Notations for Footings Subjected to Eccentric, Inclined Loads (after Kulhawy, 1983)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 8-16, p. 8-25. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.



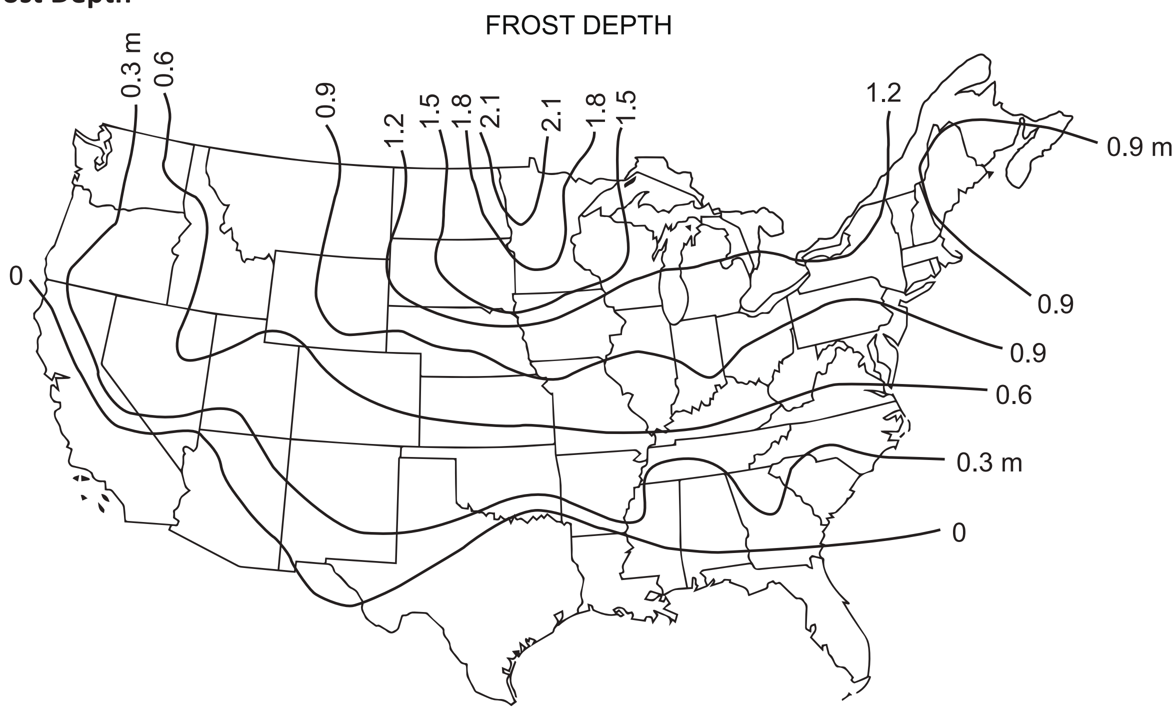
Eccentrically Loaded Footing with: (a) Linearly varying pressure distribution (structural design), (b) Equivalent uniform pressure distribution (sizing the footing)

Figure Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 8-17, p. 8-26. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

$$q_{max} = \frac{P}{BL}\left(1 + \frac{6e}{B}\right), q_{min} = \frac{P}{BL}\left(1 - \frac{6e}{B}\right) \qquad \text{for } e < \frac{B}{6}$$

$$q_{max} = \frac{4P}{3L(B - 2e)} \qquad \text{for } e > \frac{B}{6}$$

3.4.3 Frost Depth

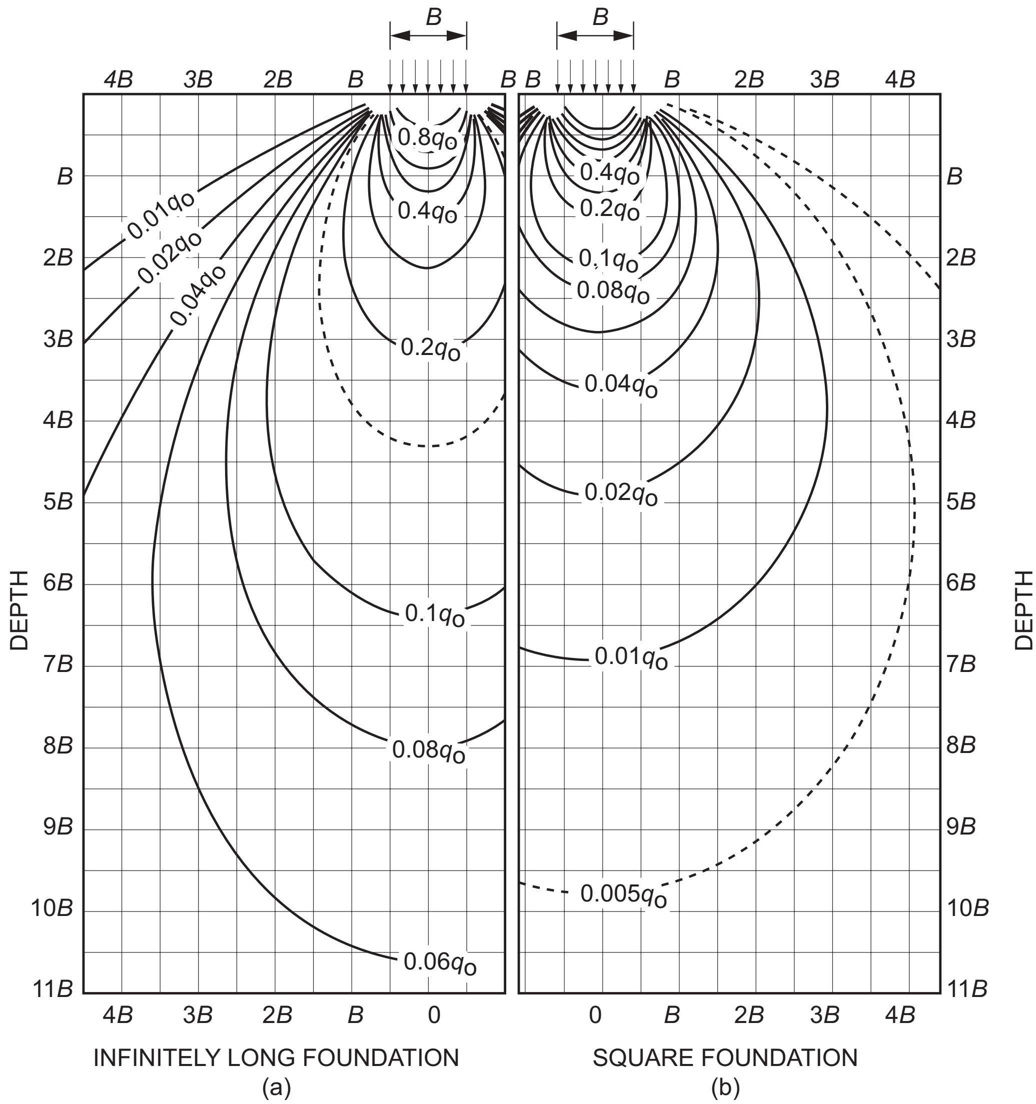


Approximate Frost Depth Map for United States (Bowles, 1996)
1 m = 3.28 ft

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-29, p. 5-71. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

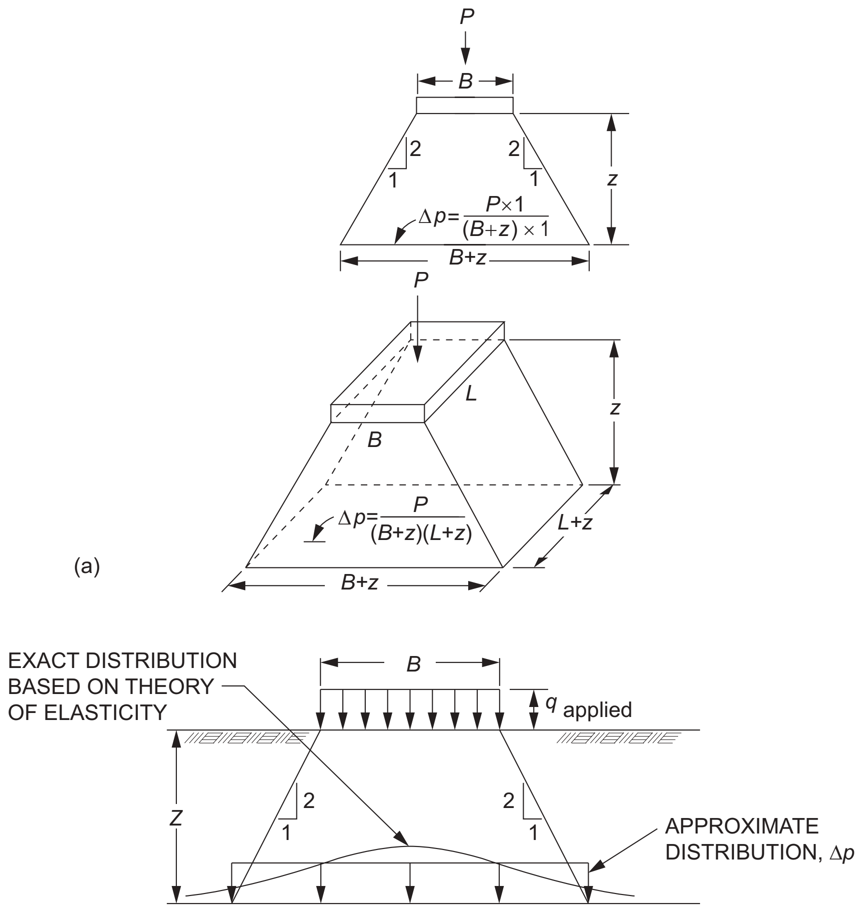
3.5 Foundation Settlement

3.5.1 Stress Distribution



Vertical Stress Contours (Isobars) Based on Boussinesq's Theory for Continuous and Square Footings (modified after Sowers, 1979; AASHTO, 2002)

Source: From *Standard Specifications for Highway Bridges*, 2002, by the American Association of State Highway and Transportation Officials, Washington, DC Used with permission.



Distribution of Vertical Stress by the 2:1 Method (after Perloff and Baron, 1976)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 2-10, p. 2-27. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.5.2 Settlement (Elastic Method)

$$\delta_v = \frac{C_d \Delta p B_f \left(1 - \nu^2\right)}{E_m}$$

where

δ_v = vertical settlement at surface

C_d = shape and rigidity factors (see following table)

Δp = change in stress at base of footing

B_f = footing diameter or width

ν = Poisson's ratio of soil or rock

E_m = elastic or Young's modulus of soil or rock

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Equation 8-19, p. 8-58. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

Shape and Rigidity Factors, C_d, for Calculating Settlements of Points on Loaded Areas at the Surface of a Semi-Infinite, Elastic Half Space

Shape	Center	Corner	Middle of Short Side	Middle of Long Side	Average
Circle	1.00	0.64	0.64	0.64	0.85
Circle (rigid)	0.79	0.79	0.79	0.79	0.79
Square	1.12	0.56	0.76	0.76	0.95
Square (rigid)	0.99	0.99	0.99	0.99	0.99
Rectangle (length/width):					
1.5	1.36	0.67	0.89	0.97	1.15
2	1.52	0.76	0.98	1.12	1.30
3	1.78	0.88	1.11	1.35	1.52
5	2.10	1.05	1.27	1.68	1.83
10	2.53	1.26	1.49	2.12	2.25
100	4.00	2.00	2.20	3.60	3.70
1,000	5.47	2.75	2.94	5.03	5.15
10,000	6.90	3.50	3.70	6.50	6.60

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Table 8-13, p. 8-59. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

Elastic Constants of Various Soils (after AASHTO 2004, with 2006 interims)

Soil Type	Typical Range of Young's Modulus Values, E_s (tsf)	Poisson's Ratio, ν
Clay:		
Soft sensitive	25–150	0.4–0.5 (undrained)
Medium stiff to stiff	150–500	
Very stiff	500–1,000	
Loess	150–600	0.1–0.3
Silt	20–200	0.3–0.35
Fine Sand:		
Loose	80–120	0.25
Medium dense	120–200	
Dense	200–300	
Sand:		
Loose	100–300	0.20–0.36
Medium dense	300–500	
Dense	500–800	0.30–0.40
Gravel:		
Loose	300–800	0.20–0.35
Medium dense	800–1,000	
Dense	1,000–2,000	0.30–0.40
Estimating E_s from SPT *N*-value		
Soil Type		**E_s (tsf)**
Silts, sandy silts, slightly cohesive mixtures		$4\ N1_{60}$
Clean fine to medium sands and slightly silty sands		$7\ N1_{60}$
Coarse sands and sands with little gravel		$10\ N1_{60}$
Sandy gravel and gravels		$12\ N1_{60}$
Estimating E_s (tsf) from q_c Static Cone Resistance		
Sandy soils	$2q_c$ where (q_c is in tsf)	

Note: 1 tsf = 95.76 kPa

Source: Based on information from *LRFD Bridge Design Specifications*, 3rd ed., 2004/2006 with Interims, by the American Association of State Highway and Transportation Officials, Washington, DC.

3.5.3 Settlement (Schmertmann's Method)

$$S_i = C_1 C_2 \Delta p \sum_{i=1}^{n} \Delta H_i$$

where

$$\Delta H_i = H_c \left(\frac{I_z}{XE_s} \right)$$

S_i = estimate of the immediate settlement of spread footing

I_z = strain influencer factor

H_c = thickness of layer

n = number of soil layers within zone of strain influence

Δp = net applied stress at foundation depth

E_s = elastic modulus of soil layer

X = factor for shape of footing

$X = 1.25$ for $L_f/B_f = 1$ (axisymmetric)

$X = 1.75$ for $L_f/B_f \geq 10$ (plane strain)

Interpolate for $1 < L_f/B_f < 10$

C_1 = correction factor for strain relief due to embedment

$$C_1 = 1 - 0.5\left(\frac{p_o}{\Delta p} \right) \geq 0.5$$

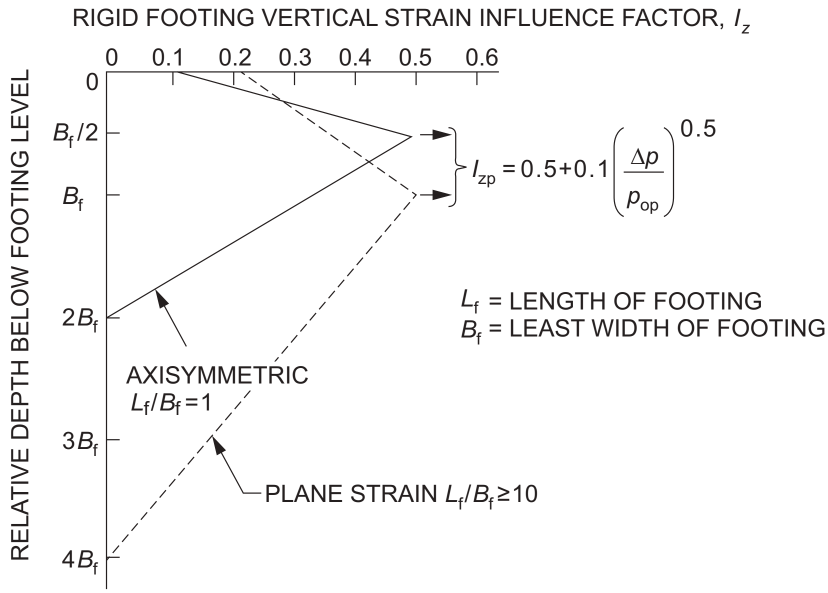
where

p_o = effective in situ overburden stress at the foundation depth

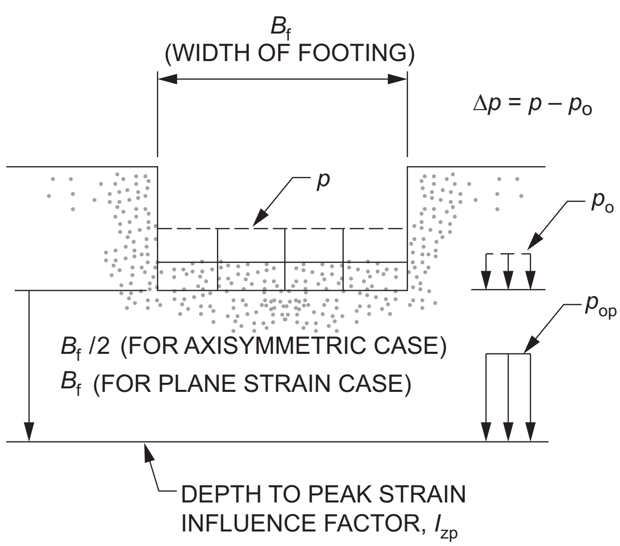
C_2 = correction factor to incorporate time dependent (creep) increase in settlement

$$C_2 = 1 + 0.2 \log_{10}\left(\frac{t}{0.1} \right)$$

t = time (years)



Simplified Vertical Strain Influence Factor Distributions

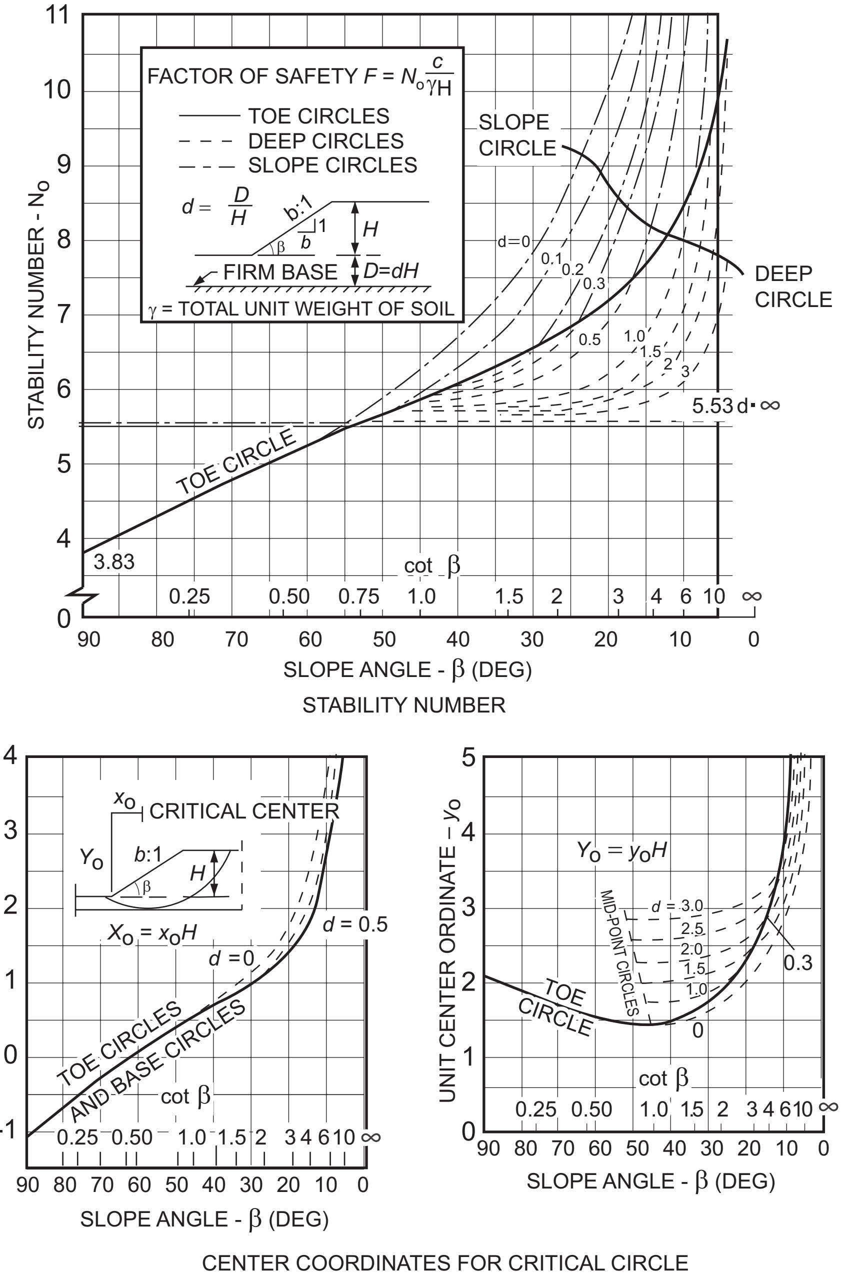


Explanation of Pressure Terms in Equation for I_{zp} (after Schmertmann *et alia*, 1978)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. II. FHWA-NHI-06-089. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 8-21, p. 8-46. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06089.pdf.

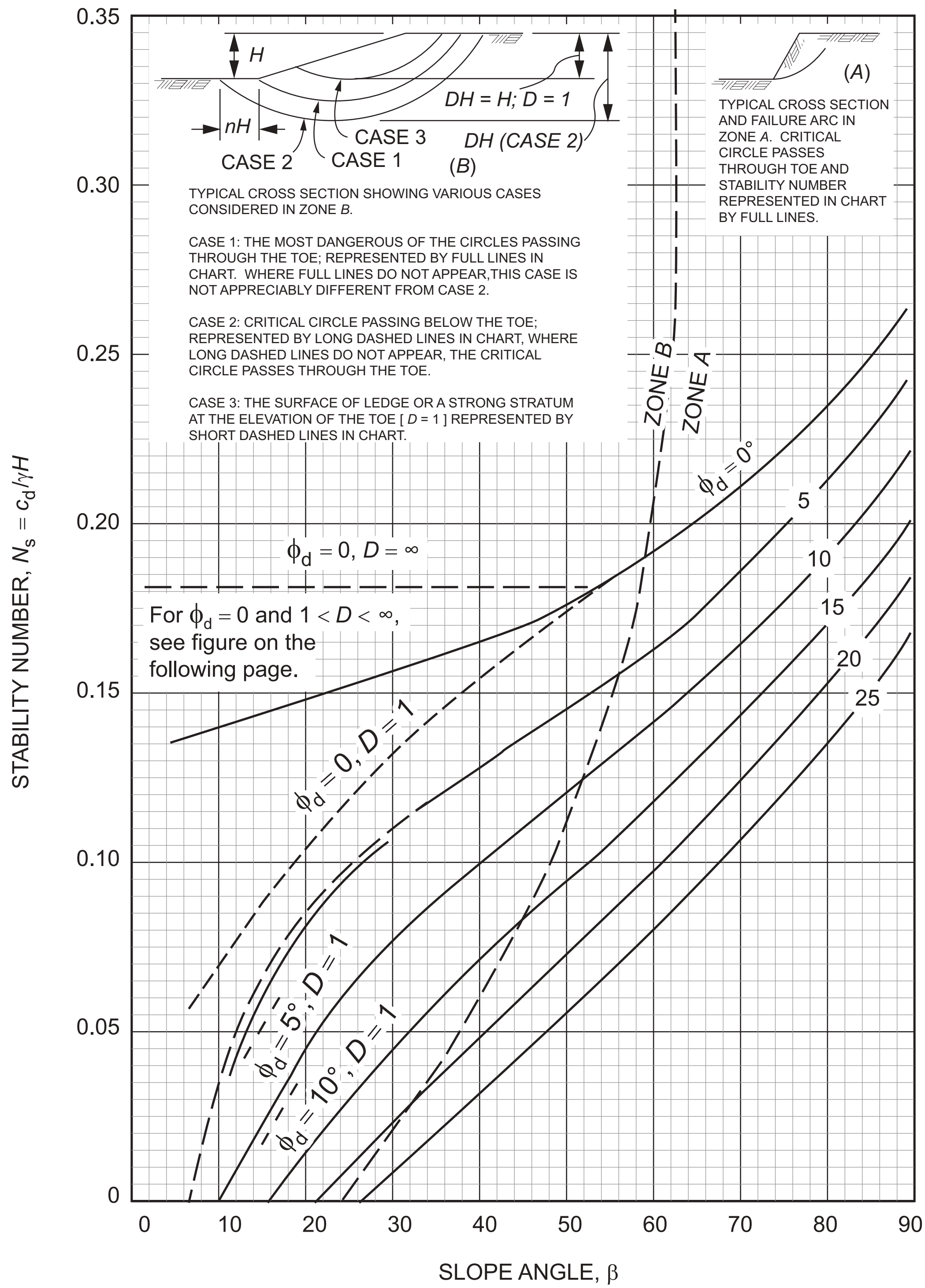
3.6 Slope Stability

3.6.1 Stability Charts



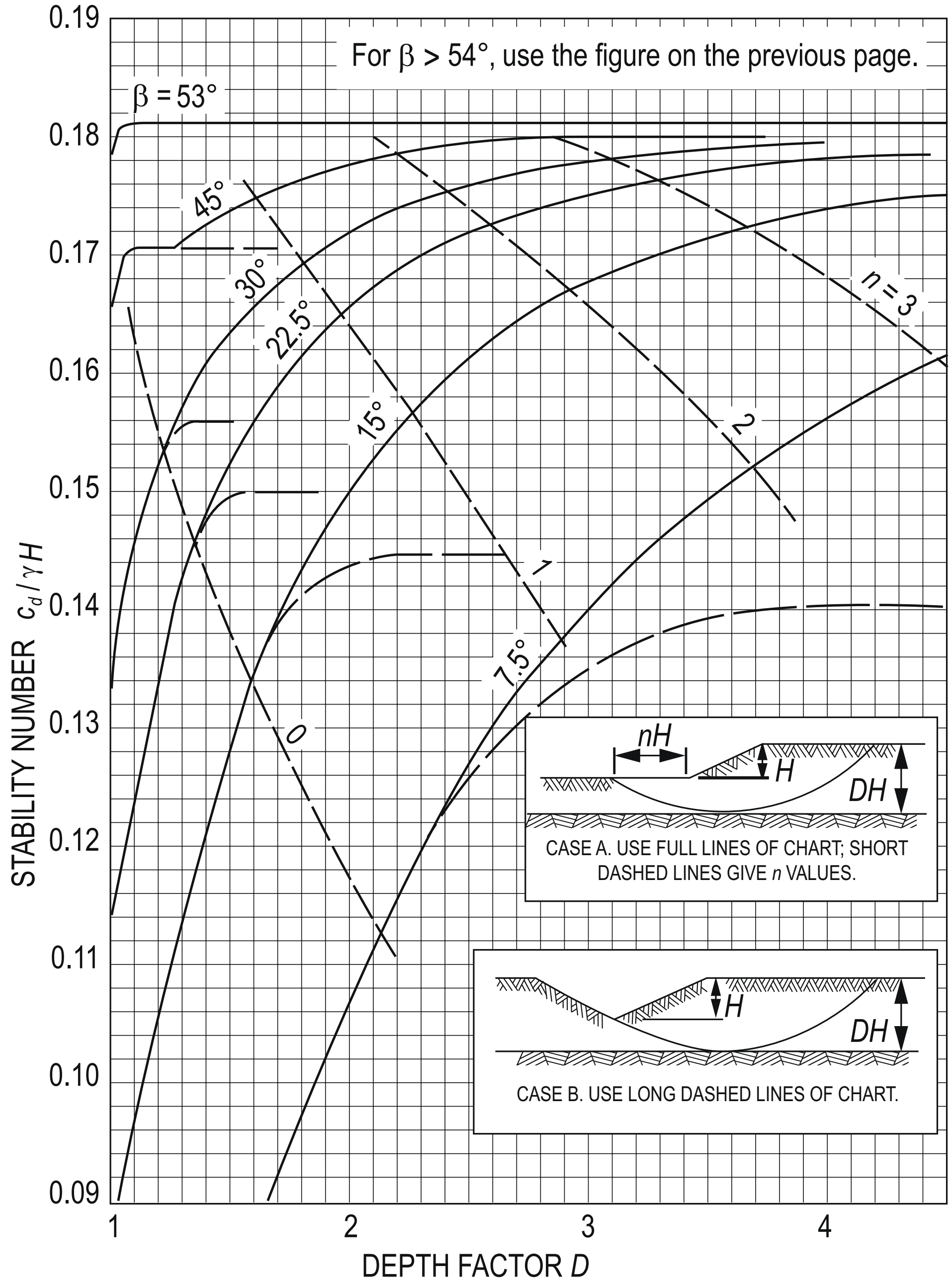
Stability Charts for $\phi = 0$ Soils (Janbu, 1968)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 6-16, p. 6-34. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Taylor's Chart for Soils with Friction Angle (after Taylor, 1948)

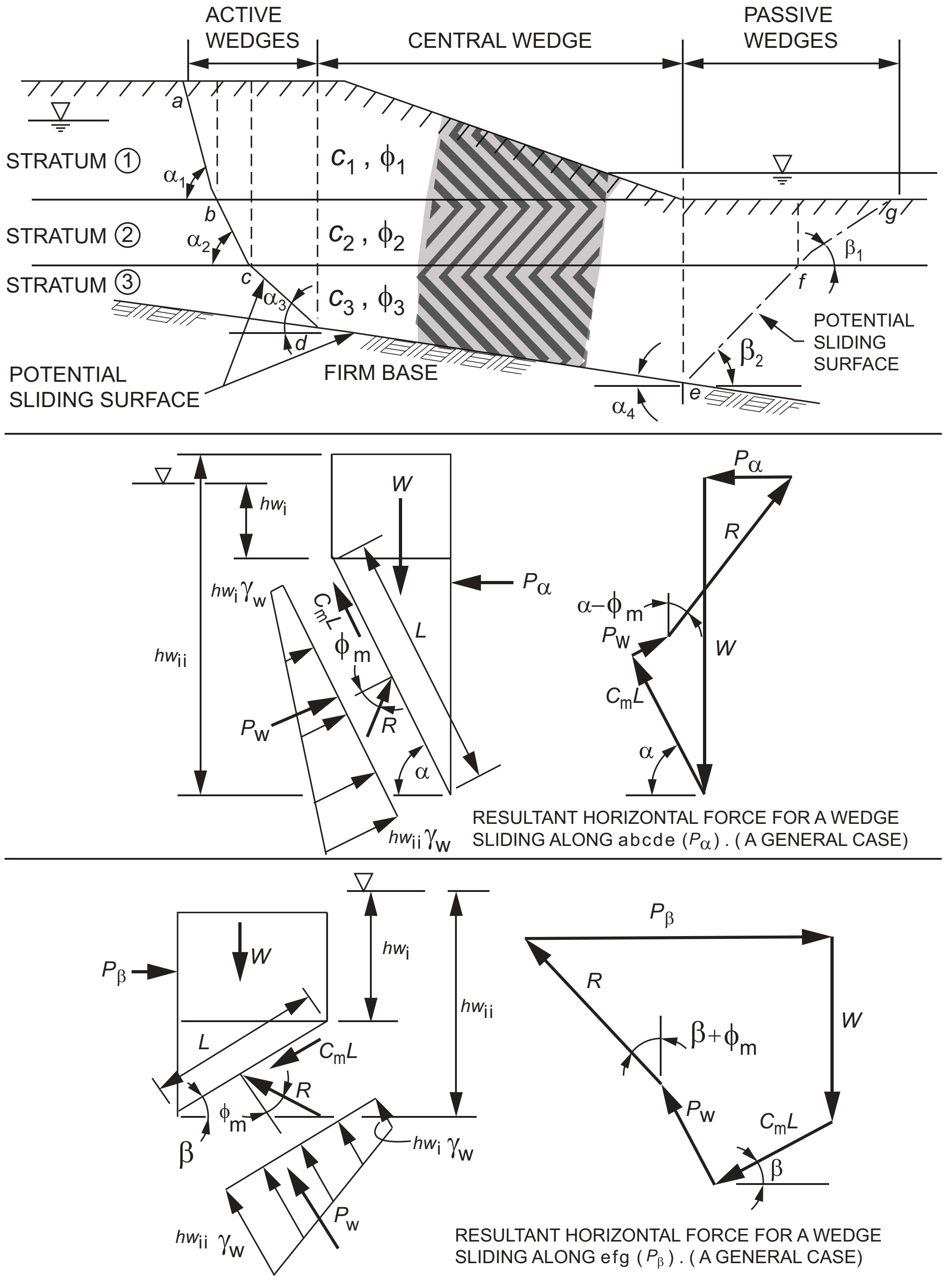
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 6-14, p. 6-27. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Taylor's Chart for ϕ′ = 0 Conditions for Slope Angles (β) less than 54° (after Taylor, 1948)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 6-15, p. 6-28. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

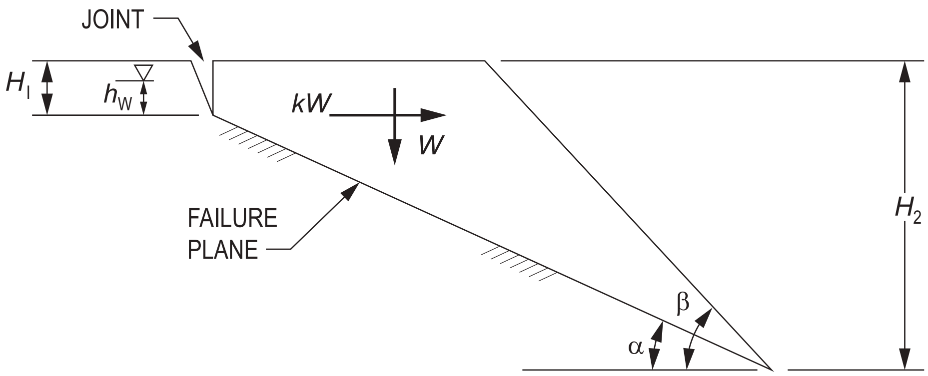
3.6.2 Translational Failure

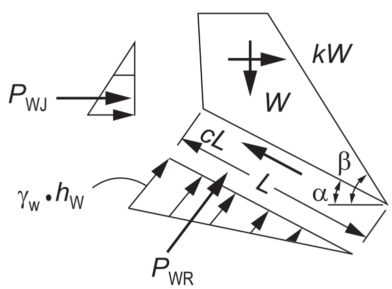


Stability Analysis of Transitional Failure

Source: Unified Facilities Criteria (UFC). *Soil Mechanics*. UFC 3-220-10N. Washington, DC: U.S. Department of Defense, June 2005, Fig. 6, p. 323. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_10n_2005.pdf.

3.6.3 Rock Slope Failure





W = WEIGHT OF WEDGE

P_{WJ} = WATER FORCE ON THE JOINT REACTION $= 1/2\ \gamma_w \cdot h_w^2$

P_{WR} = WATER FORCE ON THE ASSUMED FAILURE PLANE $= 1/2\ \gamma_w \cdot h_w \cdot L$

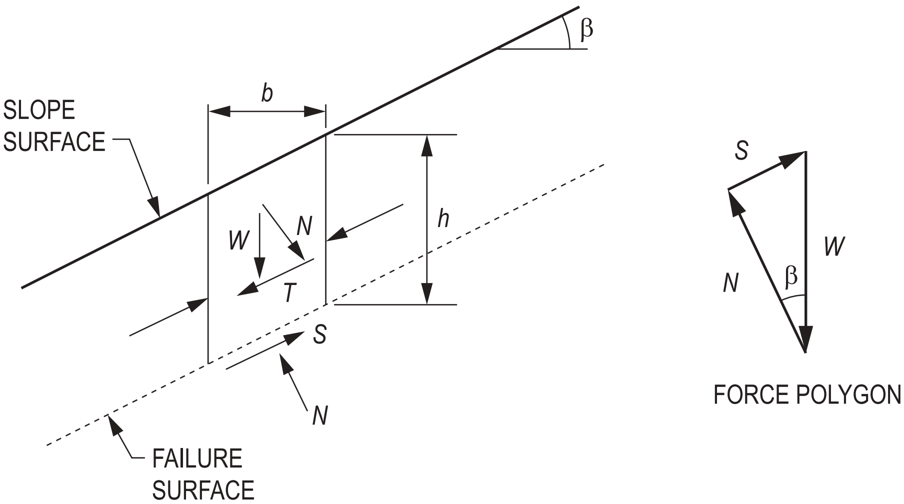
k = SEISMIC COEFFICIENT TO ACCOUNT FOR DYNAMIC HORIZONTAL FORCE

$$F_s = \frac{cL + \left(W\cos\alpha - kW\sin\alpha - P_{WR} - P_{WJ}\sin\alpha\right)\tan\phi}{W\sin\alpha + kW\cos\alpha + P_{WJ}\cos\alpha}$$

Stability of Rock Slope

Source: Unified Facilities Criteria (UFC). *Soil Mechanics*. UFC 3-220-10N. Washington, DC: U.S. Department of Defense, June 2005, Fig. 8, p. 328. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_10n_2005.pdf.

3.6.4 Infinite Slope



Infinite Slope Failure in Dry Sand

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 6-3, p. 6-6. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

γ = unit weight of soil

β = horizontal angle of slope

h = depth to slip surface

ϕ' = effective angle of friction

N = normal stress on shear surface

S = shear stress on slip surface

u = pore water pressure on slip surface

r_u = pore pressure coefficient = $\dfrac{u}{\gamma z}$

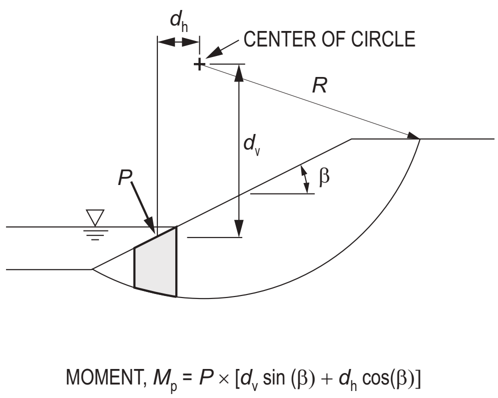
Factor of Safety (F) with pore water pressure:

$$F = \frac{\tan\phi'}{\tan\beta}\left[1 - r_u\left(1 + \tan^2\beta\right)\right]$$

Factor of Safety (F) with no pore water pressure ($r_u = 0$):

$$F = \frac{\tan\phi'}{\tan\beta}$$

3.6.5 Ordinary Method of Slices



Slice for Ordinary Method of Slices with External Water Loads

where

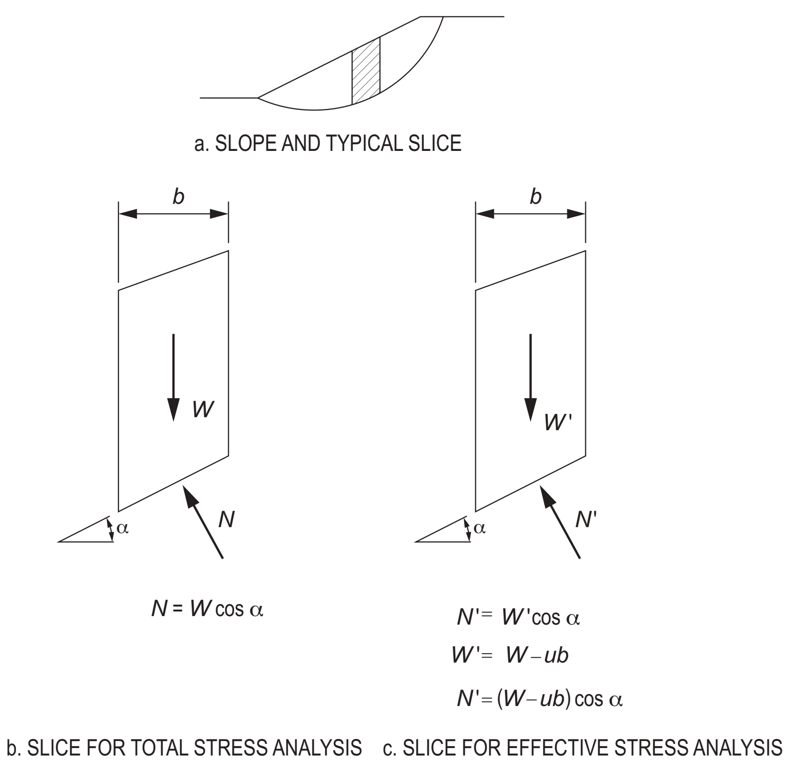
P = resultant water force acting perpendicular to top of slice

β = inclination of top of slice

M_p = moment about center of circle produced by water force acting on top of slice

R = radius of circle

Source: U.S. Army Corps of Engineers. *Engineering and Design: Slope Stability*. EM 1110-2-1902. Washington, DC: U.S. Department of the Army, October 2003, Fig. C-10, p. C-15. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1902.pdf.



Typical Slice and Forces for Ordinary Method of Slices

Source: U.S. Army Corps of Engineers. *Engineering and Design: Slope Stability*. EM 1110-2-1902. Washington, DC: U.S. Department of the Army, October 2003, Fig. C-9, p. C-13. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1902.pdf.

3.6.6 Slope Stability Guidelines

Slope Stability Guidelines for Design

Foundation Soil Type	Type of Analysis	Source of Strength Parameters	Remarks (see Note)
Cohesive	**Short-term** (embankments on soft clays—immediate end of construction: –ϕ = 0 analysis)	• UU or field vane shear test or CU triaxial test. • Use undrained strength parameters at p_o.	Use **Bishop Method**. An angle of internal friction should not be used to represent an increase of shear strength with depth. The clay profile should be divided into convenient layers and the appropriate cohesive shear strength assigned to each layer.
	Stage construction (embankments on soft clays—build embankment in stages with waiting periods to take advantage of clay-strength gain due to consolidation)	• CU triaxial test. Some samples should be consolidated to higher than existing in situ stress to determine clay-strength gain due to consolidation under staged fill heights. • Use undrained strength parameters at approximate p_o for staged height.	Use **Bishop Method** at each stage of embankment height. Consider that clay shear strength will increase with consolidation under each stage. Consolidation test data needed to estimate length of waiting periods between embankment stages. Piezometers and settlement devices should be used to monitor pore water pressure dissipation and consolidation during construction.
	Long-term (embankment on soft clays and clay cut slopes)	• CU triaxial test with pore water pressure measurements or CD triaxial test. • Use effective strength parameters.	Use **Bishop Method** with combination of cohesion and angle of internal friction (effective strength parameters from laboratory testing).
	Existing failure planes	• Direct shear or direct simple shear test. Slow strain rate and large deflection needed. • Use residual strength parameters.	Use **Bishop, Janbu, or Spencer Method** to duplicate previous shear surface.
Granular	**All types**	• Obtain effective friction angle from charts of standard penetration resistance (SPT) versus friction angle or from direct shear tests.	Use **Bishop Method** with an effective stress analysis.

Note: Methods recommended represent minimum requirement. More rigorous methods such as Spencer's method should be used when a computer program has such capabilities.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 6-1, p. 6-21. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.7 Soil Classification and Boring Log Interpretation

3.7.1 Subsurface Exploration and Planning

Guidelines for Minimum Number of Exploration Points and Depth of Exploration (modified after FHWA, 2002a)

Application	Minimum Number of Exploration Points and Location of Exploration Points	Minimum Depth of Exploration
Retaining Walls	1. A minimum of one exploration point for each retaining wall. 2. For retaining walls more than 100 ft (30 m) in length, exploration points spaced every 100 to 200 ft (30 to 60 m) with locations alternating from in front of the wall to behind the wall. 3. For anchored walls, additional exploration points in the anchorage zone spaced at 100 to 200 ft (30 to 60 m). 4. For soil-nail walls, additional exploration points at a distance of 1.0 to 1.5 times the height of the wall behind the wall spaced at 100 to 200 ft (30 to 60 m).	1. Investigate to a depth below bottom of wall between 1 and 2 times the wall height or a minimum of 10 ft (3 m) into bedrock. 2. Exploration depth should be great enough to fully penetrate soft highly compressible soils (e.g., peat, organic silt, soft fine-grained soils) into competent material of suitable bearing capacity (e.g., stiff to hard cohesive soil, compact dense cohesionless soil, or bedrock).
Embankment Foundations	1. A minimum of one exploration point every 200 ft (60 m) (erratic conditions) to 400 ft (120 m) (uniform conditions) of embankment length along the centerline of the embankment. 2. At critical locations, (e.g., maximum embankment heights, maximum depths of soft strata) a minimum of three explorations points in the transverse direction to define the existing subsurface conditions for stability analyses. 3. For bridge approach embankments, at least one exploration point at abutment locations.	1. Exploration depth should be, at a minimum, equal to twice the embankment height unless a hard stratum is encountered above this depth. 2. If soft strata are encountered extending to a depth greater than twice the embankment height, the exploration depth should be great enough to fully penetrate the soft strata into competent material (e.g., stiff to hard cohesive soil, compact to dense cohesionless soil, or bedrock).
Cut Slopes	1. A minimum of one exploration point every 200 ft (60 m) (erratic conditions) to 400 ft (120 m) (uniform conditions) of slope length. 2. At critical locations (e.g., maximum cut depths, maximum depths of soft strata) a minimum of three explorations points in the transverse direction to define the existing subsurface conditions for stability analyses. 3. For cut slopes in rock, perform geologic mapping along the length of the cut slope.	1. Exploration depth should be, at a minimum, 15 ft (4.5 m) below the minimum elevation of the cut unless a hard stratum is encountered below the minimum elevation of the cut. 2. Exploration depth should be great enough to fully penetrate through soft strata into competent material (e.g., stiff to hard cohesive soil, compact to dense cohesionless soil, or bedrock). 3. In locations where the base of the cut is below ground-water level, increase depth of exploration as needed to determine the depth of underlying pervious strata.

Guidelines for Minimum Number of Exploration Points and Depth of Exploration (cont'd)

Application	Minimum Number of Exploration Points and Location of Exploration Points	Minimum Depth of Exploration
Shallow Foundations	1. For substructure (e.g., piers or abutments) widths less than or equal to 100 ft (30 m), a minimum of one exploration point per substructure. 2. For substructure widths greater than 100 ft (30 m), a minimum to two exploration points per substructure. 3. Additional exploration points should be provided if erratic subsurface conditions or sloping rock surfaces are encountered.	Depth of exploration should be: 1. Great enough to fully penetrate unsuitable foundation soils (e.g., peat, organic silt, soft fine-grained soils) into competent material of suitable bearing capacity (e.g., stiff to hard cohesive soil, compact to dense cohesionless soil or bedrock); and 2. **At least to a depth where stress increase due to estimated footing load is less than 10% of the applied stress at the base of the footing**; and 3. In terms of the width of the footing, at least 2 times for axisymmetric case and 4 times for strip footing (interpolate for intermediate cases); and 4. If bedrock is encountered before the depth required by item 2 above is achieved, exploration depth should be great enough to penetrate a minimum of 10 ft (3 m) into the bedrock, but rock exploration should be sufficient to characterize compressibility of infill material of near-horizontal to horizontal discontinuities.
Deep Foundations	1. For substructures (e.g., bridge piers or abutments) widths less than or equal to 100 ft (30 m), a minimum of one exploration point per substructure. 2. For substructure widths greater than 100 ft (30 m), a minimum of two exploration points per substructure. 3. Additional exploration points should be provided if erratic subsurface conditions are encountered. 4. Due to large expense associated with construction of rock-socketed shafts, conditions should be confirmed at each shaft location.	1. In soil, depth of exploration should extend below the anticipated pile or shaft tip elevation a minimum of 20 ft (6 m), or a minimum of two times the maximum pile group dimensions, whichever is deeper. All borings should extend through unsuitable strata such as unconsolidated fill, peat, highly organic materials, soft fine-grained soils, and loose coarse-grained soils to reach hard to dense materials. 2. For piles bearing on rock, a minimum of 10 ft (3 m) of rock core shall be obtained at each exploration point location to verify that the boring has not terminated on a boulder. 3. For shafts supported on or extending into rock, a minimum of 10 ft (3 m) of rock core, or a length of rock core equal to at least three times the shaft diameter for isolated shafts or two times the maximum shaft group dimension, whichever is greater, shall be extended below the anticipated shaft tip elevation to determine the physical characteristics of rock within the zone of foundation influence.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 3-13, p. 3-84 and 3-85. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.7.2 Unified Soil Classification System (USCS)

Soil Classification Chart (Laboratory Method; after ASTM D 2487)

Criteria for Assigning Group Symbols and Group Names Using Laboratory Tests[a]			**Soil Classification**	
COARSE-GRAINED SOILS (Sands and Gravels): More than 50% retained in No. 200 (0.075 mm) sieve **FINE-GRAINED (Silts and Clays):** 50% or more passes the No. 200 (0.075 mm) sieve			**Group Symbol**	**Group Name[b]**
GRAVELS More than 50% of coarse fraction retained in No. 4 sieve	CLEAN GRAVELS < 5% fines	$C_u \geq 4$ and $1 \leq C_c \leq 3$[e]	GW	Well-graded gravel[f]
		$C_u < 4$ and/or $1 > C_c > 3$[e]	GP	Poorly graded gravel[f]
	GRAVELS WITH FINES > 12% of fines[c]	Fines classify as ML or MH	GM	Silty gravel[f, g, h]
		Fines classify as CL or CH	GC	Clayey gravel[f, g, h]
SANDS 50% or more of coarse fraction passes No. 4 sieve	CLEAN SANDS < 5% fines[d]	$C_u \geq 6$ and $1 \leq C_c \leq 3$[e]	SW	Well-graded sand[i]
		$C_u < 6$ and/or $1 > C_c > 3$[e]	SP	Poorly graded sand[i]
	SANDS WITH FINES > 12% fines[d]	Fines classify as ML or MH	SM	Silty sand[g, h, i]
		Fines classify as CL or CH	SC	Clayey sand[g, h, i]
SILTS AND CLAYS Liquid limit less than 50	Inorganic	*PI* > 7 and plots on or above "A" line[j]	CL	Lean clay[k, l, m]
		PI < 4 or plots below "A" line[j]	ML	Silt[k, l, m]
	Organic	$\frac{\text{Liquid limit} - \text{oven dried}}{\text{Liquid limit} - \text{not dried}} < 0.75$	OL	Organic clay[k, l, m, n]
				Organic silt[k, l, m, o]
SILTS AND CLAYS Liquid limit 50 or more	Inorganic	*PI* plots on or above "A" line	CH	Fat clay[k, l, m]
		PI plots below "A" line	MH	Elastic silt[k, l, m]
	Organic	$\frac{\text{Liquid limit} - \text{oven dried}}{\text{Liquid limit} - \text{not dried}} < 0.75$	OH	Organic clay[k, l, m, p]
				Organic silt[k, l, m, q]
Highly fibrous organic soils	Primary organic matter, dark in color, and organic odor		PT	Peat

[a] Based on the material passing the 3-in. (75-mm) sieve.
[b] If field sample contained cobbles and/or boulders, add "with cobbles and/or boulders" to group name.
[c] Gravels with 5 to 12% fines require dual symbols:
- GW-GM, well-graded gravel with silt
- GW-GC, well-graded gravel with clay
- GP-GM, poorly graded gravel with silt
- GP-GC, poorly graded gravel with clay

[d] Sands with 5 to 12% fines require dual symbols:
- SW-SM, well-graded sand with silt
- SW-SC, well-graded sand with clay
- SP-SM, poorly graded sand with silt
- SP-SC, poorly graded sand with clay

[e] $C_u = \frac{D_{60}}{D_{10}}$ $\quad C_c = \frac{(D_{30})^2}{(D_{10})(D_{60})}$ (C_u: Uniformity Coefficient; C_c: Coefficient of Curvature)

[f] If soil contains ≥ 15% sand, add "with sand" to group name.
[g] If fines classify as CL-ML, use dual symbol GC-GM, SC-SM.
[h] If fines are organic, add "with organic fines" to group name.
[i] If soil contains ≥ 15% gravel, add "with gravel" to group name.
[j] If the liquid limit and plasticity index plot in hatched area on plasticity chart, soil is a CL-ML, silty clay.

[k] If soil contains 15 to 29% plus No. 200 (0.075 mm), add "with sand" or "with gravel, " whichever is predominant.
[l] If soil contains ≥ 30% plus No. 200 (0.075 mm), predominantly sand, add "sandy" to group name.
[m] If soil contains ≥ 30% plus No. 200 (0.075 mm), predominantly gravel, add "gravelly" to group name.
[n] $PI \geq 4$ and plots on or above "A" line.
[o] $PI < 4$ and plots below "A" line.
[p] *PI* plots on or above "A" line.
[q] *PI* plots below "A" line.

Source: Reproduced with permission from ASTM D2487. *Standard Practice for Classification of Soils for Engineering Purposes* (Unified Soil Classification System). Copyright ASTM International, www.astm.org. As found in *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, 2006. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

$$PI = LL - PL$$

$$LI = \frac{w - PL}{PI}$$

where

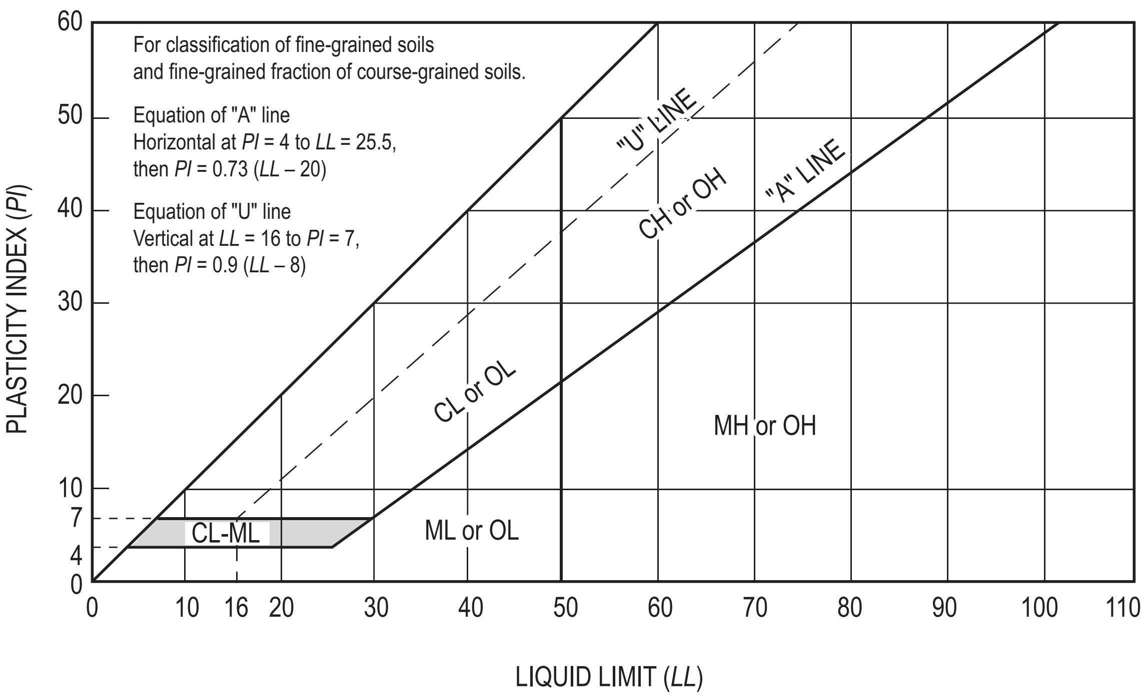
w = water content (%)

LL = Liquid Limit (%)

PL = Plastic Limit (%)

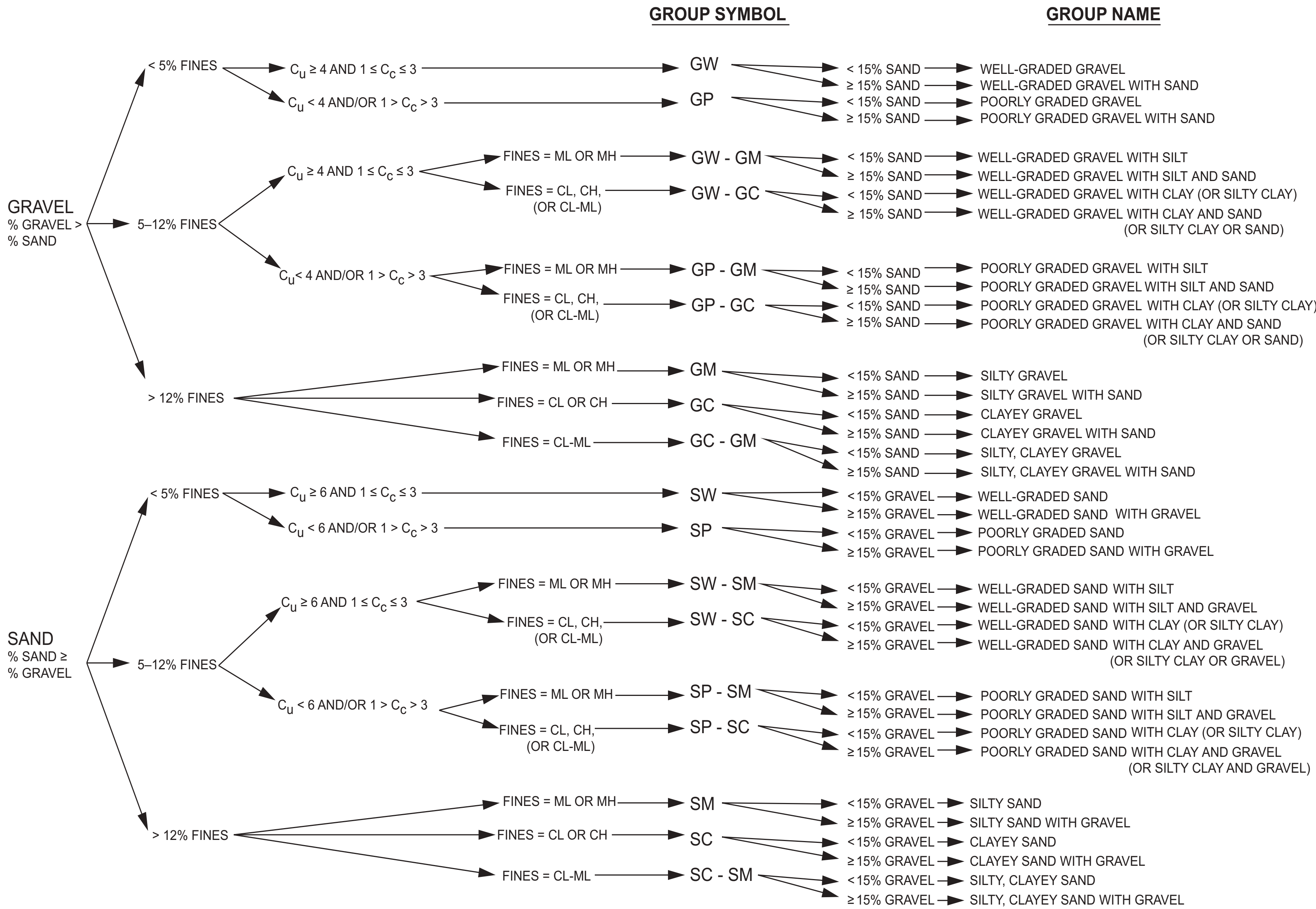
PI = Plasticity Index (%)

LI = Liquidity Index (%)



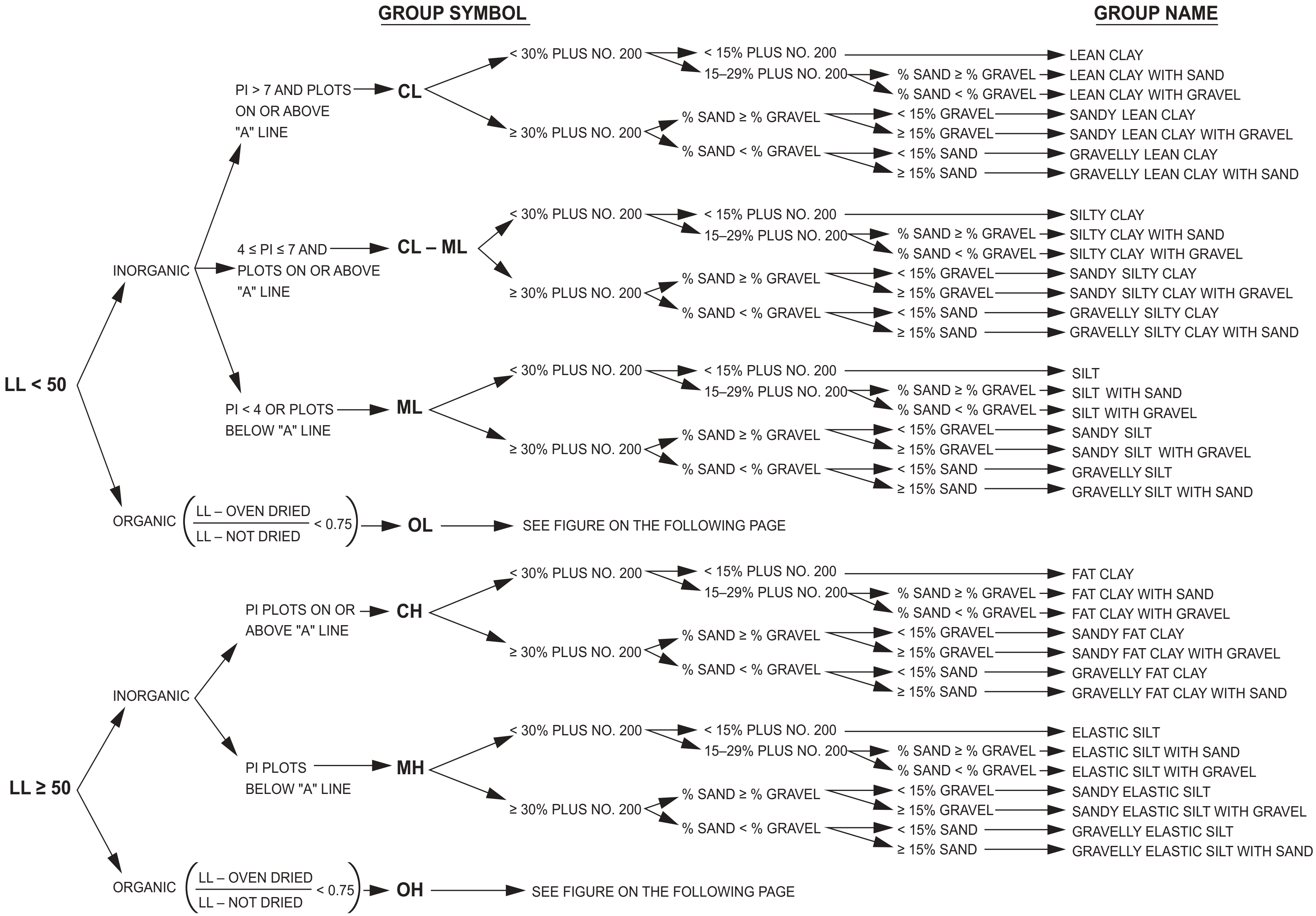
Plasticity Chart for Unified Soil Classification System (ASTM D 2487)

Source: Reproduced with permission from ASTM D2487. *Standard Practice for Classification of Soils for Engineering Purposes* (Unified Soil Classification System). Copyright ASTM International, www.astm.org. As found in *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, 2006. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



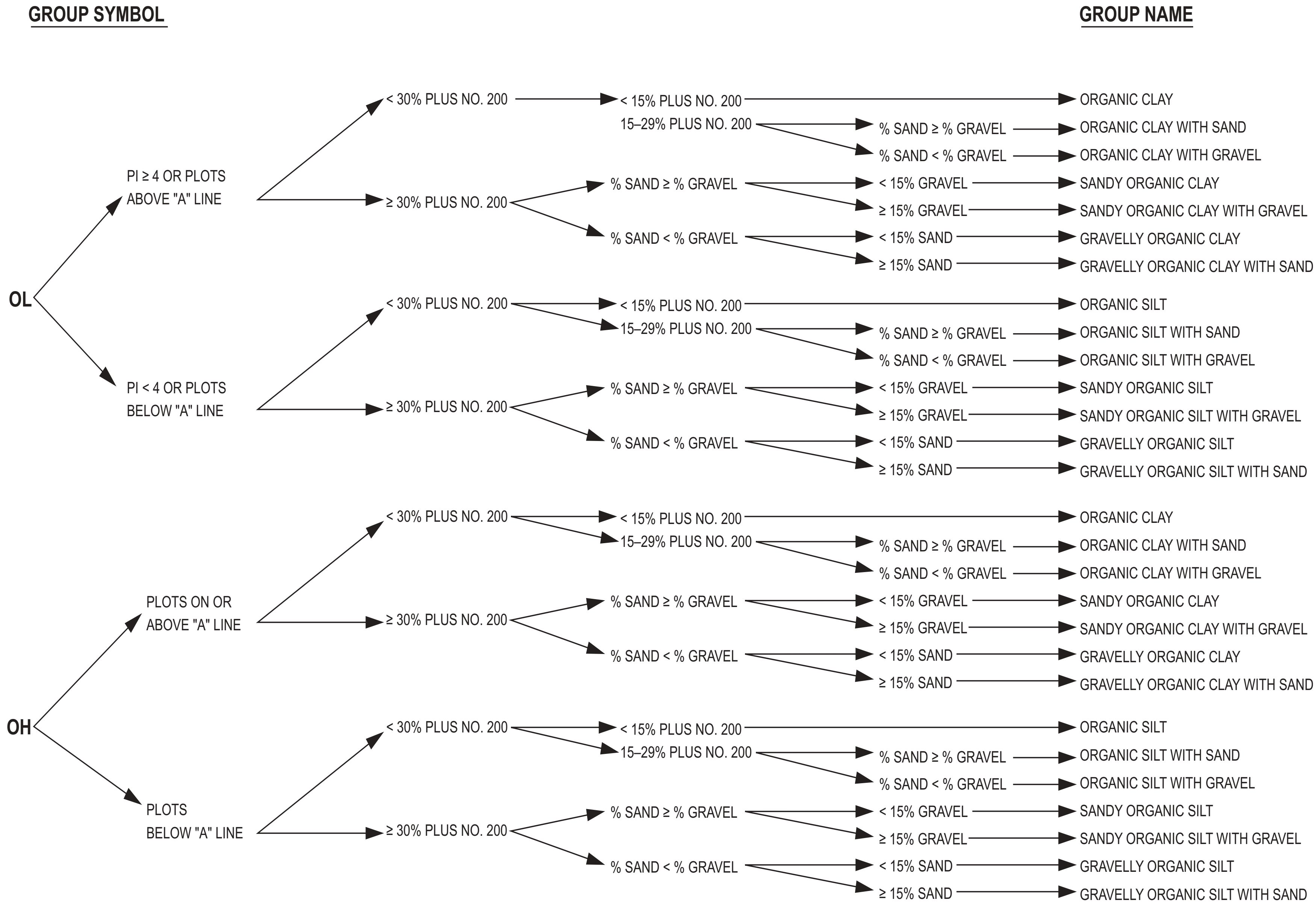
Flow Chart to Determine the Group Symbol and Group Name for Coarse-Grained Soils (ASTM D 2487)

Source: Reproduced with permission from ASTM D2487. *Standard Practice for Classification of Soils for Engineering Purposes* (Unified Soil Classification System). Copyright ASTM International, www.astm.org. As found in *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, 2006. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Flow Chart to Determine the Group Symbol and Group Name for Fine-Grained Soils (ASTM D 2487)

Source: Reproduced with permission from ASTM D2487. *Standard Practice for Classification of Soils for Engineering Purposes* (Unified Soil Classification System). Copyright ASTM International, www.astm.org. As found in *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, 2006. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Flow Chart to Determine the Group Symbol and Group Name for Organic Soils (ASTM D 2487)

Source: Reproduced with permission from ASTM D2487. *Standard Practice for Classification of Soils for Engineering Purposes* (Unified Soil Classification System). Copyright ASTM International, www.astm.org. As found in *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, 2006. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Adjectives to Describe Water Content of Soils (ASTM D 2488)

Description	Conditions
Dry	No sign of water and soil dry to touch
Moist	Signs of water and soil is relatively dry to touch
Wet	Signs of water and soil definitely wet to touch; granular soil exhibits some free water when densified

Source: Reproduced with permission from ASTM D2488. *Standard Practice for Description and Identification of Soils* (Visual-Manual Procedures). Copyright ASTM International, www.astm.org. As found in *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, 2006. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Particle Size Definition for Gravels and Sands (after ASTM D 2488)

Component	Grain Size	Determination
Boulders*	12"+ (300 mm+)	Measurable
Cobbles*	3"–12" (300 mm–75 mm)	Measurable
Gravel:		
Coarse	3/4"–3" (19 mm–75 mm)	Measurable
Fine	3/4"–#4 sieve (3/4"–0.187") (19 mm–4.75 mm)	Measurable
Sand:		
Coarse	#4–#10 sieve (0.19"–0.079") (4.75 mm–2.00 mm)	Measurable and visible to the eye
Medium	#10–#40 sieve (0.079"–0.017") (2.00 mm–0.425 mm)	Measurable and visible to the eye
Fine	#40–#200 sieve (0.017"–0.003") (0.425 mm–0.075 mm)	Measurable but barely discernible to the eye

*Boulders and cobbles are not considered soil or part of the soils classification or description, except under miscellaneous description, i.e., with cobbles at about 5 percent (volume).

Source: Reproduced with permission from ASTM D2488. *Standard Practice for Description and Identification of Soils* (Visual-Manual Procedures). Copyright ASTM International, www.astm.org. As found in *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, 2006. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Adjectives for Describing Size Distribution for Sands and Gravels (after ASTM D 2488)

Particle-Size Adjective	Abbreviation	Size Requirement
Coarse	c.	< 30% m–f sand or < 12% f. gravel
Coarse to medium	c–m	< 12% f. sand
Medium to fine	m–f	< 12% c. sand and > 30% m. sand
Fine	f.	< 30% m. sand or < 12% c. gravel
Coarse to fine	c–f	> 12% of each size[1]

[1] 12% and 30% criteria can be modified depending on fines content. The key is the shape of the particle-size distribution curve. If the curve is relatively straight or dished down, and coarse sand is present, use c-f; also use m-f sand if a moderate amount of m. sand is present. If one has any doubts, determine the above percentages base on the amount of sand or gravel present.

Source: Reproduced with permission from ASTM D2488. *Standard Practice for Description and Identification of Soils* (Visual-Manual Procedures). Copyright ASTM International, www.astm.org. As found in *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, 2006. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Field Methods to Describe Plasticity (FHWA, 2002b)

Plasticity Range	Adjective	Dry Strength	Smear Test	Thread Smallest Diameter
0	Nonplastic	None—crumbles into powder with mere pressure	Gritty or rough	ball cracks
1–10	Low plasticity	Low—crumbles into powder with some finger pressure	Rough to smooth	1/4"–1/8" (6 mm–3 mm)
> 10–20	Medium plasticity	Medium—breaks into pieces or crumbles with considerable finger pressure	Smooth and dull	1/16" (1.5 mm)
> 20–40	High plasticity	High—cannot be broken with finger pressure; specimen will break into pieces between thumb and a hard surface	Shiny	0.03" (0.75 mm)
> 40	Very plastic	Very high—cannot be broken between thumb and a hard surface	Very shiny and waxy	0.02" (0.5 mm)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-6, p. 4-10. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Descriptive Terms for Layered Soils (NAVFAC, 1986a)

Type of Layer	Thickness	Occurrence
Parting	< 1/16" (< 1.5 mm)	
Seam	1/16"–1/2" (1.5 mm–12 mm)	
Layer	1/2"–12" (12 mm–300 mm)	
Stratum	> 12" (> 300 mm)	
Pocket		Small erratic deposit
Lens		Lenticular deposit
Varved (also Layered)		Alternating seams or layers of silt and/or clay and sometimes fine sand
Occasional		One or less per 12" (300 mm) of thickness or laboratory sample inspected
Frequent		More than one per 12" (300 mm) of thickness or laboratory sample inspected

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-7, p. 4-13. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.7.3 AASHTO Classification System

AASHTO Soil Classification System Based on AASHTO M 145 (or ASTM D 3282)

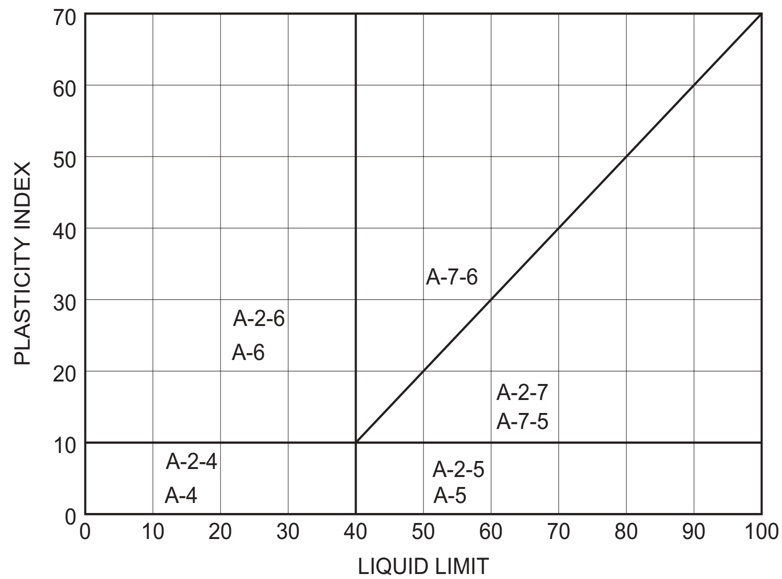
General Classification	**Granular Materials [35 percent or less of total sample passing No. 200 sieve (0.075 mm)]**							**Silt-Clay Materials [More than 35 percent of total sample passing No. 200 sieve (0.075 mm)]**			
Group Classification	A-1		A-3	A-2				A-4	A-5	A-6	A-7
	A-1-a	A-1-b		A-2-4	A-2-5	A-2-6	A-2-7				A-7-5, A-7-6
Sieve Analysis Percent Passing:											
No. 10 (2 mm)	50 max.										
No. 40 (0.425 mm)	30 max.	50 max.	51 min.								
No. 200 (0.075 mm)	15 max.	25 max.	10 max.	35 max.	35 max.	35 max.	35 max.	36 min.	36 min.	36 min.	36 min.
Characteristics of Fraction Passing No. 40 (0.425 mm):											
Liquid Limit				40 max.	41 min.	40 max.	41 min.	40 max.	41 min.	40 max.	41 min.
Plasticity Index	6 max.		NP	10 max.	10 max.	11 min.	11 min.	10 max.	10 max.	11 min.	11 min.*
Usual Significant Constituent Materials	Stone fragments, gravel and sand		Fine sand	Silty or clayey gravel and sand				Silty soils		Clayey soils	
Group Index**	0		0	0		4 max.		8 max.	12 max.	16 max.	20 max.

Classification procedure: With required test data available, proceed from left to right on chart; correct group will be found by process of elimination. The first group from left into which the test data will fit is the correct classification.

*Plasticity Index of A-7-5 subgroup is equal to less than *LL* minus 30. Plasticity Index of A-7-6 subgroup is greater than *LL* minus 30 (see AASHTO Plasticity Chart on the following page).

**See Group Index formula. Group Index should be shown in parentheses after Group Classification as A-2-4(0), A-5(11), etc.

Source: Based on information from M 145 *Standard Specification for Classification of Soils and Soil–Aggregate Mixtures for Highway Construction Purposes*, 1991, by the American Association of State Highway and Transportation Officials, Washington, DC.



AASHTO Plasticity Chart

Source: Based on information from M 145 *Standard Specification for Classification of Soils and Soil–Aggregate Mixtures for Highway Construction Purposes*, 1991, by the American Association of State Highway and Transportation Officials, Washington, DC.

Group Index

To evaluate the quality of a soil as a highway subgrade material, a number called the *group index* (*GI*) is also incorporated along with the groups and subgroups of the soil. The group index is written in parenthesis after the group or subgroup designation. The group index is given by the following equation:

$$GI = (F - 35)\,[0.2 + 0.005(LL - 40)] + 0.01(F - 15)(PI - 10)$$

where

GI = group index

F = percent passing the No. 200 sieve (0.075 mm)

LL = liquid limit (%)

PI = plasticity index (%)

When calculating the group index for soils that belong to groups A-2-6 and A-2-7, use the partial group index for *PI*, or

$$GI = 0.01\,(F - 15)\,(PI - 10)$$

Source: Based on information from M 145 *Standard Specification for Classification of Soils and Soil–Aggregate Mixtures for Highway Construction Purposes*, 1991, by the American Association of State Highway and Transportation Officials, Washington, DC

3.7.4 Rock Classification

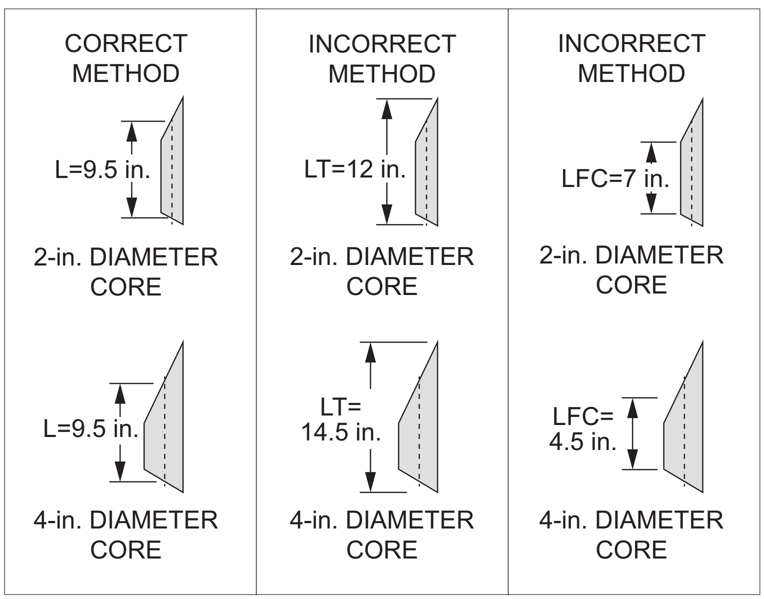
3.7.4.1 Rock Quality Designation (RQD)

$$RQD = \frac{\sum \text{Length of Sound Core Pieces} > 4 \text{ in.}}{\text{Total Core Run Length}}$$

Rock Quality Description

RQD **(Rock Quality Designation)**	**Description of Rock Quality**
0–25%	Very Poor
25–50%	Poor
50–75%	Fair
75–90%	Good
90–100%	Excellent

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 3-17, p. 3-45. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Centerline
A. Correct method for core length measurement; length of core independent of core diameter

Tip to Tip
B. Incorrect method for core length measurement; length of core dependent on core diameter

Fully Circular
C. Incorrect method for core length measurement; length of core dependent on core diameter

Length Measurements for Core *RQD* Determination (FHWA, 1997)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 3-18, p. 3-46. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Rock Groups and Types (FHWA, 1997)

Igneous		
Intrusive (Coarse Grained)	**Extrusive (Fine Grained)**	**Pyroclastic**
Granite Syenite Diorite Diabase Gabbro Peridotite Pegmatite	Rhyolite Trachyte Andesite Basalt	Obsidian Pumice Tuff
Sedimentary		
Clastic (Sediment)	**Chemically Formed**	**Organic Remains**
Shale Mudstone Claystone Siltstone Sandstone Conglomerate Limestone, oolitic	Limestone Dolomite Gypsum Halite	Chalk Coquina Lignite Coal
Metamorphic		
Foliated	**Nonfoliated**	
Slate Phyllite Schist Gneiss	Quartzite Amphibolite Marble Hornfels	

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-16, p. 4-35. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.7.4.2 Rock Descriptions

Terms to Describe Grain Size (typically for sedimentary rocks)

Description	Grain Size (mm)	Characteristic of Individual Grains
Very coarse grained	#4 (> 4.75)	Can be easily distinguished by eye
Coarse grained	#10 to #4 (2.00–4.75)	Can be easily distinguished by eye
Medium grained	#40 to #10 (0.425–2.00)	Can be distinguished by eye
Fine grained	#200 to #40 (0.075–0.425)	Can be distinguished by eye with difficulty
Very fine grained	< #200 (< 0.075)	Cannot be distinguished by unaided eye

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-17, p. 4-36. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Terms to Describe Grain Shape (for sedimentary rocks)

Description	Characteristic
Angular	Showing very little evidence of wear. Grain edges and corners are sharp. Secondary corners are numerous and sharp.
Subangular	Showing some evidence of wear. Grain edges and corners are slightly rounded off. Secondary corners are slightly less numerous and slightly less sharp than in angular grains.
Subrounded	Showing considerable wear. Grain edges and corners are rounded to smooth curves. Secondary corners are reduced greatly in number and highly rounded.
Rounded	Showing extreme wear. Grain edges and corners are smoothed off to broad curves. Secondary corners are few in number and rounded.
Well-rounded	Completely worn. Grain edges or corners are not present. No secondary edges or corners are present.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-18, p. 4-36. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Terms to Describe Stratum Thickness

Descriptive Term	Stratum Thickness (m or mm)*
Very thickly bedded	(> 1 m)
Thickly bedded	(0.5–1.0 m)
Thinly bedded	(50 mm–500 mm)
Very thinly bedded	(10 mm–50 mm)
Laminated	(2.5 mm–10 mm)
Thinly laminated	(< 2.5 mm)

* Conventionally measured in m or mm
(1 m = 3.28 ft; 25.4 mm = 1 in.)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-19, p. 4-37. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Terms to Describe Rock Weathering and Alteration (ISRM, 1981)

Grade (Term)	Description
I: Fresh	Rock shows no discoloration, loss of strength, or other effects of weathering/alteration
II: Slightly Weathered/ Altered	Rock is slightly discolored, but not noticeably lower in strength than fresh rock
III: Moderately Weathered/Altered	Rock is discolored and noticeably weakened, but less than half is decomposed; a minimum 2-in. (50-mm)-diameter sample cannot be broken readily by hand across the rock fabric
IV: Highly Weathered/ Altered	More than half of the rock is decomposed; rock is weathered so that a minimum 2-in. (50-mm)-diameter sample can be broken readily by hand across the rock fabric
V: Completely Weathered/Altered	Original minerals of rock have been almost entirely decomposed to secondary minerals even though the original fabric may be intact; material can be granulated by hand
VI: Residual Soil	Original minerals of rock have been entirely decomposed to secondary minerals, and original rock fabric is not apparent; material can be easily broken by hand

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-20, p. 4-38. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Terms to Describe Rock Strength (ISRM, 1981)

Grade (Description)	Field Identification	Approximate Range of Uniaxial Compressive Strength, psi (kPa)
R0: Extremely Weak Rock	Can be indented by thumbnail	35–150 (250–1,000)
R1: Very Weak Rock	Can be peeled by pocketknife	150–725 (1,000–5,000)
R2: Weak Rock	Can be peeled with difficulty by pocket-knife	725–3,500 (5,000–25,000)
R3: Medium-Strong Rock	Can be indented 3/16 in. (5 mm) with sharp end of pick	3,500–7,000 (25,000–50,000)
R4: Strong Rock	Requires one blow of geologist's hammer to fracture	7,000–15,000 (50,000–100,000)
R5: Very Strong Rock	Requires many blows of a geologist's hammer to fracture	15,000–36,000 (100,000–250,000)
R6: Extremely Strong Rock	Can only be chipped with blows of a geologist's hammer	> 36,000 (> 250,000)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-21, p. 4-38. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.8 Material Test Methods

3.8.1 In Situ Testing

3.8.1.1 Hammer Efficiency

$$N_{60} = \left(\frac{E_{\text{eff}}}{60}\right) N_{\text{meas}}$$

where

N_{60} = adjusted N-value for 60% efficiency

E_{eff} = efficiency of hammer, as measured

N_{meas} = actual N-values during standard penetration test (SPT)

3.8.1.2 Depth Adjustment

$$(N_1)_{60} = N_{60} C_N$$

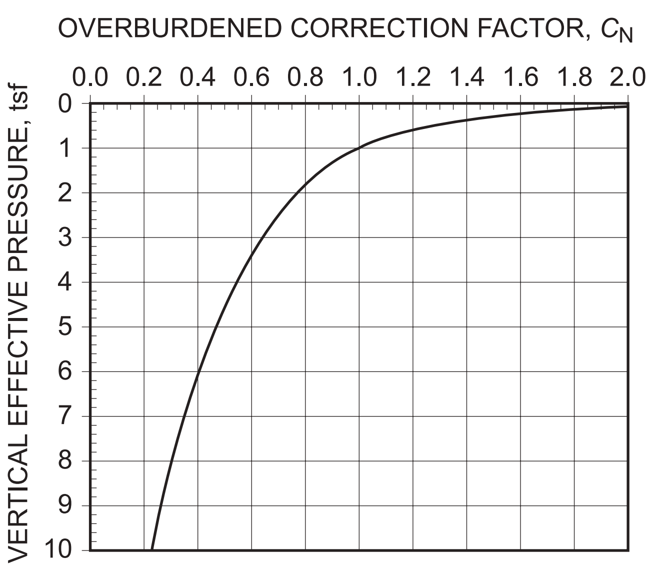
$$C_N = \left[0.77 \log_{10}\left(\frac{20}{P_o}\right)\right]$$

where

P_o = vertical effective pressure at depth where SPT is performed (tsf)

C_N = overburden correction factor

$(N_1)_{60}$ = N_{60} adjusted for overburden depth



Variation of Overburden Correction Factor, C_N, as a Function of Vertical Effective Stress

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 3-24, p. 3-57. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.8.1.3 SPT Correlations

Soil Properties Correlated with Standard Penetration Test Values (after Peck *et alia*, 1974)

Sands (Reliable)	
N_{60}	Relative Density
0–4	Very loose
5–10	Loose
11–30	Medium dense
31–50	Dense
Over 50	Very dense

Silts and Clays (Unreliable)	
N_{60}	Consistency
Below 2	Very soft
2–4	Soft
5–8	Medium
9–15	Stiff
16–30	Very stiff
Over 30	Hard

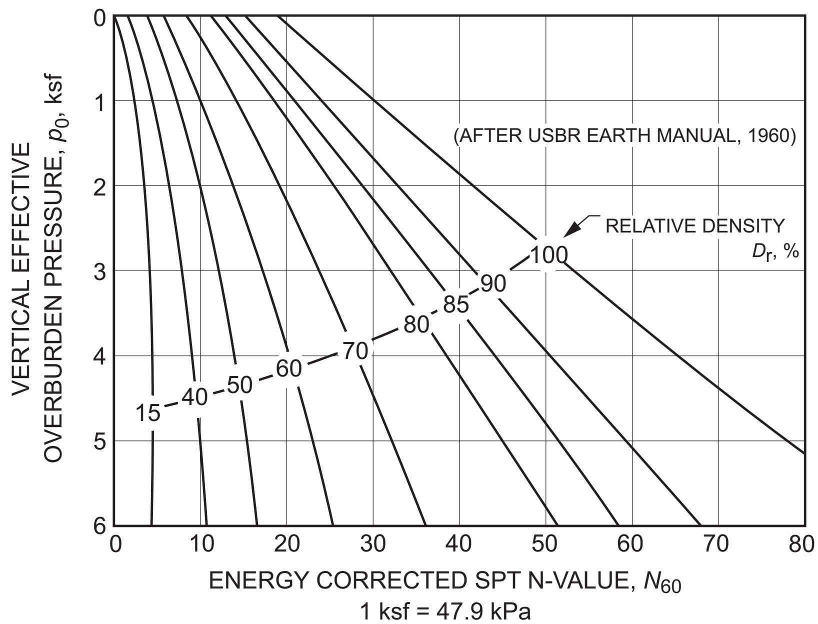
Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 3-9, p. 3-58. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Evaluation of the Consistency of Fine-Grained Soils (after Peck *et alia*, 1974)

N_{60}	Consistency	Unconfined Compressive Strength, q_u, ksf (kPa)	Results of Manual Manipulation
< 2	Very soft	< 0.5 (< 25)	Specimen (height = twice the diameter) sags under its own weight; extrudes between fingers when squeezed.
2–4	Soft	0.5–1 (25–50)	Specimen can be pinched in two between the thumb and forefinger; remolded by light finger pressure.
4–8	Medium stiff	1–2 (50–100)	Can be imprinted easily with fingers; remolded by strong finger pressure.
8–15	Stiff	2–4 (100–200)	Can be imprinted with considerable pressure from fingers or indented by thumbnail.
15–30	Very stiff	4–8 (200–400)	Can barely be imprinted by pressure from fingers or indented by thumbnail.
> 30	Hard	> 8 (> 400)	Cannot be imprinted by fingers or difficult to indent by thumbnail.

Note: N_{60} values should **not** be used to determine the design strength of fine-grained soils.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-2, p. 4-5. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Correlation Between Relative Density and SPT Resistance (NAVFAC, 1986a)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-23, p. 5-57. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Commonly Performed Laboratory Tests on Soils (after FHWA, 2002a)

Test Category	Name of Test	Test Designation	
		AASHTO	ASTM
Visual Identification	Practice for Description and Identification of Soils (Visual-Manual Procedure)	–	D 2488
	Practice for Description of Frozen Soils (Visual-Manual Procedure)	–	D 4083
Index Properties	Test Method for Determination of Water (Moisture) Content of Soil by Direct Heating Method	T 265	D 2216
	Test Method for Specific Gravity of Soils	T 100	D 854; D 5550
	Method for Particle-Size Analysis of Soils	T 88	D 422
	Test Method for Classification of Soils for Engineering Purposes	M 145	D 2487; D 3282
	Test Method for Amount of Material in Soils Finer than the No. 200 (0.075 mm) Sieve		D 1140
	Test Method for Liquid Limit, Plastic Limit, and Plasticity Index of Soils	T 89; T 90	D 4318
Compaction	Test Method for Laboratory Compaction Characteristics of Soil Using Standard Effort (12,375 ft-lb/ft^3)	T 99	D 698
	Test Method for Laboratory Compaction Characteristics of Soil Using Modified Effort (56,250 ft-lb/ft^3)	T 180	D 1557
Strength Properties	Test Method for Unconfined Compressive Strength of Cohesive Soil	T 208	D 2166
	Test Method for Unconsolidated, Undrained Compressive Strength of Cohesive Soils in Triaxial Compression	T 296	D 2850
	Test Method for Consolidated, Undrained Compressive Strength of Cohesive Soils in Triaxial Compression	T 297	D 4767
	Method for Direct Shear Test of Soils Under Consolidated Drained Conditions	T 236	D 3080
	Test Methods for Modulus and Damping of Soils by the Resonant-Column Method	–	D 4015
	Test Method for Laboratory Miniature Vane Shear Test for Saturated Fine-Grained Clayey Soil	–	D 4648
	Test Method for CBR (California Bearing Ratio) of Laboratory-Compacted Soils	–	D 1883
	Test Method for Resilient Modulus of Soils	T 294	–
	Test Method for Resistance R-Value and Expansion Pressure of Compacted Soils	T 190	D 2844

Commonly Performed Laboratory Tests on Soils (after FHWA, 2002a) (cont'd)

Test Category	Name of Test	Test Designation	
		AASHTO	ASTM
Consolidation, Swelling, Collapse Properties	Test Method for One-Dimensional Consolidation Properties of Soils	T 216	D 2435
	Test Method for One-Dimensional Consolidation Properties of Soils Using Controlled-Strain Loading	–	D 4186
	Test Methods for One-Dimensional Swell or Settlement Potential of Cohesive Soils	T 258	D 4546
	Test Method for Measurement of Collapse Potential of Soils	–	D 5333
Permeability	Test method for Permeability of Granular Soils (Constant Head)	T 215	D 2434
	Test Method for Measurement of Hydraulic Conductivity of Saturated Porous Materials Using a Flexible Wall Permeameter	–	D 5084
Corrosivity (electro-chemical)	Test Method for pH of Peat Materials	–	D 2976
	Test Method for pH of Soils	–	D 4972
	Test Method for pH of Soil for Use in Corrosion Testing	T 289	G 51
	Test Method for Sulfate Content	T 290	D 4230
	Test Method for Resistivity	T 288	D 1125; G 57
	Test Method for Chloride Content	T 291	D 512
Organic Content	Test Methods for Moisture, Ash, and Organic Matter of Peat and Other Organic Soils	T 194	D 2974

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 5-1, p. 5-6. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Methods for Index Testing of Soils (after FHWA, 2002a)

Test	Procedure	ASTM and/ or AASHTO	Applicable Soil Types	Applicable Soil Properties	Limitations/ Remarks
Moisture content, w_n	Dry soil in oven at 100 ± 5°C.	D 2216 T 265	Gravel, sand, silt, clay, peat	e_o, γ	Simple index test for all materials
Unit weight and density	Extract a tube sample; measure dimensions and weight.	D 2216 T 265	Soils where undisturbed samples can be taken, i.e., silt, clay, peat	γ_t, γ_{dry}, ρ_{tot}, ρ_{dry}, p_t	Not appropriate for clean granular materials where undisturbed sampling is not possible; very useful index test
Atterberg limits, *LL*, *PL*, *PI*, *SL*, *LI*	*LL*—Moisture content associated with closure of the groove at 25 blows of specimen in Casagrande cup. *PL*—Moisture content associated with crumbling of rolled soil at 1/8 in. (3 mm).	D 4318 T 89 T 90	Clays, silts, peat; silty and clayey sands to determine whether SM or SC	Soil classification and used in consolidation parameters	Not appropriate for nonplastic granular soil; recommended for all plastic materials

Methods for Index Testing of Soils (after FHWA, 2002a) (cont'd)

Test	Procedure	ASTM and/ or AASHTO	Applicable Soil Types	Applicable Soil Properties	Limitations/ Remarks
Mechanical sieve	Place air dried material on a series of successively smaller screens of known opening size and vibrate to separate particles of a specific equivalent diameter.	D 422 T 88	Gravel, sand, silt	Soil classification	Not appropriate for clay soils; useful, particularly in clean and dirty granular materials
Wash sieve	Flush fine particles through a U.S. No. 200 (0.075 mm) sieve with water.	C 117 D 1140 T 88	Sand, silt, clay	Soil classification	Needed to assess fines content in dirty granular materials
Hydrometer	Allow particles to settle and measure specific gravity of the solution with time.	D 422 D 1140 T 88	Fine sand, silt, clay	Soil classification	Helpful to assess relative quantities of silt and clay
Sand equivalent	Separate sample passing No. 4 (4.75 mm) sieve into sand and clay size particles.	D 2419 T 176	Gravel, sand, silt, clay	Aggregate classification compaction	Useful for aggregates
Specific gravity of solids	Compare the volume of a known mass of soil to the known volume of water in a calibrated pycnometer.	D 854 D 5550 T 100	Sand silt, clay, peat	Used in calculation of e_o	Particularly helpful in cases where unusual solid minerals are encountered
Organic content	After performing a moisture content test at 110°C (230°F), ignite the sample in a muffle furnace at 440°C (824°F) to measure the ash content.	D 2974 T 194	All soil types where organic matter is suspected to be a concern	Not related to any specific performance parameters, but samples high in organic content will likely have high compressibility	Recommended on all soils suspected to contain organic materials

Symbols used in table above:
e_o: in situ void ratio; γ_{dry}: dry unit weight; γ: unit weight; p_t: total vertical stress
ρ_{dry}: dry density; ρ_{tot}: total density; γ_t: total unit weight

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 5-2, p. 5-7. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Methods for Performance Testing of Soils (after FHWA, 2006)

Test	Procedure	Applicable Soil Types	Soil Properties	Limitations/ Remarks
1-D oedometer	Incremental loads are applied to a soil specimen confined by a rigid ring; deformation values are recorded with time; loads are typically doubled for each increment and applied for 24 hours each.	Primarily clays and silts; granular soils can be tested, but typically are not.	p_c, OCR, C_c, $C_{c\varepsilon}$, C_r, $C_{r\varepsilon}$, C_α, $C_{\alpha\varepsilon}$, c_v, k	Recommended for fine-grained soils. Results can be useful index to other critical parameters.
Constant rate of strain oedometer	Loads are applied such that Δu is between 3 and 30 percent of the applied vertical stress during testing.	Clays and silts; not applicable to free-draining granular soils.	p_c, C_c, $C_{c\varepsilon}$, C_r, $C_{r\varepsilon}$, c_v, k	Requires special testing equipment but can reduce testing time significantly.
Unconfined compression (UC)	A specimen is placed in a loading apparatus and sheared under axial compression with no confinement.	Clays and silts; cannot be performed on granular soils or fissured and varved materials.	$s_{u,UC}$	Provides rapid means to approximate undrained shear strength, but disturbance effects, test rate, and moisture migration will affect results.
Unconsolidated, undrained (UU) triaxial shear	The specimen is not allowed to consolidate under the confining stress, and the specimen is loaded at a quick enough rate to prevent drainage.	Clays and silts	$s_{u,UU}$	Sample must be nearly saturated. Sample disturbance and rate effects will affect measured strength.
Isotropic, consolidated, drained compression (CIDC)	The specimen is allowed to consolidate under the confining stress, and then is sheared at a rate slow enough to prevent build-up of pore water pressures.	Sands, silts, clays	ϕ', c', E	Can be run on clay specimen, but time consuming. Best triaxial test to obtain deformation properties.
Isotropic, consolidated, undrained compression (CIUC)	The specimen is allowed to consolidate under the confining stress with drainage allowed, and then is sheared with no drainage allowed, but pore water pressures measured.	Sands, silts, clays, peats	ϕ', c', $s_{u,CIUC}$, E	Recommended to measure pore pressures during test. Useful test to assess effective stress strength parameters. Not recommended for measuring deformation properties.
Direct shear	The specimen is sheared on a forced failure plane at a constant rate, which is a function of the hydraulic conductivity of the specimen.	Compacted fill materials; sands, silts, and clays	ϕ', ϕ'_r	Requires assumption of drainage conditions. Relatively easy to perform.

Methods for Performance Testing of Soils (cont'd)

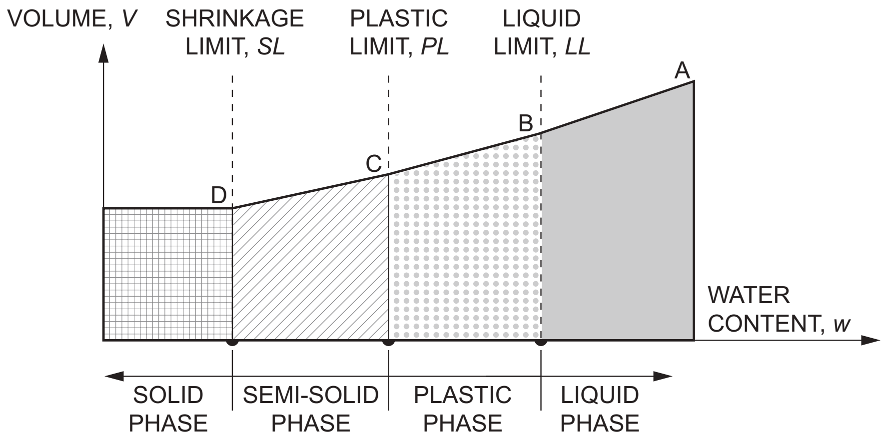
Test	Procedure	Applicable Soil Types	Soil Properties	Limitations/ Remarks
Flexible wall permeameter	The specimen is encased in a membrane, consolidated, backpressure saturated, and measurements of flow with time are recorded for a specific gradient.	Relatively low permeability materials ($k \leq 1 \times 10^{-3}$ cm/s); clays and silts	k	Recommended for fine-grained materials. Backpressure saturation required. Confining stress needs to be provided. System permeability must be at least an order of magnitude greater than that of the specimen. Time needed to allow inflow and outflow to stabilize.
Rigid wall permeameter	The specimen is placed in a rigid wall cell, vertical confinement is applied, and flow measurements are recorded with time under constant-head or falling-head conditions.	Relatively high permeability materials; sands, gravels, and silts	k	Need to control gradient. Not for use in fine-grained soils. Monitor for sidewall leakage.

Symbols used in table above:

- ϕ': peak effective stress friction angle
- ϕ'_r: residual effective stress friction angle
- c': effective stress cohesion intercept
- s_u: undrained shear strength
- p_c: preconsolidation stress
- OCR: overconsolidation ratio
- c_v: vertical coefficient of consolidation
- E: Young's modulus
- k: hydraulic conductivity
- C_c: compression index
- $C_{c\varepsilon}$: modified compression index
- C_r: recompression index
- $C_{r\varepsilon}$: modified recompression index
- C_α: secondary compression index
- $C_{\alpha\varepsilon}$: modified secondary compression index

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 5-3, p. 5-8 and 5-9. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.8.2 Atterberg Limits



Conceptual Changes in Soil Phases as a Function of Water Content

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 2-5, p. 2-16. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

$$PI = LL - PL$$

$$LI = \frac{w - PL}{PI}$$

A-line: $PI = 0.73\,(LL - 20)$

U-line: $PI = 0.9\,(LL - 8)$

where

PI = plasticity index

PL = plastic limit

LL = liquid limit

LI = liquidity index

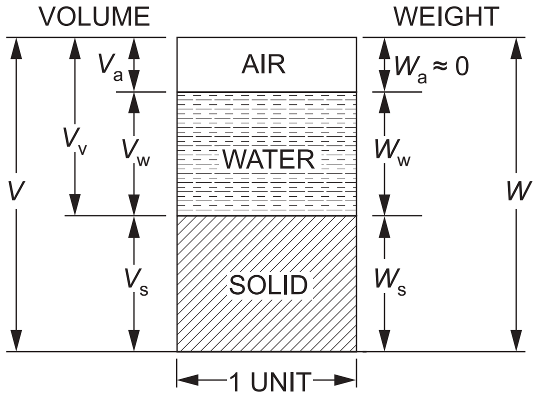
w = water content

Concept of Soil Phase, Soil Strength, and Soil Deformation Based on Liquidity Index

Liquidity Index, *LI*	Soil Phase	Soil Strength (Soil Deformation)
$LI \geq 1$	Liquid	Low strength (Soil deforms like a viscous fluid.)
$0 < LI < 1$	Plastic	Intermediate strength • At $w \approx LL$, soil is considered soft and very compressible. • At $w \approx PL$, soil is considered stiff. (Soil deforms like a plastic material.)
$LI \leq 0$	Semisolid to Solid	High strength (Soil deforms as a brittle material, i.e., sudden fracture of material.)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual.* Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 2-4, p. 2-17. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.8.3 Weight-Volume Relationships



A Unit of Soil Mass and Its Idealization

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 2-1, p. 2-2. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Weight-Volume Relationships (after Das, 1990)

Unit-Weight Relationship	Dry Unit Weight (No Water)	Saturated Unit Weight (No Air)
$\gamma_t = \dfrac{(1+w)G_s\gamma_w}{1+e}$ $\gamma_t = \dfrac{(G_s+Se)\gamma_w}{1+e}$ $\gamma_t = \dfrac{(1+w)G_s\gamma_w}{1+\dfrac{wG_s}{S}}$ $\gamma_t = G_s\gamma_w(1-n)(1+w)$	$\gamma_d = \dfrac{\gamma_t}{1+w}$ $\gamma_d = \dfrac{G_s\gamma_w}{1+e}$ $\gamma_d = G_s\gamma_w(1-n)$ $\gamma_d = \dfrac{G_s\gamma_w}{1+\dfrac{wG_s}{S}}$ $\gamma_d = \dfrac{eS\gamma_w}{(1+e)w}$ $\gamma_d = \gamma_{sat} - n\gamma_w$ $\gamma_d = \gamma_{sat} - \left(\dfrac{e}{1+e}\right)\gamma_w$	$\gamma_{sat} = \dfrac{(G_s+e)\gamma_w}{1+e}$ $\gamma_{sat} = [(1-n)G_s+n]\gamma_w$ $\gamma_{sat} = \left(\dfrac{1+w}{1+wG_s}\right)G_s\gamma_w$ $\gamma_{sat} = \left(\dfrac{e}{w}\right)\left(\dfrac{1+w}{1+e}\right)\gamma_w$ $\gamma_{sat} = \gamma_d + n\gamma_w$ $\gamma_{sat} = \gamma_d + \left(\dfrac{e}{1+e}\right)\gamma_w$

In above relationships, γ_w refers to the unit weight of water, 62.4 pcf (9.81 kN/m^3).

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 2-2, p. 2-8. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

where

γ_t = total unit weight

γ_d = dry unit weight

γ_{sat} = saturated unit weight

γ_w = unit weight of water

G_s = specific gravity

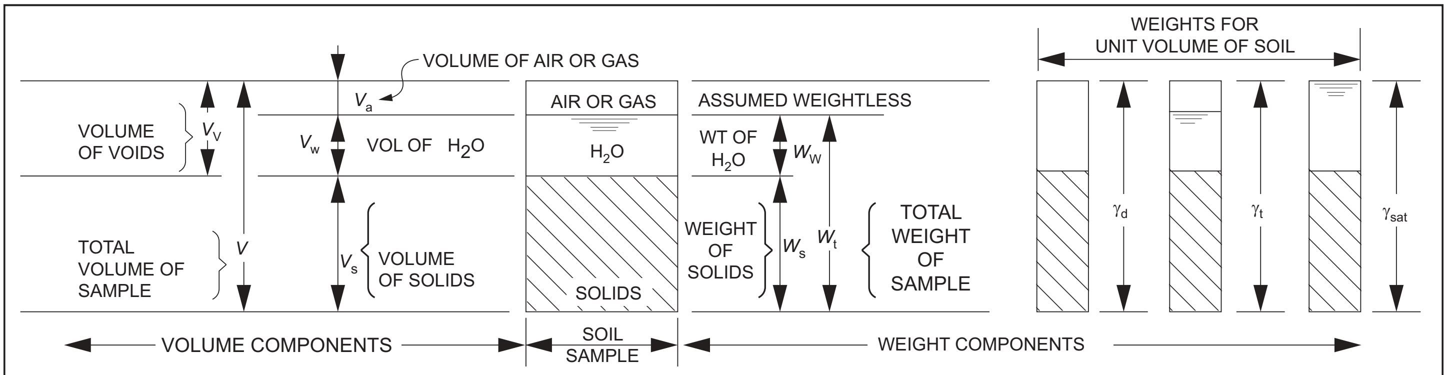
e = void ratio

n = porosity

S = saturation (ratio)

w = water content (ratio)

Volume and Weight Relationships



Property		Saturated Sample (W_s, W_w, G are known)	Unsaturated Sample (W_s, W_w, G, V are known)	Supplementary Formulas Relating Measured and Computed Factors			
Volume Components	V_s (volume of solids)	$\frac{W_s}{G\gamma_w}$		$V-(V_a+V_w)$	$V(1-n)$	$\frac{V}{1+e}$	$\frac{V_v}{e}$
	V_w (volume of water)	$\frac{W_w}{\gamma_w}$		V_v-V_a	SV_v	$\frac{SVe}{1+e}$	SV_se
	V_a (volume of air or gas)	Zero	$V-(V_s+V_w)$	V_v-V_w	$(1-S)V_v$	$\frac{(1-S)Ve}{1+e}$	$(1-S)V_se$
	V_v (volume of voids)	$\frac{W_w}{\gamma_w}$	$V-\frac{W_s}{G\gamma_w}$	$V-V_s$	$\frac{V_sn}{1-n}$	$\frac{Ve}{1+e}$	V_se
	V (total volume of sample)	V_s+V_w	Measured	$V_s+V_a+V_w$	$\frac{V_s}{1-n}$	$V_s(1+e)$	$\frac{V_v(1+e)}{e}$
	n (porosity)	$\frac{V_v}{V}$		$1-\frac{V_s}{V}$	$1-\frac{W_s}{GV\gamma_w}$	$\frac{e}{1+e}$	
	e (void ratio)	$\frac{V_v}{V_s}$		$\frac{V}{V_s}-1$	$\frac{GV\gamma_w}{W_s}-1$	$\frac{W_wG}{W_sS}$	$\frac{n}{1-n}$; $\frac{wG}{S}$
Weights for Specific Sample	W_s (weight of solids)	Measured		$\frac{W_t}{1+w}$	$GV\gamma_w(1-n)$	$\frac{W_wG}{eS}$	
	W_w (weight of water)	Measured		wW_s	$S\gamma_wV_v$	$\frac{eW_sS}{G}$	
	W_t (total weight of sample	W_s+W_w		$W_s(1+w)$			

Volume and Weight Relationships (cont'd)

Property		Saturated Sample (W_s, W_w, G are known)	Unsaturated Sample (W_s, W_w, G, V are known)	Supplementary Formulas Relating Measured and Computed Factors			
Weights for Sample of Unit Volume	γ_d (dry unit weight	$\frac{W_s}{V_s + V_w}$	$\frac{W_s}{V}$	$\frac{W_t}{V(1+w)}$	$\frac{G\gamma_w}{1+e}$	$\frac{G\gamma_w}{1+\frac{wG}{S}}$	
	γ_t (wet unit weight)	$\frac{W_s + W_w}{V_s + V_w}$	$\frac{W_s + W_w}{V}$	$\frac{W_t}{V}$	$\frac{(G+Se)\gamma_w}{1+e}$	$\frac{(1+w)\gamma_w}{\frac{w}{S}+\frac{1}{G}}$	
	γ_{sat} (saturated unit weight)	$\frac{W_s + W_w}{V_s + V_w}$	$\frac{W_s + V_v\gamma_w}{V}$	$\frac{W_s}{V} + \left(\frac{e}{1+e}\right)\gamma_w$	$\frac{(G+e)\gamma_w}{1+e}$	$\frac{(1+w)\gamma_w}{w+\frac{1}{G}}$	
	γ_{sub} [submerged (buoyant) unit weight]	$\gamma_{sat} - \gamma_w$		$\frac{W_s}{V} - \left(\frac{1}{1+e}\right)\gamma_w$	$\left(\frac{G+e}{1+e} - 1\right)\gamma_w$	$\left(\frac{1-\frac{1}{G}}{w+\frac{1}{G}}\right)\gamma_w$	
Combined Relations	w (moisture content)	$\frac{W_w}{W_s}$		$\frac{W_t}{W_s} - 1$	$\frac{Se}{G}$	$S\left(\frac{\gamma_w}{\gamma_d} - \frac{1}{G}\right)$	
	S (degree of saturation)	1.00	$\frac{V_w}{V_v}$	$\frac{W_w}{V_v\gamma_w}$	$\frac{wG}{e}$	$\frac{w}{\left(\frac{\gamma_w}{\gamma_d} - \frac{1}{G}\right)}$	
	G (specific gravity)	$\frac{W_s}{V_s\gamma_w}$		$\frac{Se}{w}$			

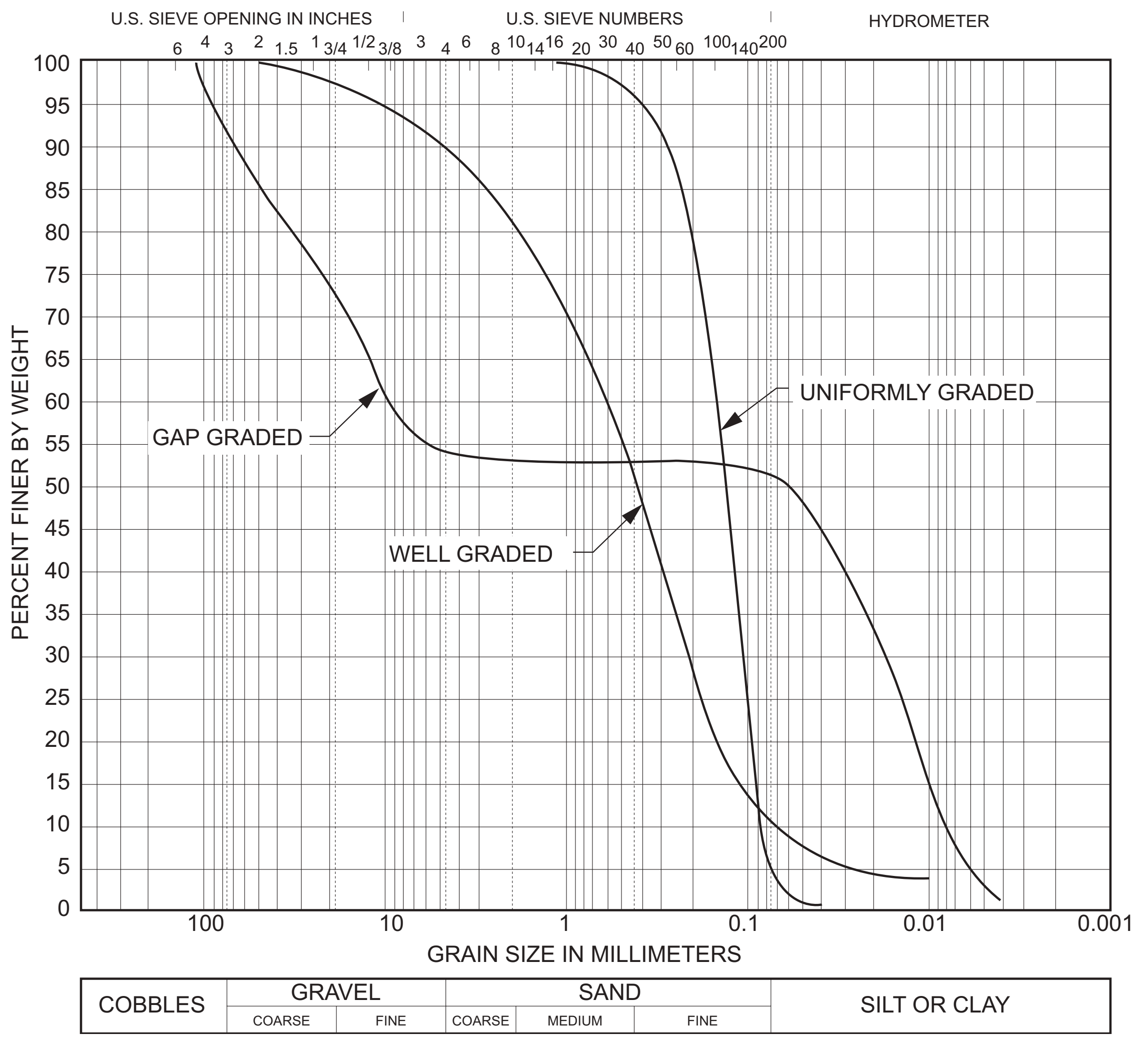
Source: Unified Facilities Criteria (UFC). *Soil Mechanics*. UFC 3-220-10N. Washington, DC: U.S. Department of Defense, June 2005, Table 6, p. 135-136. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_10n_2005.pdf.

3.8.4 Gradation Tests

U.S. Standard Sieve Sizes and Corresponding Opening Dimension

U.S. Standard Sieve No.	Sieve Opening (in.)	Sieve Opening (mm)	Comment [Based on the Unified Soil Classification System (USCS)]
3	0.2500	6.35	
4	0.1870	4.75	• Breakpoint between fine gravels and coarse sands • Soil passing this sieve is used for compaction test
6	0.1320	3.35	
8	0.0937	2.36	
10	0.0787	2.00	• Breakpoint between coarse and medium sands
12	0.0661	1.70	
16	0.0469	1.18	
20	0.0331	0.850	
30	0.0234	0.600	
40	0.0165	0.425	• Breakpoint between medium and fine sands • Soil passing this sieve is used for Atterberg limits
50	0.0117	0.300	
60	0.0098	0.250	
70	0.0083	0.212	
100	0.0059	0.150	
140	0.0041	0.106	
200	0.0029	0.075	• Breakpoint between fine sand and silt or clay
270	0.0021	0.053	
400	0.0015	0.038	

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 2-3, p. 2-10. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Sample Grain Size Distribution Curves

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 2-3, p. 2-12. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

$$C_u = \frac{D_{60}}{D_{10}}$$

$$C_c = \frac{{D_{30}}^2}{D_{60} \times D_{10}}$$

where

D_{60} = diameter at 60% passing

D_{30} = diameter at 30% passing

D_{10} = diameter at 10% passing

C_u = coefficient of uniformity

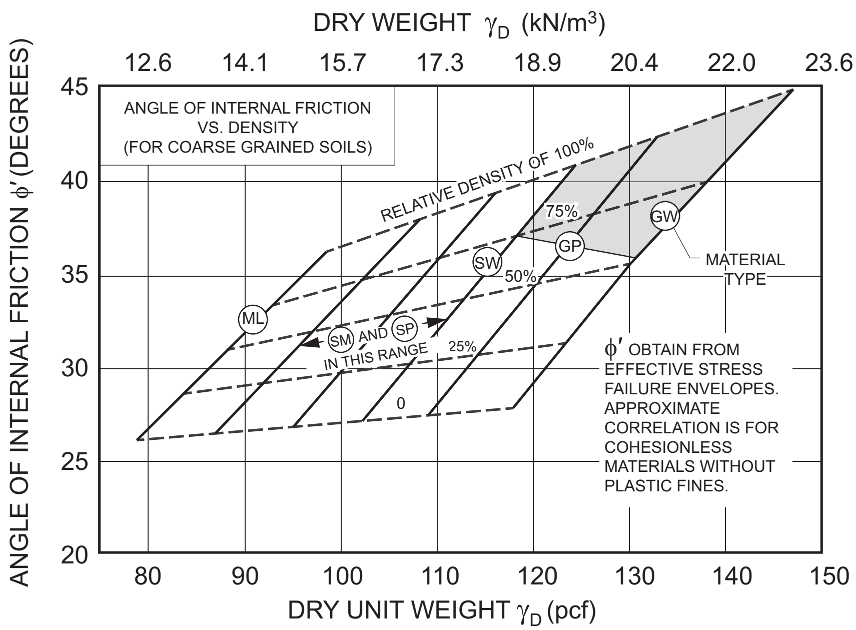
C_c = coefficient of curvature

Gradation Based on C_u and C_c Parameters

Gradation	Gravels	Sands
Well-graded	$C_u \geq 4$ and $1 \leq C_c \leq 3$	$C_u \geq 6$ and $1 \leq C_c \leq 3$
Poorly graded	$C_u < 4$ and/or $1 > C_c > 3$	$C_u < 6$ and/or $1 > C_c > 3$
Gap graded*	C_c not between 1 and 3	C_c not between 1 and 3

*Gap-graded soils may be well-graded or poorly graded. In addition to the C_c value, it is recommended that the shape of the GSD be the basis for definition of gap graded.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 4-10, p. 4-19. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



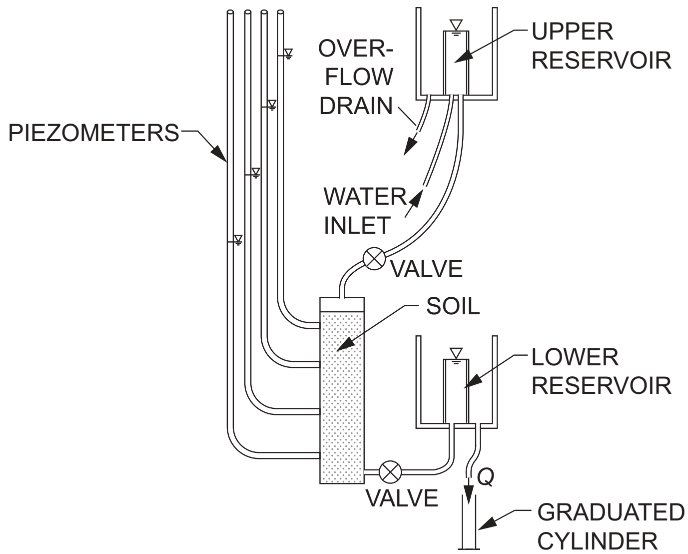
Correlation Between Relative Density, Material Classification, and Angle of Internal Friction for Coarse-Grained Soils (NAVFAC, 1986a)

Note: Use caution in the shaded portion of the chart due to the potential for unreliable SPT *N*-values in gravels.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-22, p. 5-57. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.8.5 Permeability Testing Properties of Soil and Rock

3.8.5.1 Laboratory Permeability Tests



Schematic of a Constant Head Permeameter (Coduto, 1999)
Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 5-38, p. 5-102. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/05d.cfm.

$$k = \frac{QL}{tAh}$$

where

k = coefficient of permeability

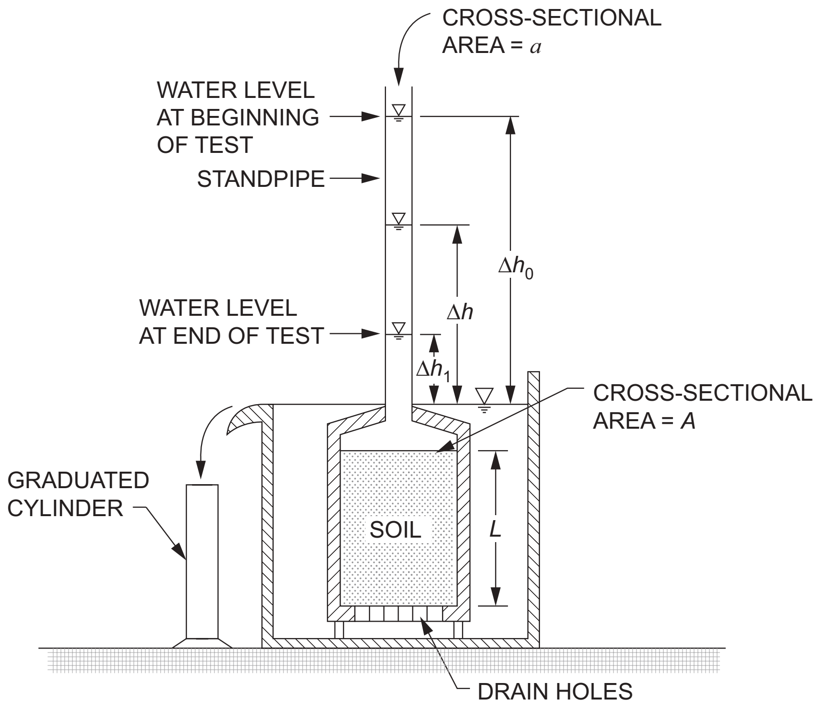
Q = volume of flow

L = length of sample

t = time of flow

A = cross-sectional area of sample

h = constant hydraulic head



Schematic of a Falling Head Permeameter (Coduto, 1999)

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 5-39, p. 5-104. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/05d.cfm.

$$k = \frac{aL}{A\left(t_1 - t_2\right)} \ln\left(\frac{h_1}{h_2}\right)$$

where

k = coefficient of permeability

a = cross-sectional area of standpipe

L = length of sample

A = cross-sectional area of sample

t_1 = time at start of flow

t_2 = time at end of flow

h_1 = hydraulic head at t_1

h_2 = hydraulic head at t_2

Source: Unified Facilities Criteria (UFC). *Dewatering and Groundwater Control*. UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, Fig. 3-2, p. 3-3. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf.

3.8.5.2 Hazen's Equation for Permeability

$$k = C\left(D_{10}\right)^2$$

where

k = permeability (cm/s)

C = coefficient from 0.4 to 1.2 depending on sand size

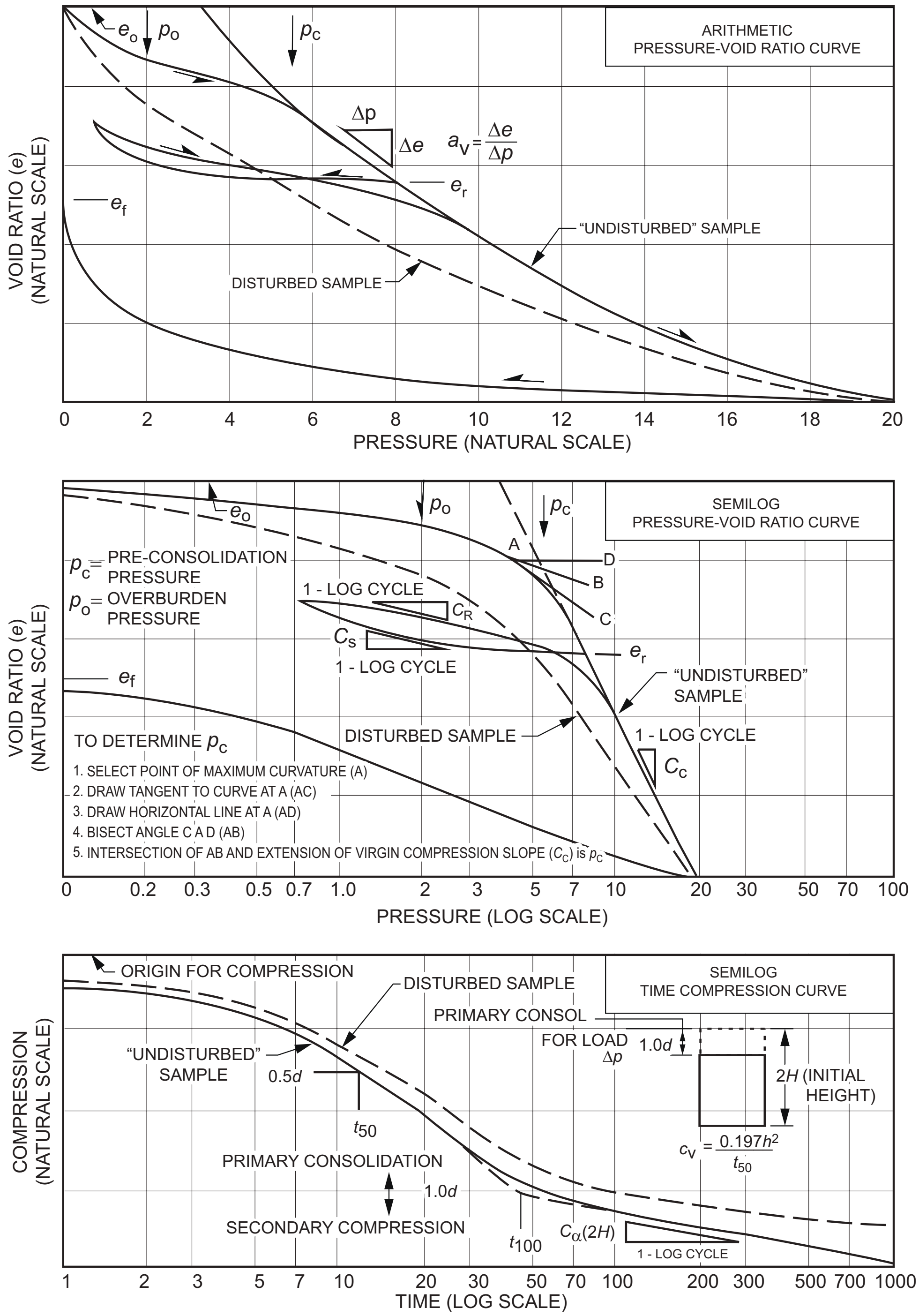
D_{10} = effective grain size, in mm, passing at 10% by weight

3.8.5.3 Consolidation Testing

Hydraulic Conductivity Unit Conversion Factors

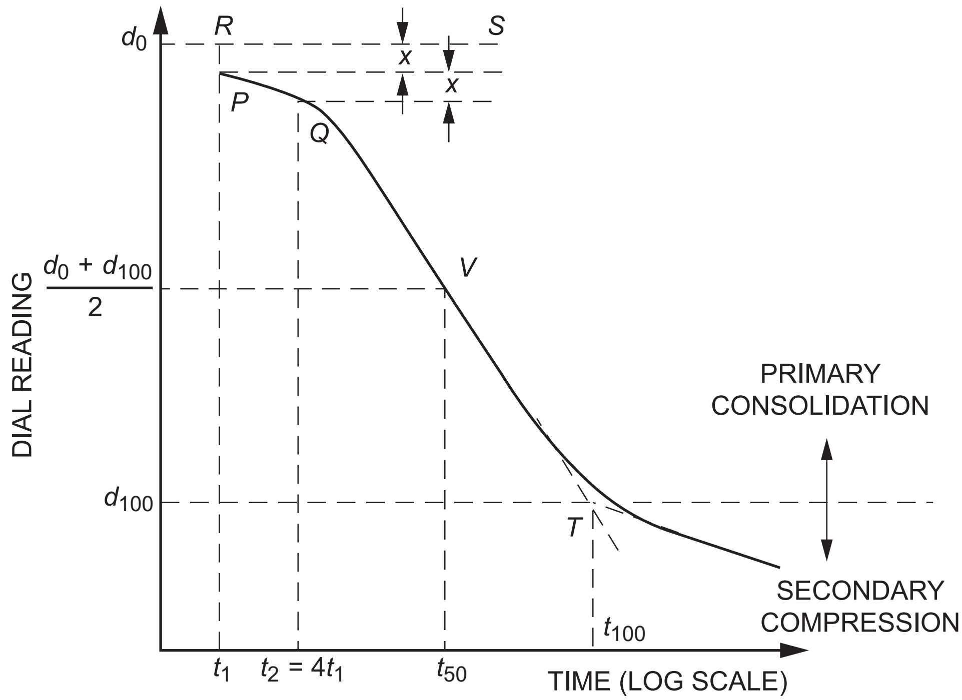
Starting Unit	Desired Unit	Multiplication Factor
cm/s	ft/min	1.97
cm/s	ft/day	2,834.6
cm/s	Lugeon	76,923
ft/min	cm/s	0.508
ft/min	ft/day	1,440
ft/min	Lugeon	39,047
ft/day	cm/s	3.53×10^{-4}
ft/day	ft/min	6.94×10^{-4}
ft/day	Lugeon	27.1
Lugeon	cm/s	1.3×10^{-5}
Lugeon	ft/min	2.6×10^{-5}
Lugeon	ft/day	3.7×10^{-2}

Source: U.S. Army Corps of Engineers. *Engineering and Design: Grouting Technology*. EM 1110-2-3506. Washington, DC: U.S. Department of the Army, March 2017, Table 5-1, p. 5-3. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-3506.pdf.



Consolidation Test Relationships (after NAVFAC, 1986a)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-8, p. 5-27. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Logarithm-of-Time Method for Determination of c_v

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 7-15, p. 7-34. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

$$c_v = \frac{0.197\, H_d^2}{t_{50}}$$

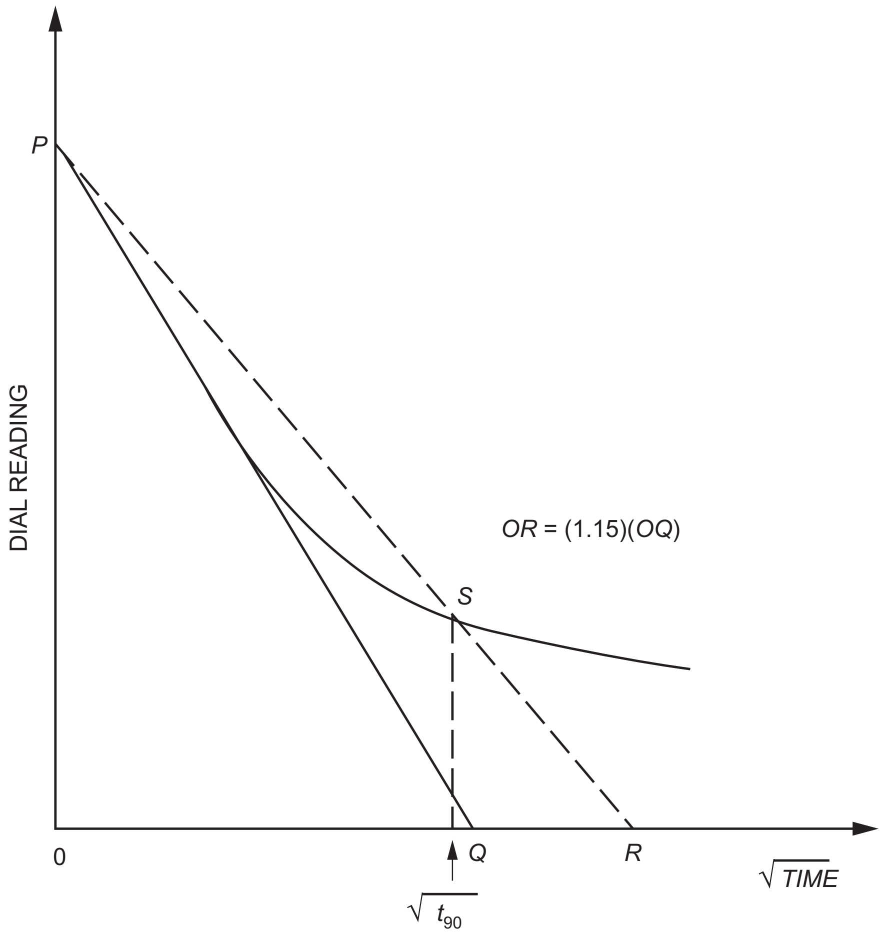
where

c_v = coefficient of vertical consolidation

H_d = thickness of compressible layer

t_{50} = time to 50% of primary consolidation

t_{90} = time to 90% of primary consolidation



Square-Root-of-Time Method for Determination of c_v

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 7-16, p. 7-35. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

$$c_v = \frac{0.848\ H_d^2}{t_{90}}$$

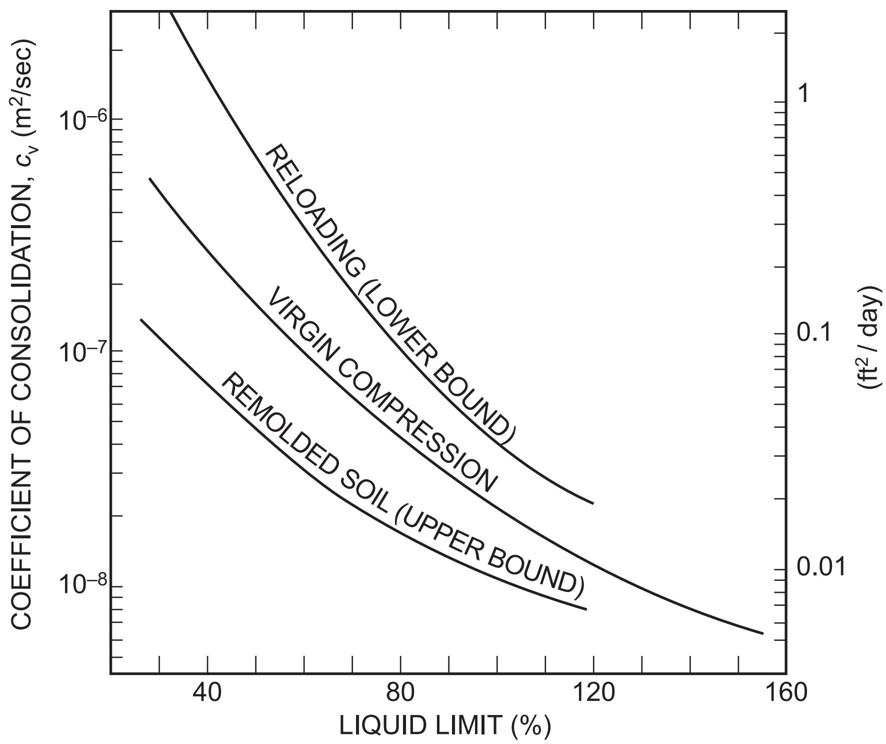
where

c_v = coefficient of vertical consolidation

H_d = thickness of compressible layer

t_{50} = time to 50% of primary consolidation

t_{90} = time to 90% of primary consolidation



Approximate Correlations Between c_v and *LL* (NAVFAC, 1986a)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-10, p. 5-36. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.8.5.4 Secondary Consolidation

$$C_{\alpha\varepsilon} = \frac{\Delta\varepsilon}{\Delta(\log t)}$$

$$C_{\alpha e} = \frac{\Delta e}{\Delta(\log t)}$$

$$C_{\alpha\varepsilon} = \frac{C_{\alpha e}}{(1 + e_o)}$$

where

$C_{\alpha\varepsilon}$ = coefficient of secondary consolidation (strain)

$C_{\alpha e}$ = coefficient of secondary consolidation (void ratio)

e_o = initial void ratio

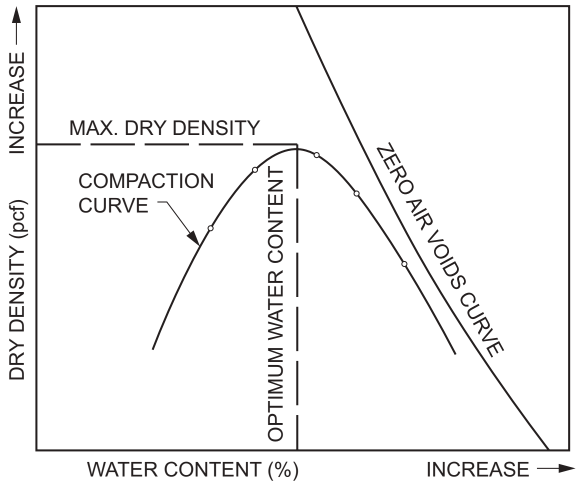
t = time

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Equation 5-7, p. 5-37. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Source: Unified Facilities Criteria (UFC). *Soil Mechanics*. UFC 3-220-10N. Washington, DC: U.S. Department of Defense, June 2005, Fig. 17, p. 7.1-245. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_10n_2005.pdf.

3.9 Compaction: Laboratory and Field Compaction

3.9.1 Laboratory Compaction Tests



Moisture-Density Relationship for Compaction

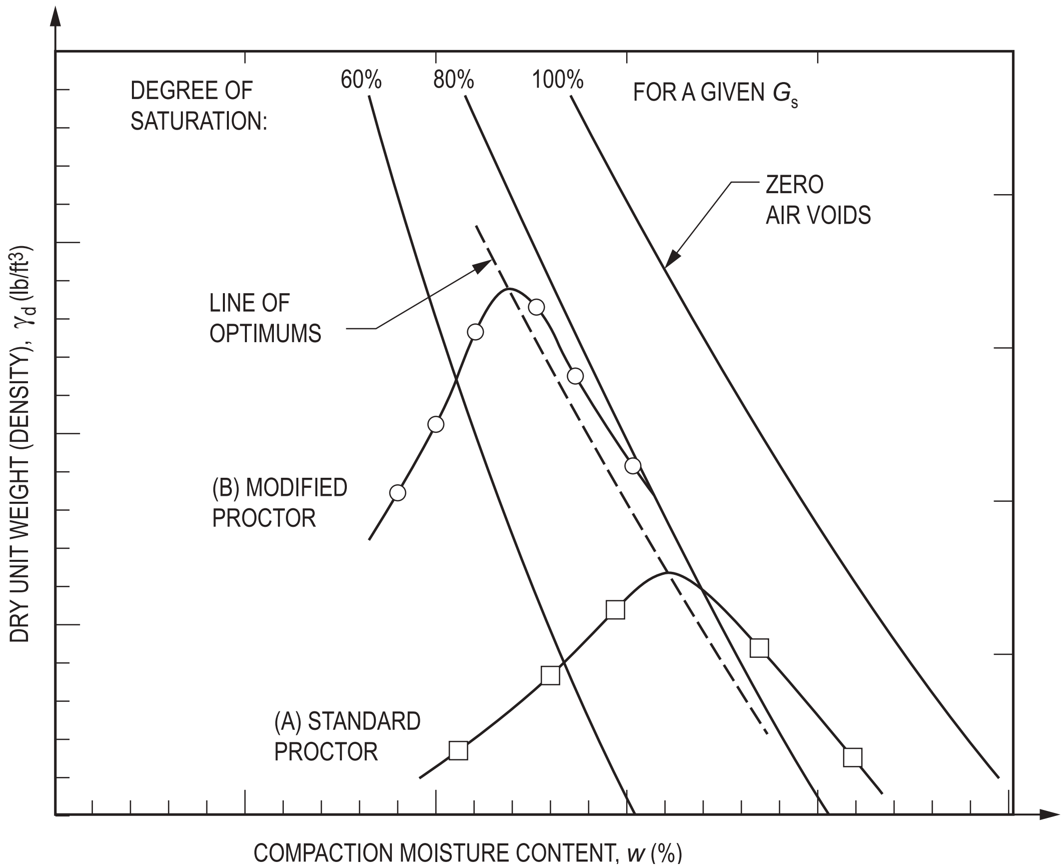
Source: Unified Facilities Criteria (UFC). *Backfill for Subsurface Structures*. UFC 3-220-04FA. Washington, DC: U.S. Department of Defense, January 2004, Fig. B-1, p. B-2. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_04fa_2004.pdf.

Characteristics of Laboratory Compaction Tests

Common Name	ASTM (AASHTO) Designations	Mold Dimensions			Hammer		No. of Layers	Blows/ Layer	Energy (ft-lb/ft^3)
		Diam. (in.)	Height (in.)	Vol. (ft^3)	Wt. (lb)	Drop Ht. (in.)			
Standard Proctor	D 698 (T 99)	4	4.5	1/30	5.5	12	3	25	12,375
Modified Proctor	D 1557 (T 180)	4	4.5	1/30	10	18	5	25	56,250

Note: Both tests are performed on minus No. 4 (4.75 mm) fraction of the soil.

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Table 5-13, p. 5-73. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Compaction Curves (after Holtz and Kovacs, 1981)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-31, p. 5-74. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Moisture content and dry density relationship:

$$\gamma_d = \frac{\gamma_t}{(1 + w)}$$

Zero air voids (100% saturation):

$$\gamma_d = \frac{G_s \gamma_w}{(1 + e)}$$

where

γ_t = total unit weight

γ_d = dry unit weight

γ_w = unit weight of water

w = water content

G_s = specific gravity

e = void ratio

3.9.2 Field Compaction

Field Density Testing

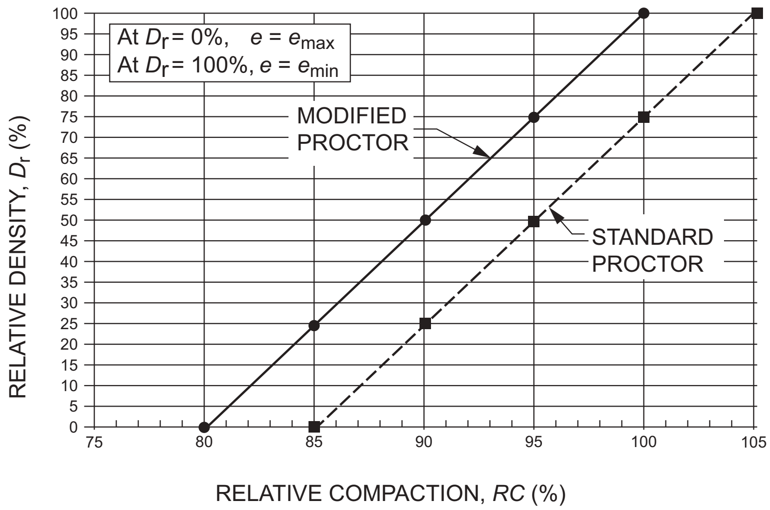
$$RC = \frac{\gamma_{\text{d field}}}{\gamma_{\text{d max}}} \times 100$$

where

RC = relative compaction (%)

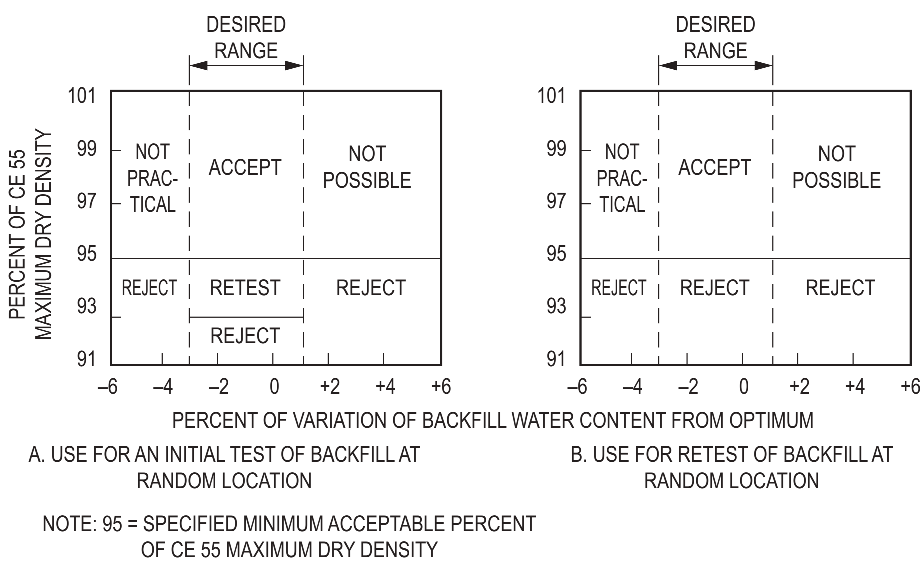
$\gamma_{\text{d field}}$ = dry unit weight from field density test

$\gamma_{\text{d max}}$ = maximum dry unit weight from laboratory test (standard or modified Proctor)



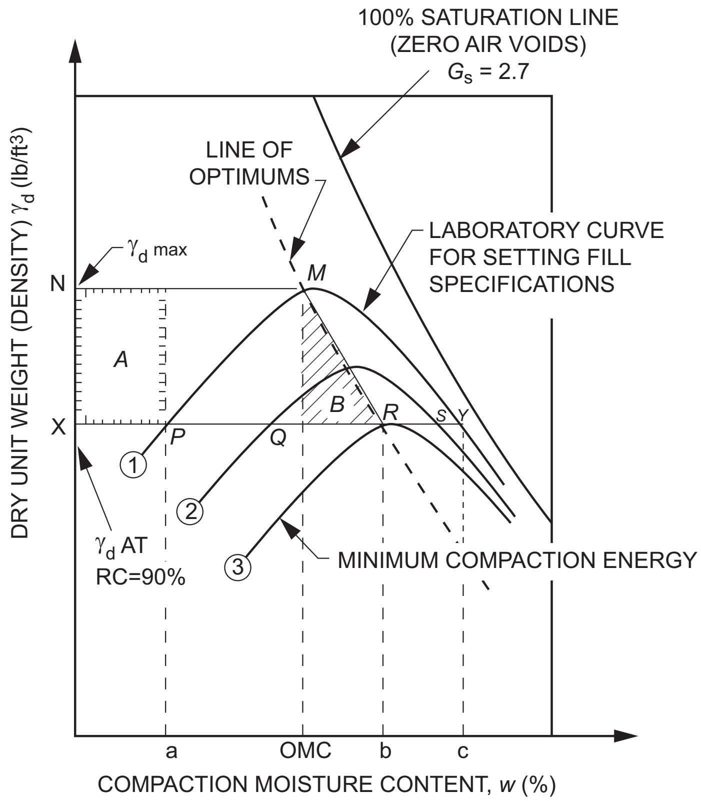
Relative Density, Relative Compaction, and Void Ratio Concepts

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-33, p. 5-78. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.



Acceptance Criteria for Compaction

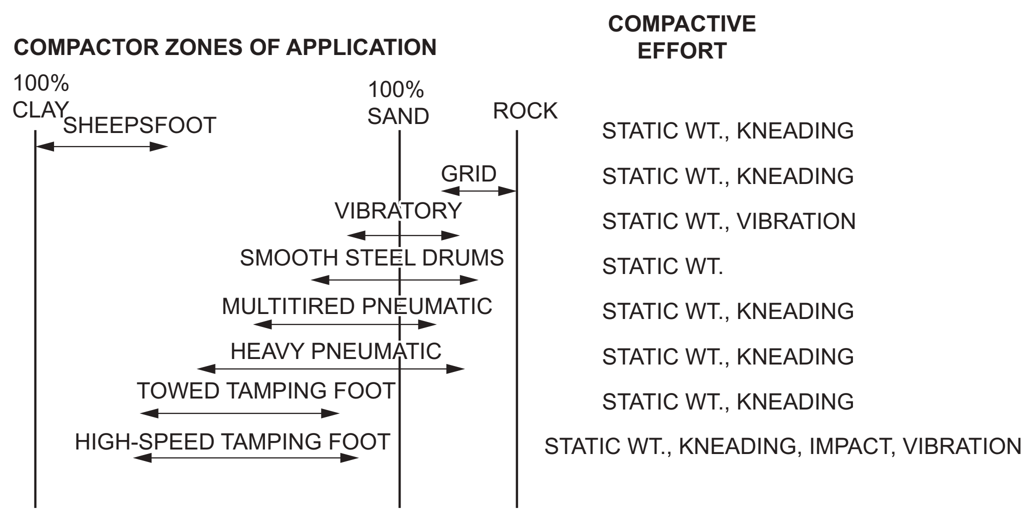
Source: Unified Facilities Criteria (UFC). *Backfill for Subsurface Structures*. UFC 3-220-04FA. Washington, DC: U.S. Department of Defense, January 2004, Table 7-1, p. 7-8. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_04fa_2004.pdf.



Example Evaluation of Economical Field Compaction Conditions (after Bowles, 1979)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-34, p. 5-81. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

3.9.3 Compaction Equipment



Compactors Recommended for Various Types of Soil and Rock (Schroeder, 1980)

Source: Federal Highway Administration. National Highway Institute. *Soils and Foundations, Reference Manual*. Vol. I. FHWA-NHI-06-088. Washington, DC: U.S. Department of Transportation, December 2006, Fig. 5-32, p. 5-77. www.fhwa.dot.gov/engineering/geotech/pubs/nhi06088.pdf.

Summary of Compaction Criteria[a]

Soil Group	Soil Types	Degree of Compaction	Fill and Backfill: Typical Equipment and Procedures for Compaction — Equipment	No. of Passes or Coverages	Comp. Lift Thick., in.	Placement Water Content	Field Control
Pervious (Free Draining)	GW GP SW SP	Compacted • 90 to 95% of CE 55 maximum density • 75 to 85% of relative density	Vibratory rollers and compactors Rubber-tired roller[b] Crawler-type tractor[c] Power hand tamper[d]	Indefinite 2–5 coverages 2–5 coverages Indefinite	Indefinite 12 8 6	Saturate by flooding	Control tests at intervals to determine degree of compaction or relative density
		Semicompacted • 85 to 90% of CE 55 maximum density • 65 to 75% of relative density	Rubber-tired roller[b] Crawler-type tractor[c] Power hand tamper[d] Controlled routing of construction equipment	2–5 coverages 1–2 coverages Indefinite Indefinite	14 10 8 8–10	Saturate by flooding	Control tests as noted above, if needed
Semipervious and Impervious	GM GC SM SC ML CL OL OH MH CH	Compacted • 90 to 95% of CE 55 maximum density	Rubber-tired roller[b] Sheepsfoot roller[e] Power hand tamper[d]	2–5 coverages 4–8 passes Indefinite	8 6 4	Optimum water content	Control tests at intervals to determine degree of compaction
		Semicompacted • 85 to 90% of CE 55 maximum density	Rubber-tired roller[b] Sheepsfoot roller[e] Crawler-type tractor[c] Power hand tamper[d] Controlled routing of construction equipment	2–4 coverages 4–8 passes 3 coverages Indefinite Indefinite	10 8 6 6 6–8	(A) Optimum water content (B) By observation; wet side-maximum water content at which material can satisfactorily operate, dry side-minimum water content required to bond particles and which will not result in voids or honeycombed material	(A) Control tests as noted above, if needed (B) Field control exercised by visual inspection of action of compaction equipment

Notes: The above requirements will be adequate in relation to most construction. In special cases where tolerable settlements are unusually small, it may be necessary to employ additional compaction, equivalent to 95% to 100% of compaction effort. A coverage consists of one application of the wheel of a rubber-tired roller or the threads of a crawler-type tractor over each point in the area being compacted. For a sheepsfoot roller, one pass consists of one movement of a sheepsfoot roller drum over the area being compacted.

[a] From TM 3-818-1, *Soils and Geology: Procedures for Foundation Design of Buildings and Other Structures (Except Hydraulic Structures)*

[b] Rubber-tired rollers having a wheel load between 18,000 and 25,000 lb and a tire pressure between 80 and 100 psi

[c] Crawler-type tractors weighing not less than 20,000 lb and exerting a foot pressure not less than 6.5 psi

[d] Power hand tampers weighing more than 100 lb; pneumatic or operated by gasoline engine

[e] Sheepsfoot rollers having a foot pressure between 250 and 500 psi and tamping feet 7 to 10 inches in length with a face area between 7 and 16 in^2

Source: Unified Facilities Criteria (UFC). *Backfill for Subsurface Structures*. UFC 3-220-04FA. Washington, DC: U.S. Department of Defense, January 2004, Table 5-1, p. 5-3. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_04fa_2004.pdf.

3.10 Trench and Excavation Construction Safety

3.10.1 Determination of Soil Type

OSHA categorizes soil and rock deposits into four types, A through D:

A. Stable Rock is natural solid mineral matter than can be excavated with vertical sides and remain intact while exposed. It is usually identified by a rock name such as granite or sandstone. Determining whether a deposit is of this type may be difficult unless it is known whether cracks exist and whether the cracks run into or away from the excavation.

B. Type A Soils are cohesive soils with an unconfined compressive strength of 1.5 tons per square foot (tsf) (144 kPa) or greater. Examples of Type A cohesive soils are often: clay, silty clay, sandy clay, clay loam, and, in some cases, silty clay loam and sandy clay loam. (No soil is Type A if it is fissured, is subject to vibration of any type, has previously been disturbed, is part of a sloped, layered system where the layers dip into the excavation on a slope of 4 horizontal to 1 vertical (4H:1V) or greater, or has seeping water.

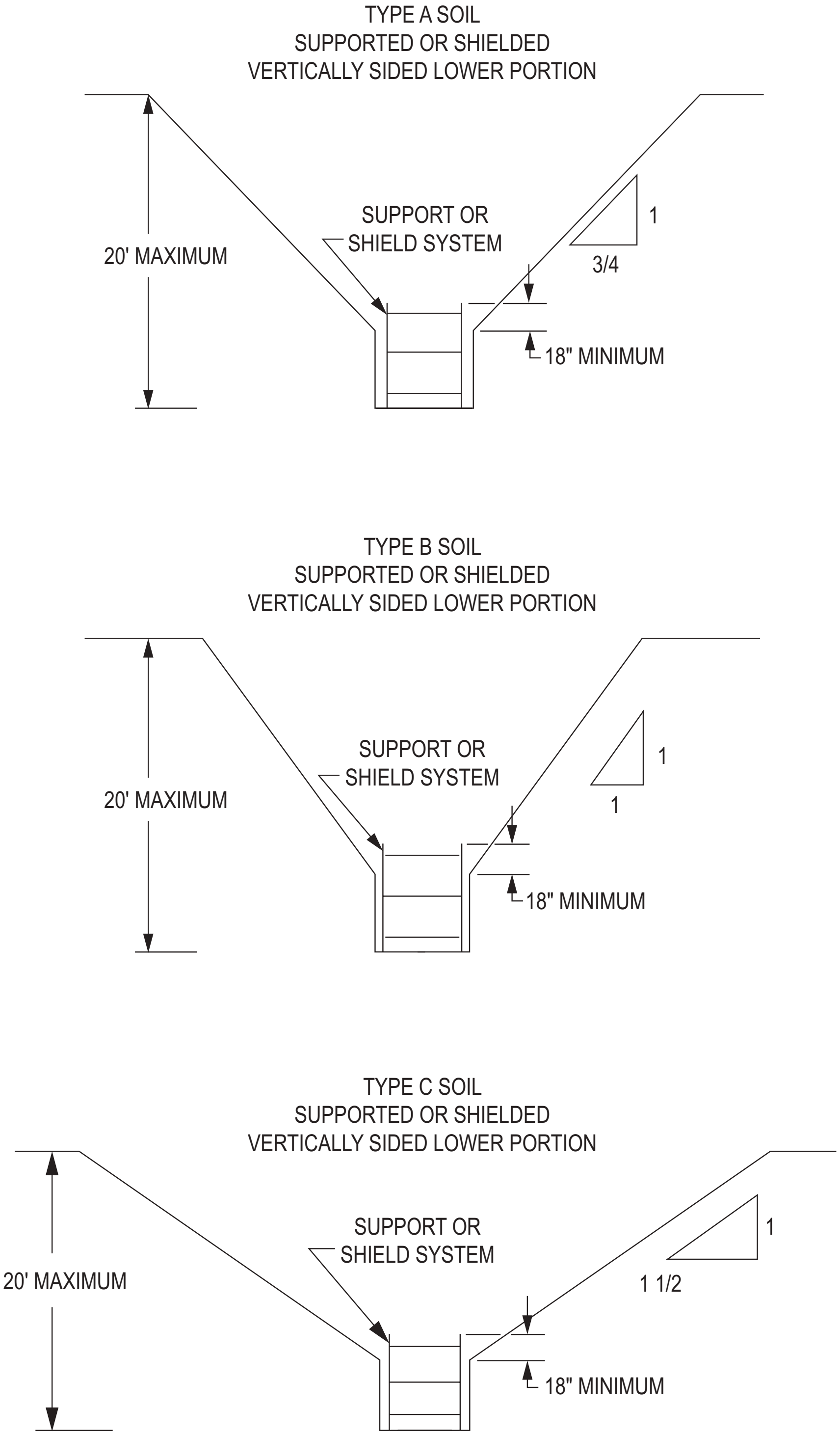
C. Type B Soils are cohesive soils with an unconfined compressive strength greater than 0.5 tsf (48 kPa) but less than 1.5 tsf (144 kPa). Examples of other Type B soils are: angular gravel, silt, silt loam, previously disturbed soils unless otherwise classified as Type C, soils that meet the unconfined compressive strength or cementation requirements of Type A soils but are fissured or subject to vibration, dry unstable rock, and layered systems sloping into the trench at a slope less than 4H:1V (only if the material would be classified as a Type B soil).

D. Type C Soils are cohesive soils with an unconfined compressive strength of 0.5 tsf (48 kPa) or less. Other Type C soils include granular soils such as gravel, sand and loamy sand, submerged soil, soil from which water is freely seeping, and submerged rock that is not stable. Also included in this classification is material in a sloped, layered system where the layers dip into the excavation or have a slope of four horizontal to one vertical (4H:1V) or greater.

E. Layered Geological Strata. Where soils are configured in layers, i.e., where a layered geological structure exists, the soil must be classified on the basis of the soil classification of the weakest soil layer. Each layer may be classified individually if a more stable layer lies below a less stable layer, i.e., where a Type C soil rests on top of stable rock.

Source: Occupational Safety and Health Administration. *OSHA Technical Manual (OTM)*. TED 01-00-015.
Washington, DC: U.S. Department of Labor, Sec. V, Ch. 2.
https://www.osha.gov/dts/osta/otm/otm_v/otm_v_2.html#4.

3.10.2 Slope and Shield Configurations



OSHA Slope and Shield Configurations

Source: Occupational Safety and Health Administration. *OSHA Technical Manual (OTM)*. TED 01-00-015. Washington, DC: U.S. Department of Labor, Sec. V, Ch. 2, Fig. V:2-12. https://www.osha.gov/dts/osta/otm/otm_v/otm_v_2.html#4.

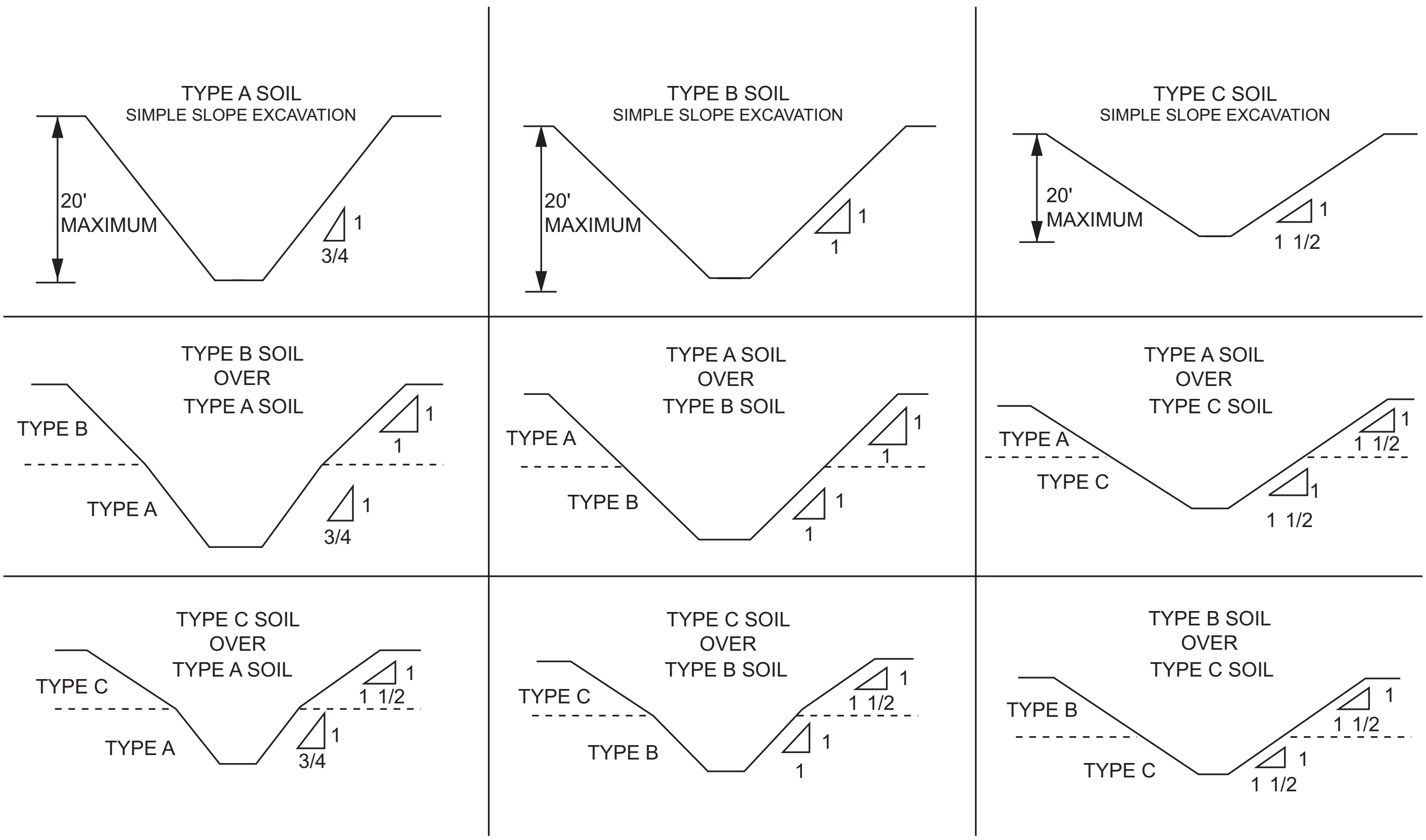
Allowable Slopes

Soil Type	Height: Depth Ratio	Slope Angle
Stable Rock	Vertical	90°
Type A	3/4:1	53°
Type B	1:1	45°
Type C	1 1/2:1	34°
Type A (short-term)*	1/2:1	63°

*For a maximum excavation depth of 12 ft for short-term conditions

Source: Occupational Safety and Health Administration. *OSHA Technical Manual (OTM)*. TED 01-00-015. Washington, DC: U.S. Department of Labor, Sec. V, Ch. 2, Table V:2. https://www.osha.gov/dts/osta/otm/otm_v/otm_v_2.html#4.

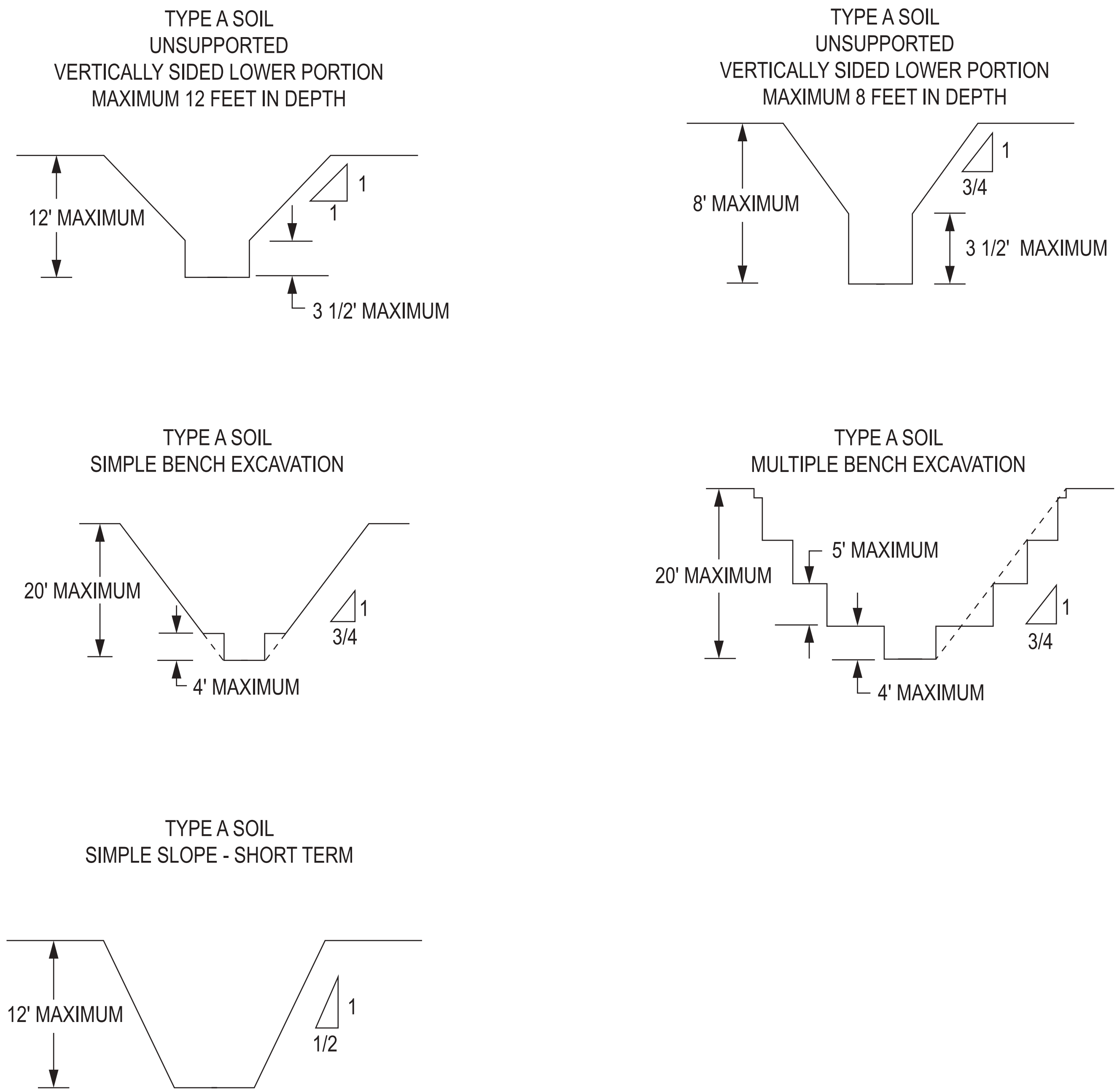
3.10.3 Slope Configurations: Excavations in Layered Soils



This figure illustrates the different types of slope excavations in layered soils. This includes Type A, B, and C soils in single-slope excavations, and different permutations such as Type A soil over Types B and C individually, Type B soil over Types A and C individually, and Type C soil over Types A and B individually.

Source: Occupational Safety and Health Administration. *OSHA Technical Manual (OTM)*. TED 01-00-015. Washington, DC: U.S. Department of Labor, Sec. V, Ch. 2, Fig. V:2-13. https://www.osha.gov/dts/osta/otm/otm_v/otm_v_2.html#4.

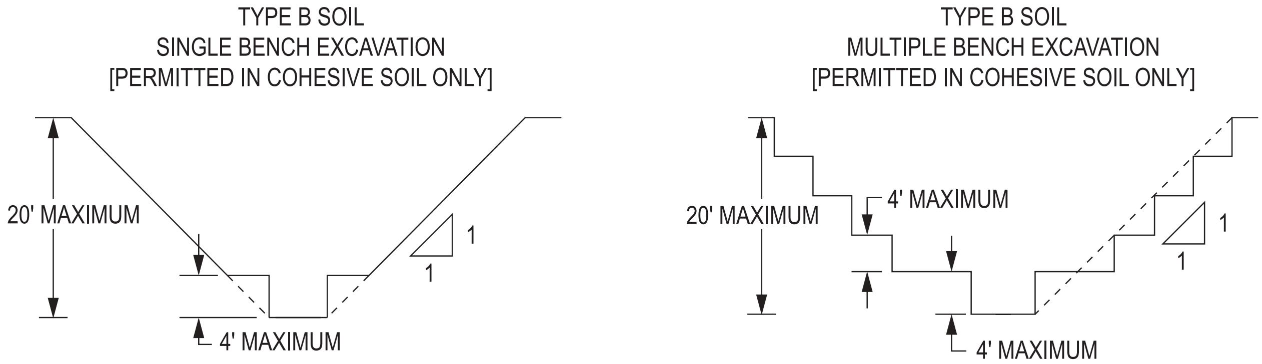
3.10.4 Excavations Made in Type A Soil



This figure illustrates the types of excavations made in Type A soil: Two types use an unsupported, vertically sided, lower portion (with maximum depths of 8 feet and 12 feet), and the other three types are single bench excavation, multiple bench excavation, and simple slope–short-term excavation.

Source: Occupational Safety and Health Administration. OSHA CFR Part 1926 Subpart P, App. B, *Sloping and Benching,* Fig. B-1.1 http://www.osha.gov/laws-regs/regulations/standardnumber/1926/1926SubpartPAppB.

3.10.5 Excavations Made in Type B Soil



This figure illustrates two types of excavations made in Type B soil that are permitted in cohesive soil only: single-bench excavation and multiple-bench excavation

Source: Occupational Safety and Health Administration. *OSHA Technical Manual (OTM)*. TED 01-00-015. Washington, DC: U.S. Department of Labor, Sec. V, Ch. 2, Fig. V:2-15. https://www.osha.gov/dts/osta/otm/otm_v/otm_v_2.html#4.

3.11 Geotechnical Instrumentation

Instruments for Measuring Piezometric Pressure

Instrument Type	Advantages	Limitations[a]
Observation well	Easy installation Field-readable	Provides vertical connection between strata and should only be used in continuously permeable strata
Open standpipe piezometer	Reliable Long successful performance record Self-de-airing if inside diameter of standpipe is adequate Integrity of seal can be checked after installation Can be used to determine permeability Readings can be made by installing pressure transducer or sonic sounder in standpipe	Time lag can be a factor Subject to damage by construction equipment and by vertical compression of soil around standpipe Extension of standpipe through embankment fill interrupts construction and may cause inferior compaction Possible freezing problems Porous filter can plug owing to repeated water inflow and outflow
Twin-tube hydraulic piezometer	Buried components have no moving parts Reliable when maintained Long successful performance record When installed in fill, integrity can be checked after installation Piezometer cavity can be flushed Can be used to determine permeability Short time lag Can be used to read negative pore water pressures	Application generally limited to long-term monitoring of pore water pressure in embankment dams Elaborate terminal arrangements needed Tubing must not be significantly above minimum piezometric elevation Periodic flushing required Possible freezing problems Attention to many details necessary
Pneumatic piezometer (embedded)	Short time lag Calibrated part of system accessible Minimum interferences to construction; level of tubes and readout independent of level of tip No freezing problems	Requires gas supply Installation, calibration, and maintenance require care

Instruments for Measuring Piezometric Pressure (cont'd)

Instrument Type	Advantages	Limitations[a]
Vibrating wire piezometer (embedded)	Easy to read Short time lag Minimum interference to construction; level of lead wires and readout independent of level of tip Lead wire effects minimal Can be used to read negative pore water pressures No freezing problems	Potential for zero drift (special manufacturing techniques required to minimize zero drift) Need for lightning protection should be evaluated
Electrical resistance piezometer (embedded)	Easy to read Short time lag Minimum interference to construction; level of lead wires and readout independent of level of tip Can be used to read negative pore water pressures No freezing problems	Potential lead wire effects unless converted to 4 to 20 milliamps Errors caused by moisture and corrosion are possible Need for lightning protection should be evaluated

[a] Diaphragm piezometer readings indicate the head above the piezometer and the elevation of the piezometer must be measured or estimated if piezometric elevation is required. All diaphragm piezometers, except those provided with a vent to the atmosphere, are sensitive to barometric pressure change. If piezometer pipes, tubes, or cables are carried up through fill, there will be significant interruption to construction and the probability of inferior compaction of the fill around the piezometer pipe.

Source: U.S. Army Corps of Engineers. *Engineering and Design: Instrumentation of Embankment Dams and Levees.* EM 1110-2-1908. Washington, DC: U.S. Department of the Army, June 1995, Table 4-1, p. 4-8. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1908.pdf.

Categories of Instruments for Measuring Deformation

Category	Type of Measured Deformation (Dunnicliff, 1988)					
	Horizontal ↔	Vertical ↕	Axial ⤢	Rotational ∘	Surface	Subsurface
Surveying Methods Optical and other methods Benchmarks Horizontal control stations Surface measuring points	●	●	●		●	
Probe Extensometers Mechanical probe gauges Electrical probe gauges Combined probe extensometers and inclinometer casings	●	●	●			●
Fixed Embankment Extensometers Settlement platforms Burled plates Gauges with electrical linear displacement transducers	●	●	●			●
Subsurface Settlement Points		●				●
Fixed Borehole Extensometers Single-point extensometers Multipoint extensometers	●	●	●			●
Inclinometers Probe inclinometers In-place inclinometers	●	●	●	●		●
Liquid Level Gauges Single-point gauges Full-profile gauges		●				●

Source: U.S. Army Corps of Engineers. *Engineering and Design: Instrumentation of Embankment Dams and Levees*. EM 1110-2-1908. Washington, DC: U.S. Department of the Army, June 1995, Table 4-2, p. 4-10. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1908.pdf.

Measurements and Instruments for Long-Term Performance Monitoring

Measurement, in Priority Order	Recommended Instruments
Condition of entire structure	Visual observations
Seepage	Seepage weirs or flumes Precipitation gauge Pool elevation
Pore water pressure[1]	Open standpipe piezometers Twin-tube hydraulic piezometers Vibrating wire piezometers Pneumatic piezometers Electrical resistance piezometers
Seismic events	Strong motion accelerographs Microseismographs
Surface vertical or lateral deformations	Surveying techniques Global positioning system
Subsurface vertical or lateral deformations	Liquid level gauges Inclinometers Extensometers Subsurface settlement points
Total stress at structure contacts	Contact earth pressure cells

[1] Listed in priority order, vibrating wire, pneumatic, and electrical resistance piezometers are only used in special cases. Source: Dunnicliff (1988)

Source: U.S. Army Corps of Engineers. *Engineering and Design: Instrumentation of Embankment Dams and Levees.* EM 1110-2-1908. Washington, DC: U.S. Department of the Army, June 1995, Table 9-1, p. 9-4. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1908.pdf.

3.12 Ground Improvement

3.12.1 Types of Ground Improvement

General Applicability of Technologies

Category	Technologies	Applicability
Vertical Drains and Accelerated Consolidation	PVDs, with and without fill preloading	Compressible clays, saturated low strength clays
Lightweight Fills	Compressive Strength Fills: Geofoam, Foamed Concrete	Broad applicability; no geologic or geometric limitations
Lightweight Fills	Granular Fills: Wood Fiber, Blast Furnace Slag, Fly Ash; Boiler Slag; Expanded Shale, Clay, and Slate; Tire Shreds	Broad applicability; no geologic or geometric limitations
Deep Compaction	Deep Dynamic Compaction	Loose pervious and semi-pervious soils with fines contents less than 15%, materials containing large voids, spoils, and waste areas
Deep Compaction	Vibro-Compaction	Cohesionless soils, clean sands with less than 15% silts and/or less than 2% clay
Aggregate Columns	Stone Columns	Clays, silts, loose silty sands, and uncompacted fill
Aggregate Columns	Rammed Aggregate Piers	Clays, silts, loose silty sands, and uncompacted fill
Column Supported Embankments	Column Supported Embankments	Soft compressible clay, peats, and organic soils where settlement and global stability are concerns
Column Supported Embankments	Reinforced Soil Load Transfer Platform	Soft compressible clay, peats, and organic soils where settlement and global stability are concerns
Column Supported Embankments	Columns: Non-compressible	All soil types, in particular weak soils that cannot support surface loads
Column Supported Embankments	Columns: Compressible	All soil types except very soft soils low undrained shear strength
Soil Mixing	Deep Mixing	Suitable in large range of soils, ones that can be stabilized with cement, lime, slag, or other binders
Soil Mixing	Mass Mixing	Peat, soft clay, dredged soil, soft silt, sludges, contaminated soils
Grouting	Chemical (Permeation) Grouting	Wide range of soil types including weakly cemented rock-fill materials

Source: Federal Highway Administration. National Highway Institute. *Ground Modification Methods, Reference Manual*. Vol. I. FHWA-NHI-16-027 and FHWA GEC 013. Washington, DC: U.S. Department of Transportation, April 2017, Table1-2, p. 1-10. https://www.fhwa.dot.gov/engineering/geotech/pubs/nhi16027.pdf.

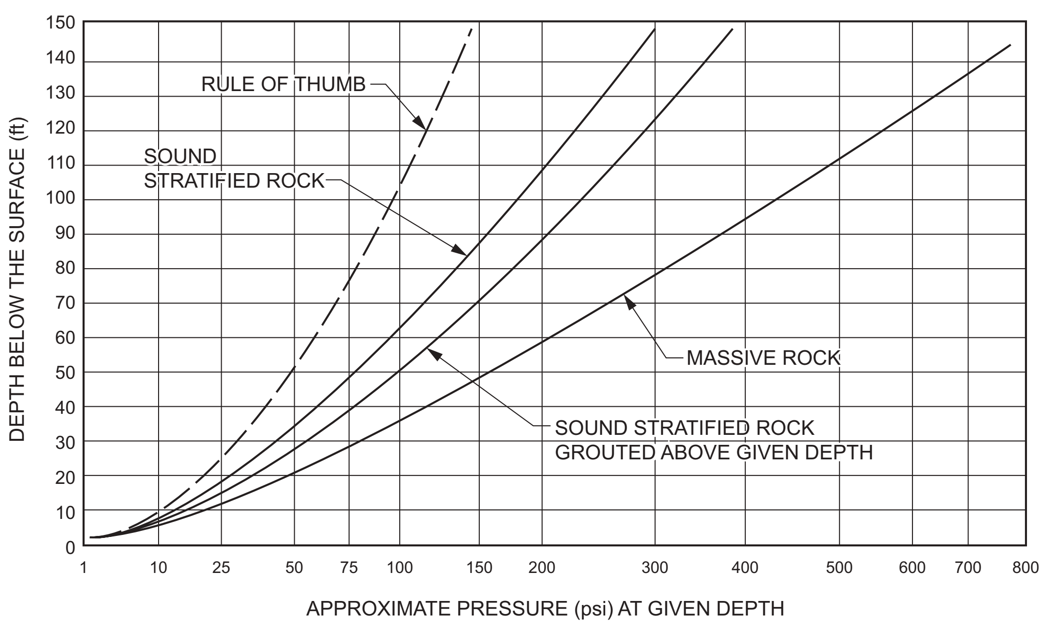
3.12.2 Grouting

Physical Properties of Chemical Grouts

Class	Example	Viscosity (centipoise)	Gel Time Range (min.)	Unconfined Compressive Strength (psi)
Precipitated grouts:				
Silicate (low concentration)	Silicate-bicarbonate	1.5	0.1–300	Under 50
Silicate (high concentration)	Silicate-formamide (Siroc)[a]	4–40	5–300	Over 500
	Silicate-chloride (Joosten)	30–50	0	Over 500
Chrome lignin	TDM	2.5–4	5–120	50 to 500
	Terra Firma[b]	2–5	10–300	Under 50
	Blox-All[c]	8	3–90	Under 50
	Lignosol[d]	50	10–1000	–
Polymerized grouts:				
Vinyl polymer	AM-9[e]	1.2–1.6	0.1–1000	50 to 500
Methylol bridge polymer	Urea formaldehyde	6	5–300	Over 500
	Herculox[c]	13	4–60	Over 500
	Cyanaloc 62[e]	13	1–60	Over 500
	Resorcinol-formaldehyde	3–5	–	Over 500
Oil-based unsaturated fatty-acid polymers	Polythixon FRD	10–80	25–360	Over 500
Epoxy resin	62E2[f]	2–18	–	Over 500

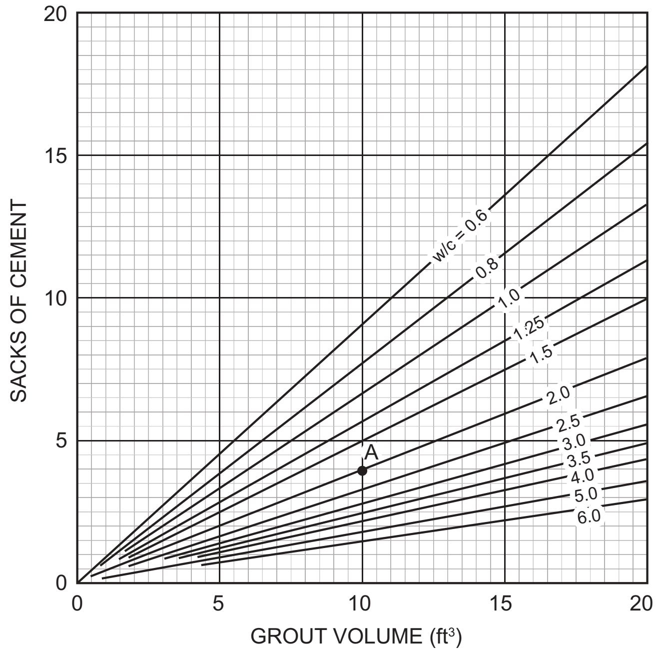
[a] Diamond Alkali Company
[b] Intrusion Prepakt, Inc.
[c] Halliburton Company
[d] Lignosol Chemical, Ltd.
[e] American Cyanamid Company
[f] George W. Whitesides Company

Source: Unified Facilities Criteria (UFC). *Grouting Methods and Equipment*. UFC 3-220-06. Washington, DC: U.S. Department of Defense, January 2004, Table 1, p. 18. https://www.wbdg.org/FFC/DOD/UFC/ufc_3_220_06_2004.pdf.



Rough Guide for Grouting Pressures

Source: Creager, Justin and Hinds. *Engineering for Dams*. Vol. 1. John Wiley and Sons, 1945. As found in Unified Facilities Criteria (UFC). *Grouting Methods and Equipment*. UFC 3-220-06. Washington, DC: U.S. Department of Defense, January 2004, Fig. 3, p. 24. https://www.wbdg.org/FFC/DOD/UFC/ufc_3_220_06_2004.pdf.

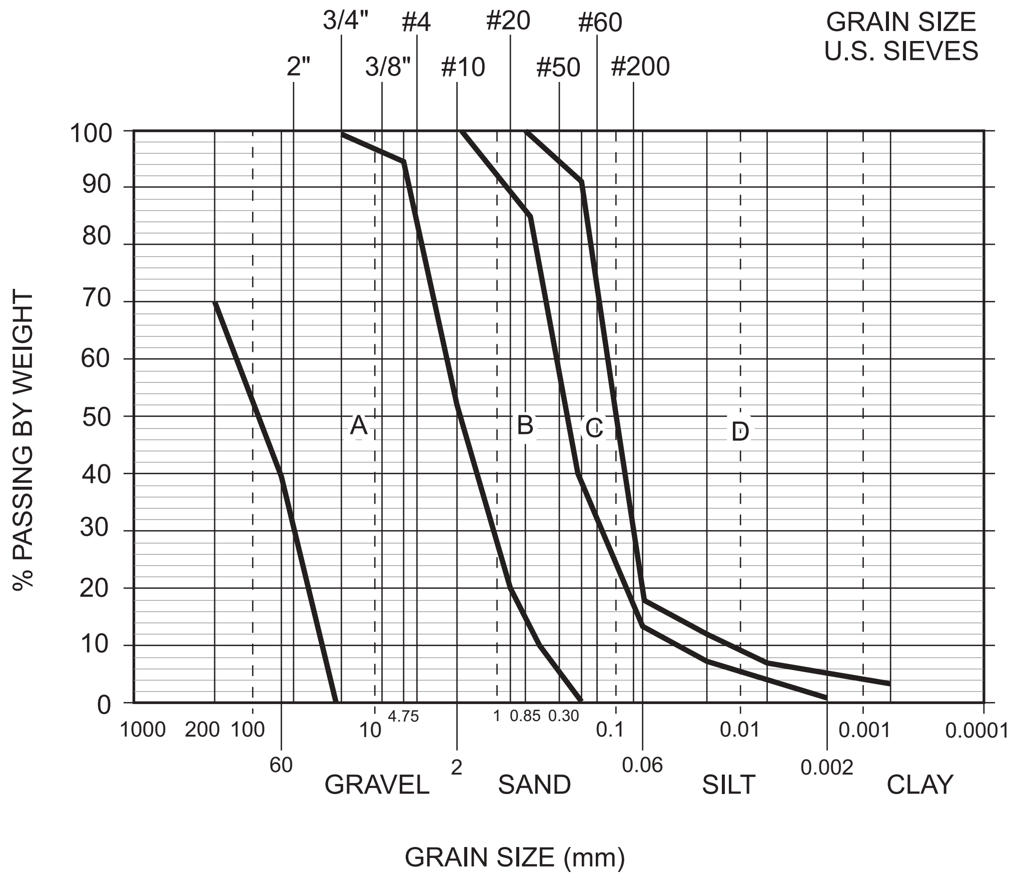


Cement Content of Portland-Cement Grout Mixes

Example: 10 ft^3 of 2.0 w/c grout (A) = 4.0 sacks of cement
Note: Water-cement ratio (w/c) = cubic feet water/sacks of cement

Source: Unified Facilities Criteria (UFC). *Grouting Methods and Equipment*. UFC 3-220-06. Washington, DC: U.S. Department of Defense, January 2004, Fig. 5, p. 28. https://www.wbdg.org/FFC/DOD/UFC/ufc_3_220_06_2004.pdf.

3.12.3 Vibrocompaction

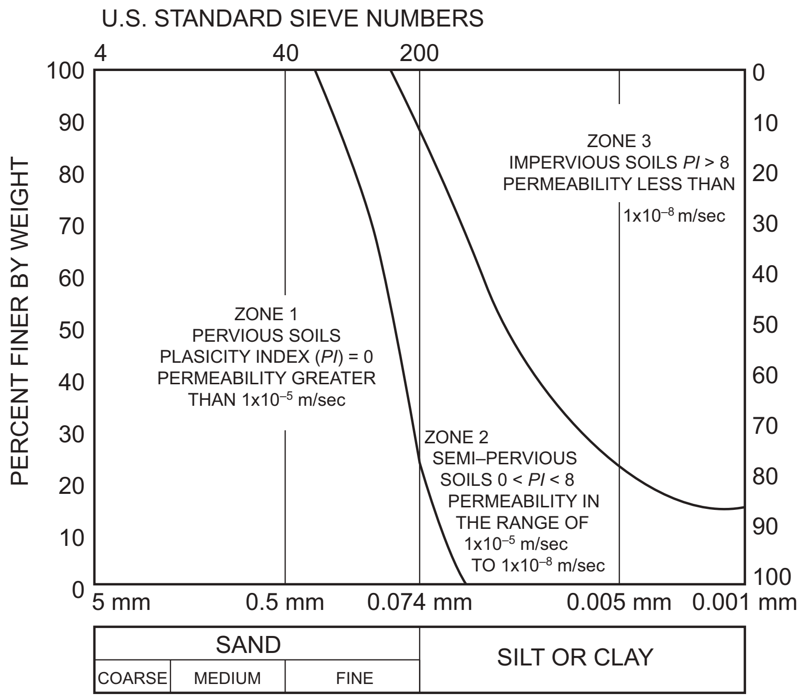


Range of Soil Types Treated by Vibrocompaction

Source: Federal Highway Administration. National Highway Institute. *Ground Modification Methods, Reference Manual.* Vol. I. FHWA-NHI-16-027 and FHWA GEC 013. Washington, DC: U.S. Department of Transportation, April 2017, Fig. 4-27, p. 4-66. https://www.fhwa.dot.gov/engineering/geotech/pubs/nhi16027.pdf.

Areas A and B – Vibrocompaction with Sand Backfill
Area C – Vibrocompaction with Gravel Backfill
Area D – Vibroreplacement

3.12.4 Dynamic Compaction

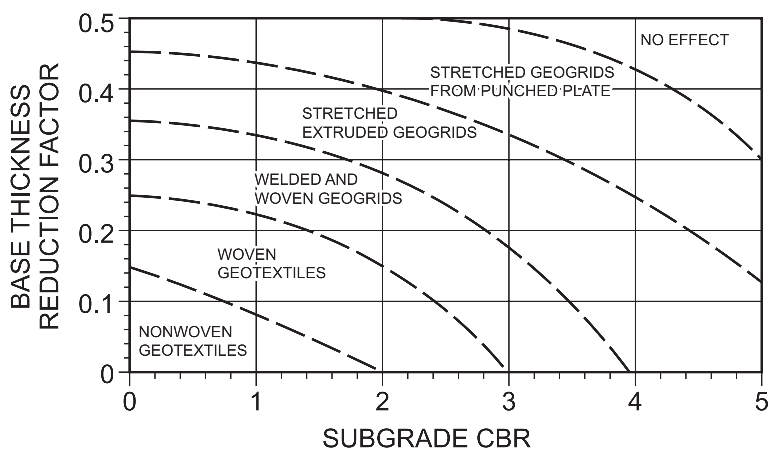


Grouping of Soils for Dynamic Compaction

Source: Federal Highway Administration. Geotechnical Engineering Circular No. 1: Dynamic Compaction. FHWA-SA-95-037. Washington, DC: U.S. Department of Transportation, October 1995, Fig. 5, p. 10. https://www.fhwa.dot.gov/engineering/geotech/pubs/009754.pdf.

3.13 Geosynthetics

3.13.1 Types of Geosynthetics



Reduction of Roadway Base Course Thickness Using Various Geosynthetics (after van Gurp and van Leest)

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 3.10, p. 410. Used with permission of the Geosynthetic Institute.

Identification of Usual Primary Function Versus Type of Geosynthetic

Type of Geosynthetic (GS)	Primary Function				
	Separation	Reinforcement	Filtration	Drainage	Containment
Geotextile (GT)	●	●	●	●	
Geogrid (GG)		●			
Geonet (GN)				●	
Geomembrane (GM)					●
Geosynthetic Clay Liner (GCL)					●
Geofoam (GF)	●				
Geocomposite (GC)	●	●	●	●	●

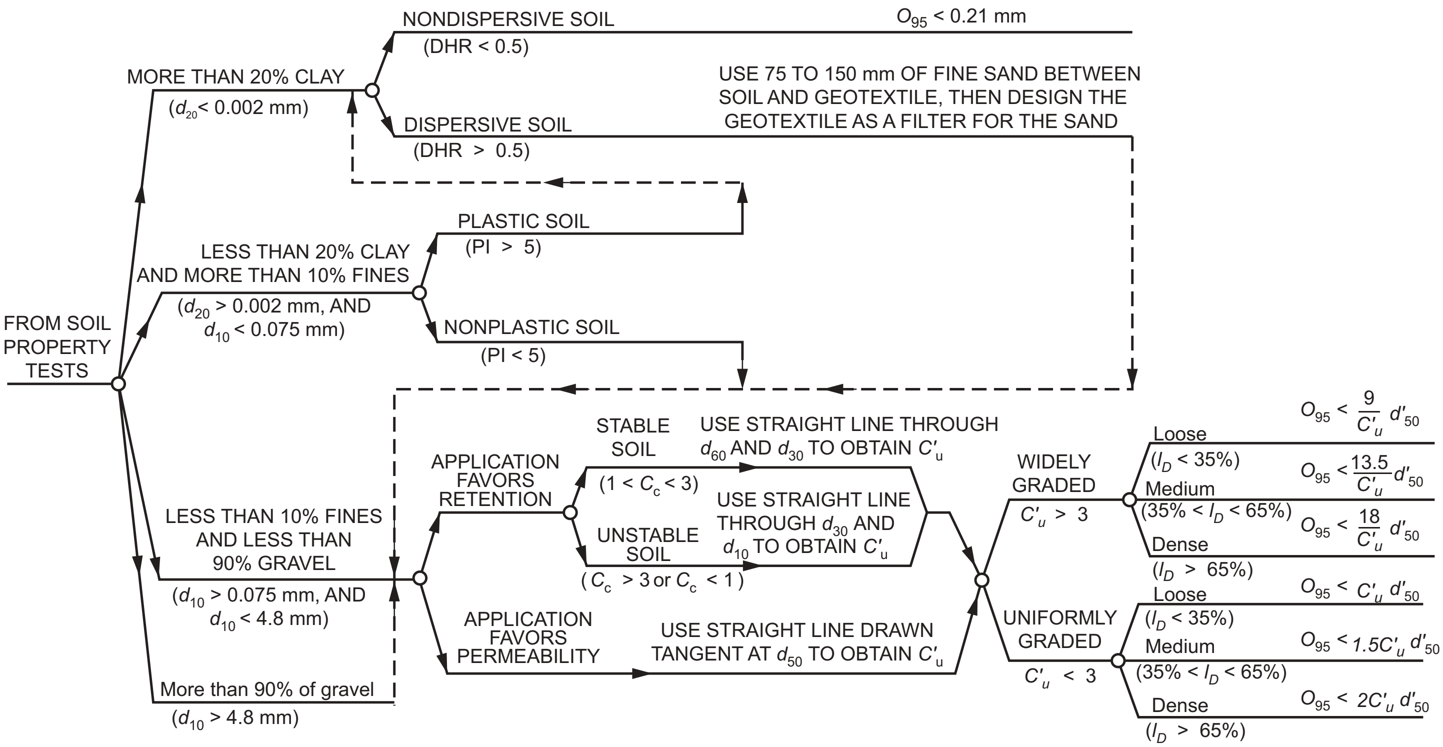
Source: Koerner, Robert M. *Designing with Geosynthetics.* Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Table 1.1, p. 9.
Used with permission of the Geosynthetic Institute.

The following are the most commonly used polymers in the manufacturing of geosynthetics:

- High-density polyethylene (HDPE) – developed in 1941
- Linear low-density polyethylene (LLDPE) – developed in 1956
- Polypropylene (PP) – developed in 1957
- Polyvinyl chloride (PVC) – developed in 1927
- Polyester (PET) – developed in 1950
- Expanded polystyrene (EPS) – developed in 1950
- Chlorosulphonated polyethylene (CSPE) – developed around 1965
- Thermoset polymers such as ethylene propylene diene terpolymer (EPDM) – developed in 1960

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, p. 11.
Used with permission of the Geosynthetic Institute.

3.13.2 Filter Criteria



Notes:

d_x = particle of which x percent is smaller

$C'_u = \sqrt{\dfrac{d'_{100}}{d'_0}}$, where d'_{100} and d'_0 are the extremities of a straight line drawn through the particle-size distribution, as directed above; and d'_{50} is the midpoint of this line

$$C_c = \frac{(d_{30})^2}{d_{60} \times d_{10}}$$

I_D = relative density of the soil

PI = plasticity index of the soil

DHR = double-hydrometer ratio of the soil

Soil Retention Criteria for Geotextile Filter Design Using Steady-State Flow Conditions (after Luettich et alia)

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 2.4a, p. 111. Used with permission of the Geosynthetic Institute.

Conversions of U.S. Standard Sieve Sizes to Equivalent Square Opening Sizes, in millimeters

Sieve Size (No.)	Opening (mm)
4	4.750
6	3.350
8	2.360
10	2.000
16	1.180
20	0.850
30	0.600
40	0.425
50	0.300
60	0.250
70	0.210
80	0.180
100	0.150
140	0.106
170	0.088
200	0.075
270	0.053
400	0.037

Source: Koerner, Robert M. *Designing with Geosynthetics.* Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Table 2.5, p. 144. Used with permission of the Geosynthetic Institute.

3.13.3 Strength Criteria

Allowable Tensile Strength Equation

$$T_{allow} = T_{ult}\left(\frac{1}{RF_{ID} \times RF_{CR} \times RF_{CBD}}\right)$$

$$T_{allow} = T_{ult}\left(\frac{1}{\Pi RF}\right)$$

where

T_{allow} = allowable tensile strength

T_{ult} = ultimate tensile strength

RF_{ID} = reduction factor for installation damage (≥ 1.0)

RF_{CR} = reduction factor for creep (≥ 1.0)

RF_{CBD} = reduction factor for chemical and biological degradation (≥ 1.0)

ΠRF = cumulative reduction factors (≥ 1.0)

Source: Koerner, Robert M. *Designing with Geosynthetics.* Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Equations 2.24a and 2.24b, p. 174. Used with permission of the Geosynthetic Institute.

Recommended Strength-Reduction Factor Values for Use in Allowable Tensile Strength Equation

Area	Range of Reduction Factors		
	Installation Damage	Creep*	Chemical/Biological Degradation
Separation	1.1 to 2.5	1.5 to 2.5	1.0 to 1.5
Cushioning	1.1 to 2.0	1.2 to 1.5	1.0 to 2.0
Unpaved roads	1.1 to 2.0	1.5 to 2.5	1.0 to 1.5
Walls	1.1 to 2.0	2.0 to 4.0	1.0 to 1.5
Embankments	1.1 to 2.0	2.0 to 3.5	1.0 to 1.5
Bearing and foundations	1.1 to 2.0	2.0 to 4.0	1.0 to 1.5
Slope stabilization	1.1 to 1.5	2.0 to 3.0	1.0 to 1.5
Pavement overlays	1.1 to 1.5	1.0 to 2.0	1.0 to 1.5
Railroads (filter/sep.)	1.5 to 3.0	1.0 to 1.5	1.5 to 2.0
Flexible forms	1.1 to 1.5	1.5 to 3.0	1.0 to 1.5
Silt fences	1.1 to 1.5	1.5 to 2.5	1.0 to 1.5

*The low end of the range refers to applications that have relatively short service lifetimes and/or situations where creep deformations are not critical to the overall system performance.

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Table 2.8a, p. 175. Used with permission of the Geosynthetic Institute.

Recommended Reduction Factor Values for Use in Determining Allowable Tensile Strength of Geogrids

Application Area	Reduction Factor Values		
	RF_{ID}	RF_{CR}	RF_{CBD}
Paved roads	1.2 to 1.5	1.5 to 2.5	1.1 to 1.7
Unpaved roads	1.1 to 1.6	1.5 to 2.5	1.0 to 1.6
Embankments	1.1 to 1.4	2.0 to 3.0	1.1 to 1.5
Slopes	1.1 to 1.4	2.0 to 3.0	1.1 to 1.5
Walls	1.1 to 1.4	2.0 to 3.0	1.1 to 1.5
Foundations	1.2 to 1.5	2.0 to 3.0	1.1 to 1.6
Veneer covers	1.1 to 1.4	1.5 to 2.5	1.1 to 1.6

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Table 3.2, p. 401. Used with permission of the Geosynthetic Institute.

3.13.4 Flow Rates

$$q_{\text{allow}} = q_{\text{ult}}\left(\frac{1}{RF_{\text{SCB}} \times RF_{\text{CR}} \times RF_{\text{IN}} \times RF_{\text{CC}} \times RF_{\text{BC}}}\right)$$

$$q_{\text{allow}} = q_{\text{ult}}\left(\frac{1}{\Pi RF}\right)$$

where

q_{allow} = allowable flow rate

q_{ult} = ultimate flow rate

RF_{SCB} = reduction factor for soil clogging and binding (≥ 1.0)

RF_{CR} = reduction factor for creep reduction of void space (≥ 1.0)

RF_{IN} = reduction factor for adjacent materials intruding into the geotextile's void space (≥ 1.0)

RF_{CC} = reduction factor for chemical clogging (≥ 1.0)

RF_{BC} = reduction factor for biological clogging (≥ 1.0)

ΠRF = cumulative reduction factors (≥ 1.0)

Recommended Flow-Reduction Factor Values for Use in Allowable Flow Rates Equation

Application	Range of Reduction Factors				
	Clogging and Binding*	Creep Reduction of Voids	Intrusion into Voids	Chemical Clogging**	Biological Clogging***
Retaining wall filters	2.0 to 4.0	1.5 to 2.0	1.0 to 1.2	1.0 to 1.2	1.0 to 1.3
Underdrain filters	2.0 to 10	1.0 to 1.5	1.0 to 1.2	1.2 to 1.5	2.0 to 4.0***
Erosion control filters	2.0 to 10	1.0 to 1.5	1.0 to 1.2	1.0 to 1.2	2.0 to 4.0
Landfill filters	2.0 to 10	1.5 to 2.0	1.0 to 1.2	1.2 to 1.5	2.0 to 5.0***
Gravity drainage	2.0 to 4.0	2.0 to 3.0	1.0 to 1.2	1.2 to 1.5	1.2 to 1.5
Pressure drainage	2.0 to 3.0	2.0 to 3.0	1.0 to 1.2	1.1 to 1.3	1.1 to 1.3

* If stone riprap or concrete blocks cover the surface of the geotextile, use either the upper values or include an addition for reduction factor.
** Values can be higher, particularly for high alkalinity groundwater.
***Values can be higher for extremely high microorganism content and/or growth of organisms and plant/vegetation roots.

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Table 2.8b, p. 176. Used with permission of the Geosynthetic Institute.

3.13.5 Reinforced Walls and Slopes

3.13.5.1 Geotextile-Reinforced Walls

$$\sigma_{hs} = K_a \gamma Z$$

$$\sigma_{hq} = K_a q$$

$$\sigma_{hl} = P\frac{x^2 Z}{R^5}$$

$$\sigma_h = \sigma_{hs} + \sigma_{hq} + \sigma_{hl}$$

$$\sigma_h S_v = \frac{T_{allow}}{FS}$$

$$S_v = \frac{T_{allow}}{\sigma_h FS}$$

$$L = L_e + L_R$$

$$L_R = (H - Z)\tan\left(45 - \frac{\phi}{2}\right)$$

$$L_e = \frac{S_v \sigma_h FS}{2(c_a + \gamma Z \tan\delta)}$$

$$L_o = \frac{S_v \sigma_h FS}{4(c_a + \gamma Z \tan\delta)}$$

where

σ_h = total lateral pressure

σ_{hs} = lateral pressure from soil

σ_{hq} = lateral pressure from surcharge

σ_{hl} = lateral pressure from live load

γ = unit weight of soil

Z = depth from ground surface

x = horizontal distance from wall

R = radial distance from point load P

q = surcharge pressure = $\gamma_q D$

  γ_q = unit weight of surcharge soil

  D = depth of surcharge soil

P = concentrated load

K_a = coefficient of active earth pressure = $\tan^2(45 - \phi/2)$

  ϕ = angle of internal friction of backfill soil

S_v = vertical spacing (lift thickness)

T_{allow} = allowable stress

FS = factor of safety

H = retained height of wall

L = total geotextile length

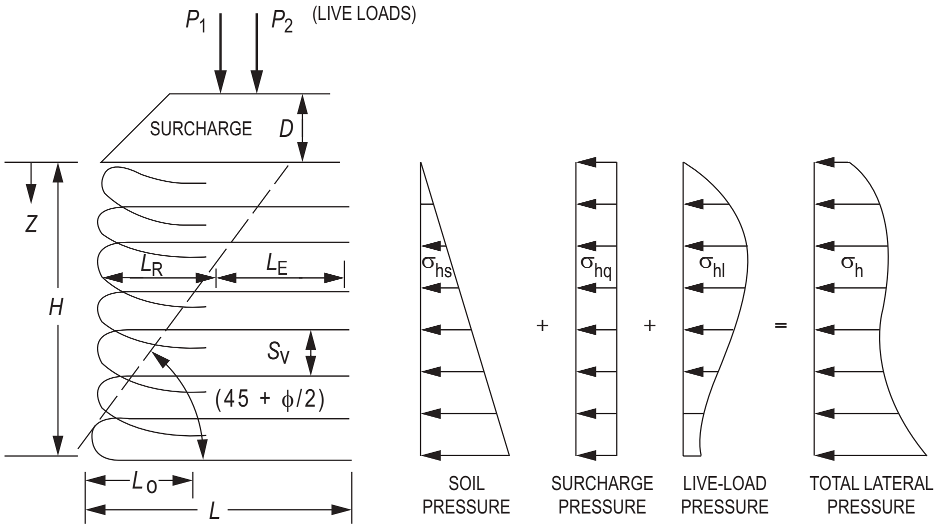
L_R = nonacting length of geotextile within the active zone for L

L_e = required embedment length (minimum 1.0 m)

L_o = required overlap length (minimum 1.0 m)

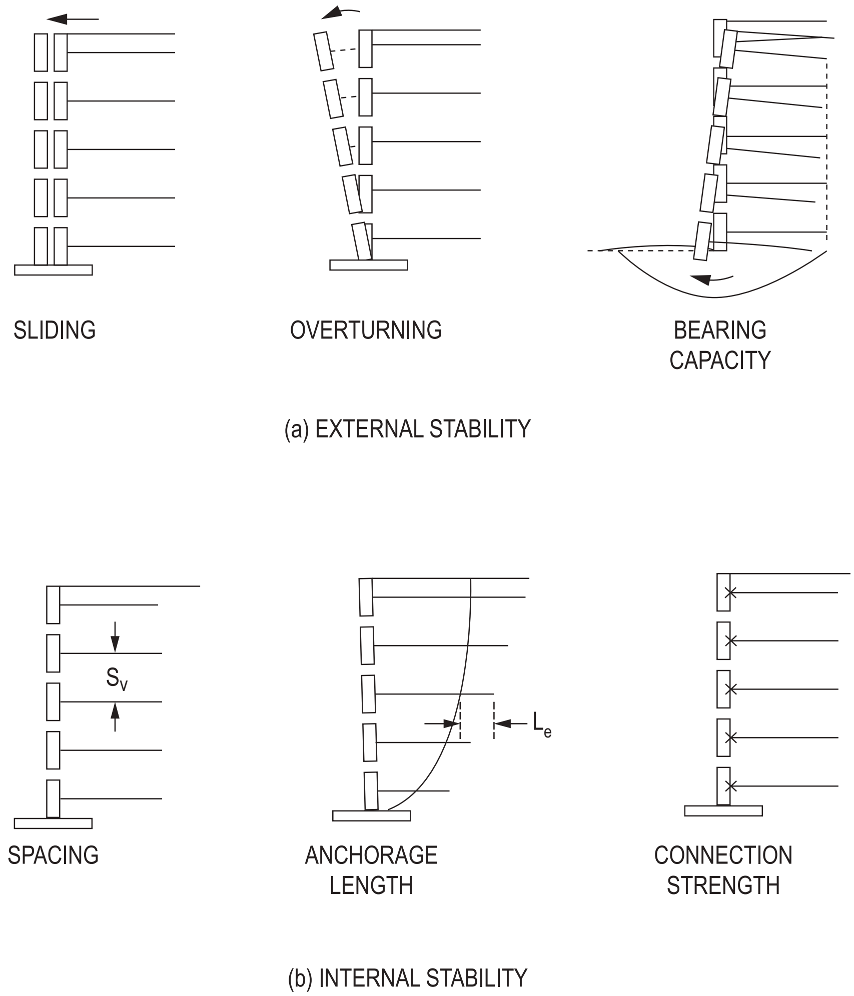
c_a = soil adhesion between soil and geotextile

δ = angle of shearing friction between soil and geotextile



Earth Pressure Concepts and Theory for Geotextile Wall Design

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 2.33, p. 212. Used with permission of the Geosynthetic Institute.



Elements of Geogrid (or Geotextile) Reinforced Wall Design

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 3.19, p. 425.
Used with permission of the Geosynthetic Institute.

3.13.5.2 Reinforced Slopes with Geotextiles

$$FS = \frac{\sum_{i=1}^{n}\left(N_i \tan\phi + c\Delta l_i\right)R + \sum_{i=1}^{m} T_i y_i}{\sum_{i=1}^{n}\left(W_i \sin\theta_i\right)R}$$

$$N_i = W_i \cos\theta_i$$

$$FS = \frac{\sum_{i=1}^{n}\left(\overline{N_i} \tan\overline{\phi} + \overline{c}\Delta l_i\right)R + \sum_{i=1}^{m} T_i y_i}{\sum_{i=1}^{n}\left(\overline{W_i} \sin\theta_i\right)R}$$

$$\overline{N_i} = N_i - u_i \Delta x_i$$

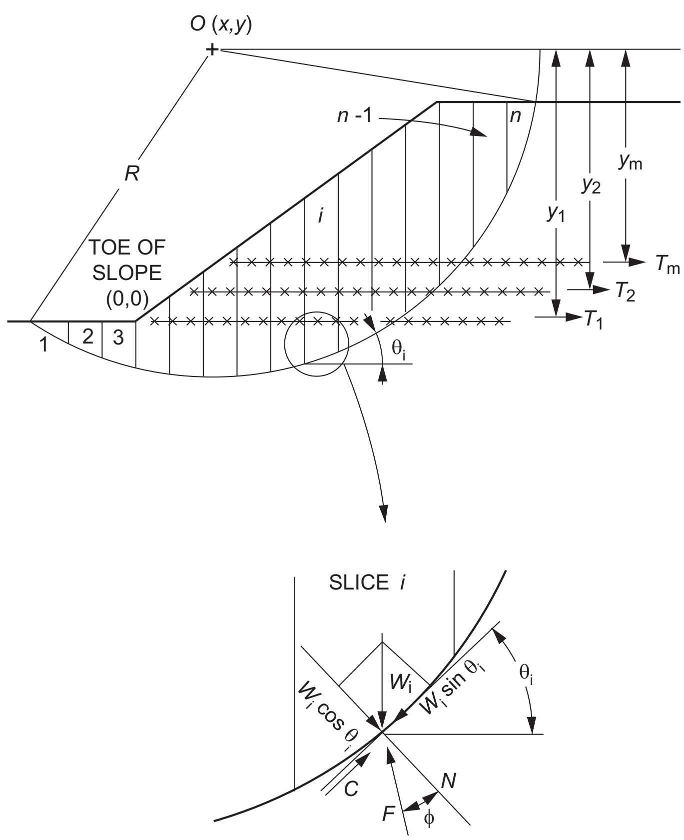
$$u_i = h_i \gamma_w$$

where

FS = (global) factor of safety

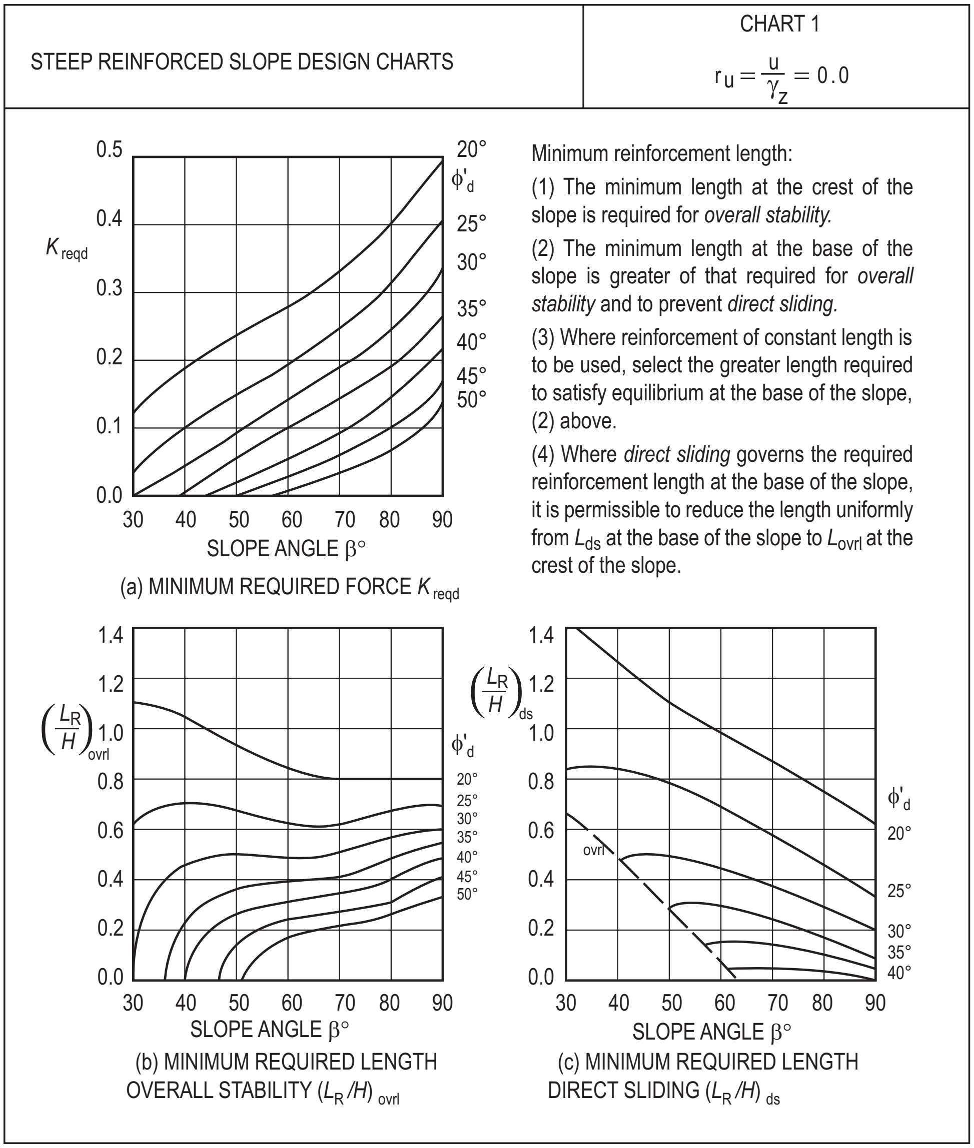
$W_i, \overline{W_i}$ = total and effective weight of each slice i

θ_i	= angle of intersection of horizontal to tangent at center of each slice
$\phi, \overline{\phi}$	= total and effective angles of shearing resistance
R	= radius of failure
$c, \overline{c}$	= total and effective cohesion
Δl_i	= arc length of each slice
$N_i, \overline{N}_i$	= total and effective normal force at the base of the slice
T_i	= allowable geotextile tensile strength
y_i	= moment arm for geotextiles
n	= number of slices
m	= number of geotextile layers
Δx_i	= width of slice
γ_w	= unit weight of water
h_i	= height of water above base of the circle for each slice
u_i	= pore water pressure



Details of Circular Arc Slope Stability Analysis for (c, ϕ) Shear Strength Soils

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 2.38, p. 233.
Used with permission of the Geosynthetic Institute.



Steep, Reinforced, Soil-Slope Design Charts for Zero-Pore Water Pressure (after Jewell)

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 3.15, p. 418. Used with permission of the Geosynthetic Institute.

where

L_R = length of reinforcement

H = height of slope

K_{reqd} = lateral load factor

$$T_{reqd} = K \gamma_z H$$

where

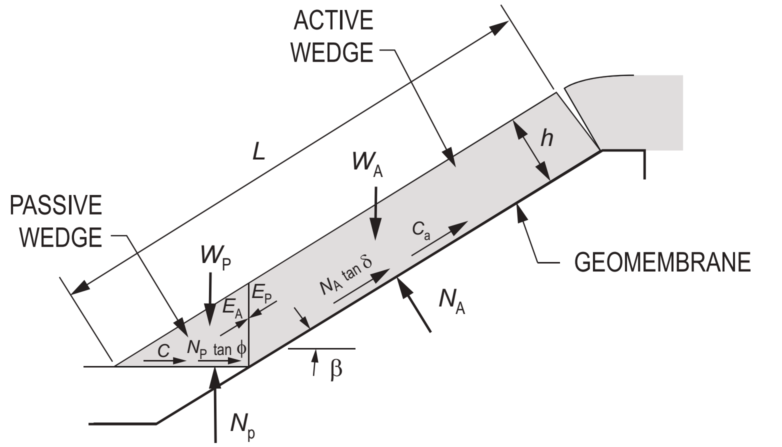
γ_z = unit weight of soil

ϕ_d = angle of internal friction

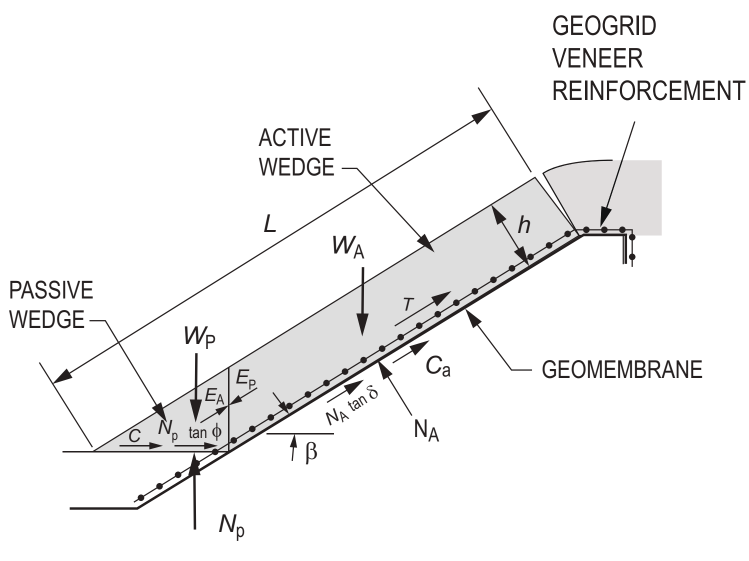
β = slope angle (from horizontal)

u = pore water pressure

r_u = pore water pressure factor



(a) WITHOUT REINFORCEMENT



(b) WITH THE USE OF GEOGRID VENEER REINFORCEMENT

Limit Equilibrium Forces Involved in a Finite Length Slope Analysis for a Uniformly Thick Cover Soil

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 1. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 3.24, p. 444. Used with permission of the Geosynthetic Institute.

$$W_P = \frac{\gamma h^2}{\sin 2\beta}$$

$$N_P = W_P + E_P \sin\beta$$

$$C = \frac{c \times h}{\sin\beta}$$

$$W_A = \gamma h^2\left(\frac{L}{h} - \frac{1}{\sin\beta} - \frac{\tan\beta}{2}\right)$$

$$N_A = W_A \cos\beta$$

$$C_a = c_a\left(L - \frac{h}{\sin\beta}\right)$$

$$E_A = \frac{(FS)(W_A - N_A\cos\beta) - (N_A\tan\delta + C_a)\sin\beta}{\sin\beta(FS)}$$

$$E_P = \frac{C + W_P\tan\phi}{\cos\beta(FS) - \sin\beta\tan\phi}$$

$$FS = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = (W_A - N_A\cos\beta)\cos\beta$$

$$b = -\left[(W_A - N_A\cos\beta)\sin\beta\tan\phi + (N_A\tan\delta + C_a)\sin\beta\cos\beta + \sin\beta(C + W_P\tan\phi)\right]$$

$$c = (N_A\tan\delta + C_a)\sin^2\beta\tan\phi$$

where

W_A = total weight of the active wedge

W_P = total weight of the passive wedge

N_A = effective force normal to the failure plane of the active wedge

N_P = effective force normal to the failure plane of the passive wedge

γ = unit weight of the cover soil

h = thickness of the cover soil

L = length of slope measured along the geomembrane

β = soil slope angle beneath the geomembrane

ϕ = friction angle over the cover soil

δ = interface friction angle between cover soil and geomembrane

C_a = adhesive force between cover soil of the active wedge and the geomembrane

c_a = adhesion between cover soil of the active wedge and geomembrane

C = cohesive force along the failure plane of the passive wedge

c = cohesion of the cover soil

E_A = interwedge force acting on the active wedge from the passive wedge

E_P = interwedge force acting on the passive wedge from the active wedge

FS = factor of safety against cover soil sliding on the geomembrane

Peak Friction Values and Efficiencies of Various Geosynthetic Interfaces*

(a) Soil-to-Geomembrane Friction Angles

Geomembrane	Soil Type					
	Concrete Sand (ϕ = 30°)		Ottawa Sand (ϕ = 28°)		Mica Schist Sand (ϕ = 26°)	
HDPE						
Textured	30°	(1.00)	26°	(0.92)	22°	(0.83)
Smooth	18°	(0.56)	18°	(0.61)	17°	(0.63)
PVC						
Rough	27°	(0.88)	–	–	25°	(0.96)
Smooth	25°	(0.81)	–	–	21°	(0.79)
fPP-R	25°	(0.81)	21°	(0.72)	23°	(0.87)

(b) Geomembrane-to-Geotextile Friction Angles

Geotextile	Geomembrane				
	HDPE		PVC		fPP-R
	Textured	Smooth	Rough	Smooth	
Nonwoven, needle-punched	32°	8°	23°	21°	15°
Nonwoven, heat-bonded	28°	11°	20°	18°	21°
Woven, monofilament	19°	6°	11°	10°	9°
Woven, slit film	32°	10°	28°	24°	13°

(c) Soil-to-Geotextile Friction Angles

Geotextile	Soil Type					
	Concrete Sand (ϕ = 30°)		Concrete Sand (ϕ = 30°)		Mica Schist Sand (ϕ = 26°)	
Nonwoven, needle-punched	30°	(1.00)	26°	(0.92)	25°	(0.96)
Nonwoven, heat-bonded	26°	(0.84)	–	–	–	–
Woven, monofilament	26°	(0.84)	–	–	–	–
Woven, slit film	24°	(0.77)	24°	(0.84)	23°	(0.87)

*Efficiency values (in parentheses) are based on the relationship $E = \tan\delta/\tan\phi$.

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 2. 6th ed. Bloomington, IN: Xlibris, 2012, Table 5.6, p. 540.
Used with permission of the Geosynthetic Institute.

3.13.6 Geofoam

Physical Property Requirements of Geofoam According to ASTM D6817

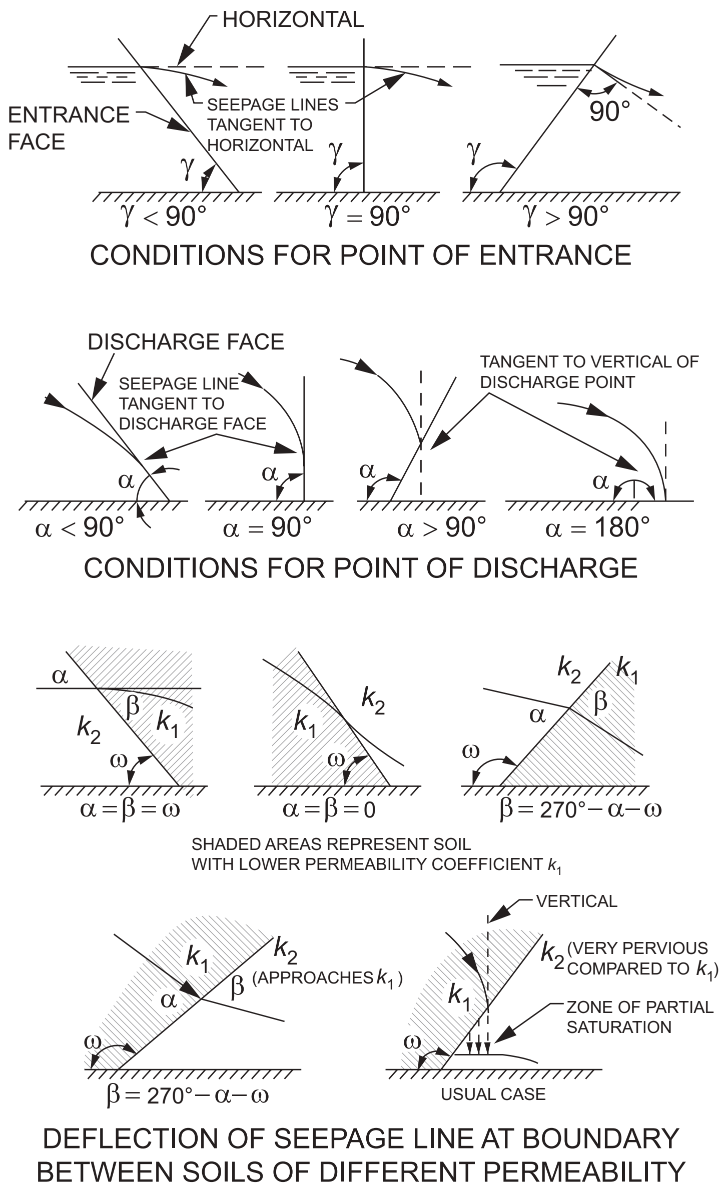
Type	EPS12	EPS15	EPS19	EPS22	EPS29	XPS20	XPS21	XPS26	XPS29	XPS36	XPS48
Density, min., kg/m^3 [lb/ft3]	11.2 [0.70]	14.4 [0.90]	18.4 [1.15]	21.6 [1.35]	28.8 [1.80]	19.2 [1.20]	20.8 [1.30]	25.6 [1.60]	28.8 [1.80]	35.2 [2.20]	48.0 [3.00]
Compressive Strength, min., kPa [psi] at 1% strain	15 [2.2]	25 [3.6]	40 [5.8]	50 [7.3]	75 [10.9]	20 [2.9]	35 [5.1]	75 [10.9]	105 [15.2]	160 [23.2]	280 [40.6]
Compressive Strength, min., kPa [psi] at 5% strain	35 [5.1]	55 [8.0]	90 [13.1]	115 [16.7]	170 [24.7]	85 [12.3]	110 [16.0]	185 [26.8]	235 [34.1]	335 [46.6]	535 [77.6]
Compressive Strength, min., kPa [psi] at 10% strain	40 [5.8]	70 [10.2]	110 [16.0]	135 [19.6]	200 [29.0]	104 [15.0]	104 [15.0]	173 [25.0]	276 [40.0]	414 [60.0]	690 [100.0]
Flexural Strength, min., kPa [psi]	69 [10.0]	172 [25.0]	207 [30.0]	240 [35.0]	345 [50.0]	276 [40.0]	276 [40.0]	345 [50.0]	414 [60.0]	517 [75.0]	689 [100.0]
Oxygen Index, min., volume %	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0

where

EPS*XX* = number designation for expanded polystyrene geofoam type(s) having a minimum density of *XX* kg/m^3
XPS*XX* = number designation for extruded polystyrene geofoam type(s) having a minimum density of *XX* kg/m^3

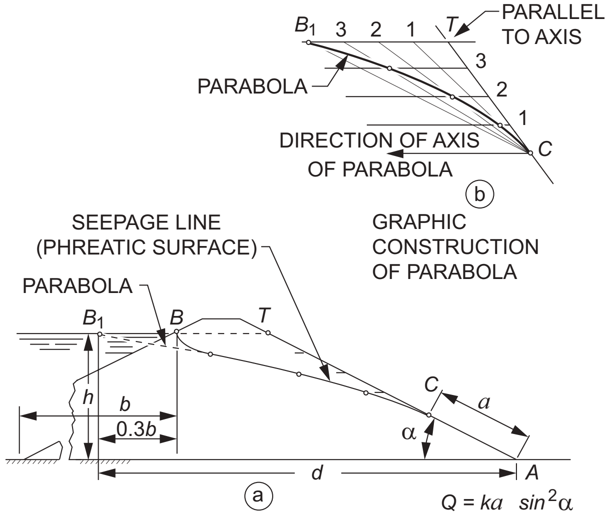
Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 2. 6th ed. Bloomington, IN: Xlibris, 2012, Table 7.1, p. 810. Used with permission of the Geosynthetic Institute. And ASTM D6817/D6817M-13a: *Standard Specification for Rigid Cellular Polystyrene Geofoam*. West Conshohocken, PA: ASTM International, May 2013.

3.14 Earth Dams, Levees, and Embankments



Entrance, Discharge, and Transfer Conditions of Seepage Line

Source: Bureau of Reclamation. *Reclamation: Managing Water in the West. Design Standards No. 13: Embankment Dams.* Chapter 8: Seepage. Phase 4 (Final). DS-13(8)-4.1. Washington, DC: U.S. Department of Interior, January 2014. Figure 8.4.4.1-1, p. 8-50. https://www.usbr.gov/tsc/techreferences/designstandards-datacollectionguides/finalds-pdfs/DS13-8.pdf.



Determination of Seepage Line for Homogeneous Section on Impervious Foundation α < 60°

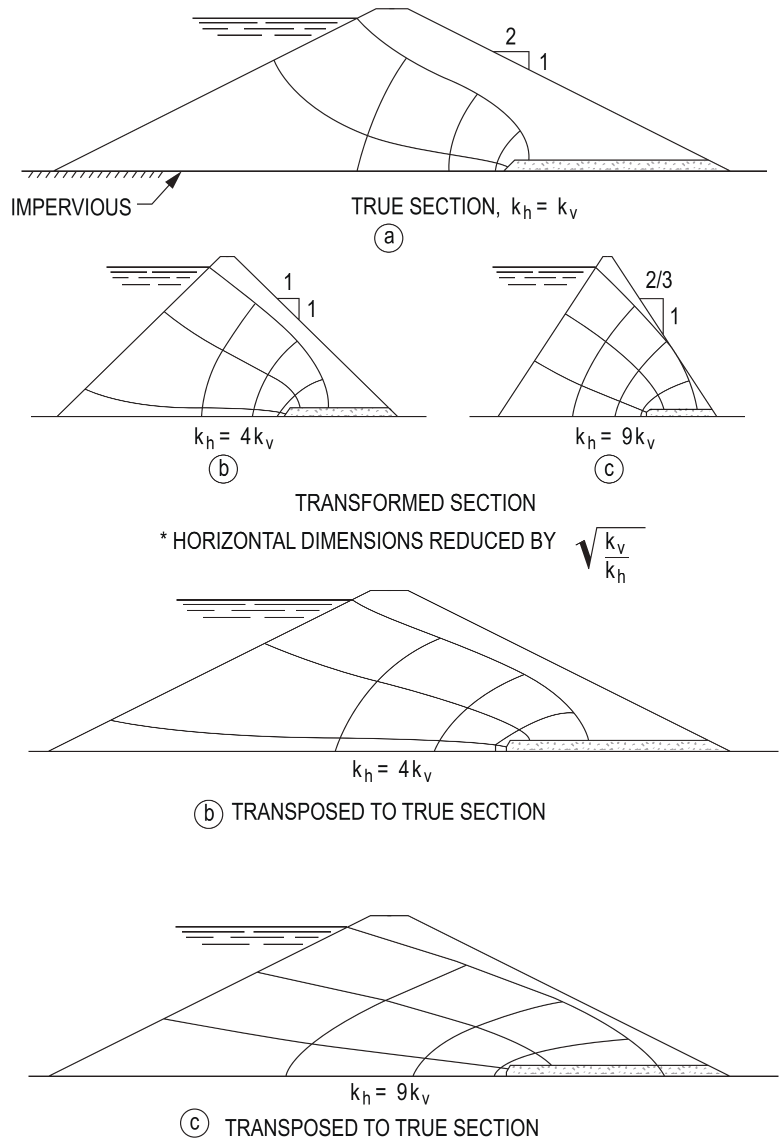
Procedure for locating seepage line:

1. Transform the section as in Transformation Method figure, if necessary (see following page).
2. Locate point B_1. $\overline{BB_1} = 0.3b$.
3. Locate discharge point C from equation:

   $a = S_o - \sqrt{S_o^2 - \frac{h^2}{\sin^2 \alpha}}$, where S_o is length of seepage line $\overline{B_1 CA}$

   $S_o = \sqrt{h^2 + d^2}$
4. Plot parabola by graphic method as shown in (b).
5. Complete seepage line by sketching a transition curve connecting point B with parabola.
6. Retransform the section, as in Transformation Method figure (see following page).

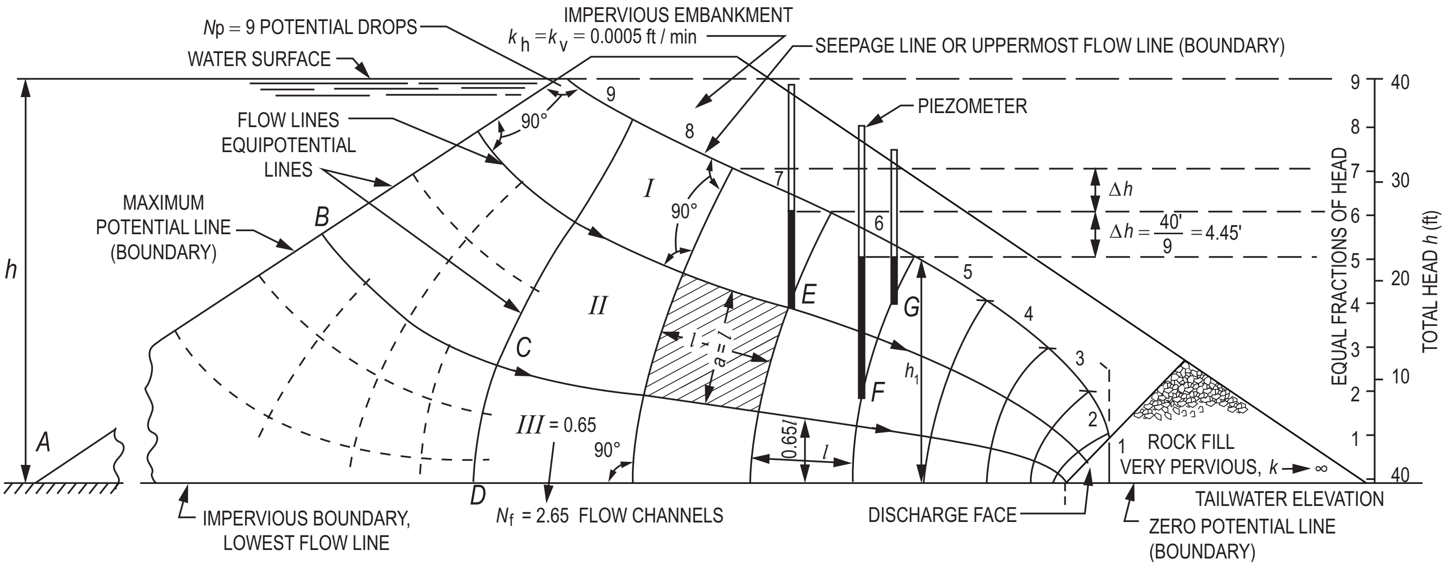
Source: Bureau of Reclamation. *Reclamation: Managing Water in the West. Design Standards No. 13: Embankment Dams*. Chapter 8: Seepage. Phase 4 (Final). DS-13(8)-4.1. Washington, DC: U.S. Department of Interior, January 2014. Figure 8.4.4.1-2, p. 8-51. https://www.usbr.gov/tsc/techreferences/designstandards-datacollectionguides/finalds-pdfs/DS13-8.pdf.



Transformation Method for Analysis of Anisotropic Embankments

Source: Bureau of Reclamation. *Reclamation: Managing Water in the West. Design Standards No. 13: Embankment Dams*. Chapter 8: Seepage. Phase 4 (Final). DS-13(8)-4.1. Washington, DC: U.S. Department of Interior, January 2014. Figure B-2, p. B-2. https://www.usbr.gov/tsc/techreferences/designstandards-datacollectionguides/finalds-pdfs/DS13-8.pdf.

Flow Nets



Computations:

1. Discharge per unit width of section:

$$Q = kh\frac{N_f}{N_p} = 0.0005 \times 40 \times \frac{2.65}{9} = 0.0059\ \frac{\text{ft}^3}{\text{min}} \text{ per foot of embankment}$$

2. Hydrostatic pressure at any point:

$h_1 = \frac{n_p}{N_p}h$, where n_p = number of potential drops between point and zero potential.

At point E, $h_1 = \frac{6}{9} \times 40 \text{ ft} = 26.7 \text{ ft}$

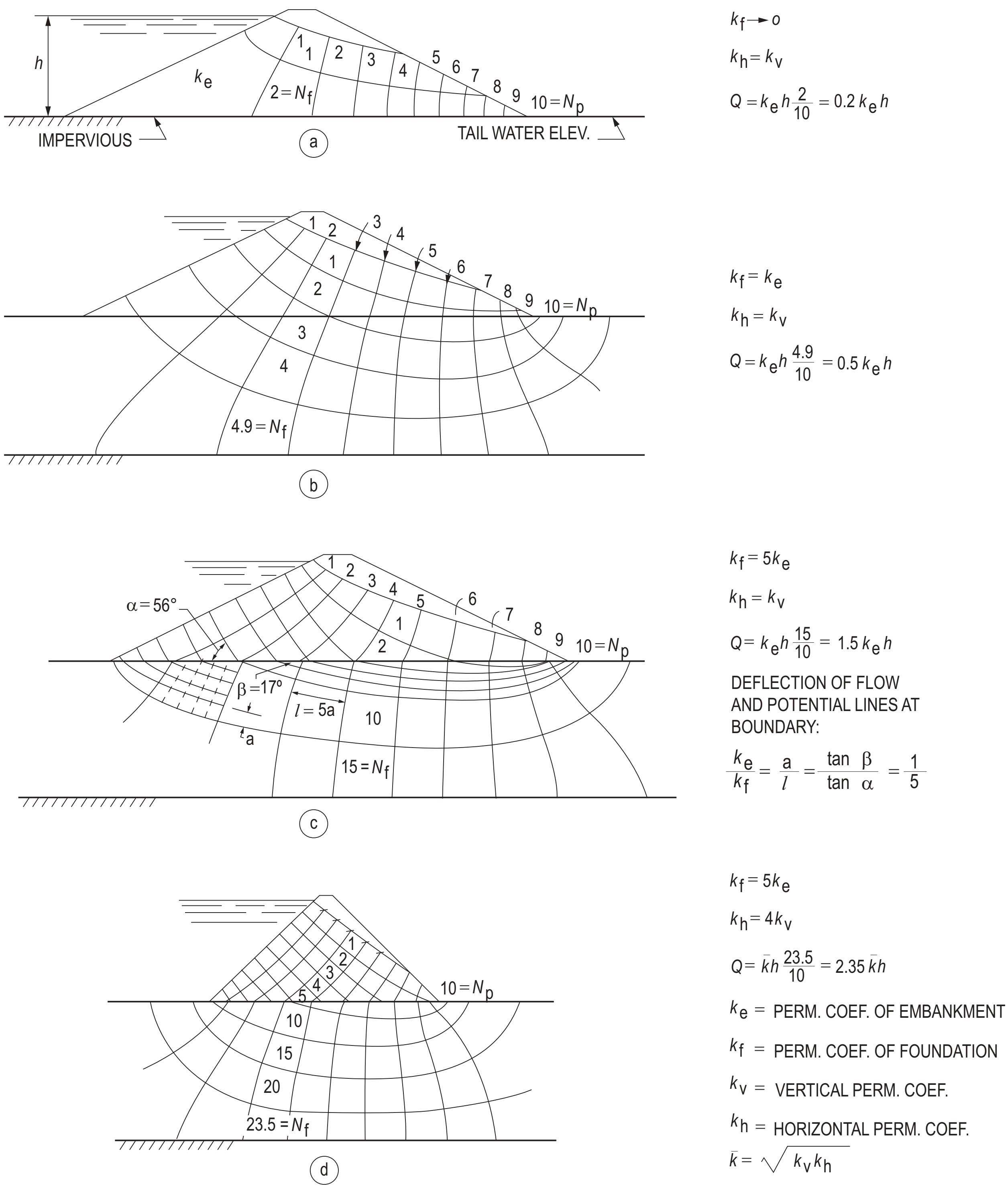
At points F and G, $h_1 = \frac{5}{9} \times 40 \text{ ft} = 22.2 \text{ ft}$

h_1 can also be determined graphically.

3. Average hydraulic gradient for any *square* element:
$i = \frac{\Delta h}{l}$, for shaded area, $i = \frac{4.45 \text{ ft}}{11.2 \text{ ft}} = 0.40$

Typical Flow Net Showing Basic Requirements and Computations

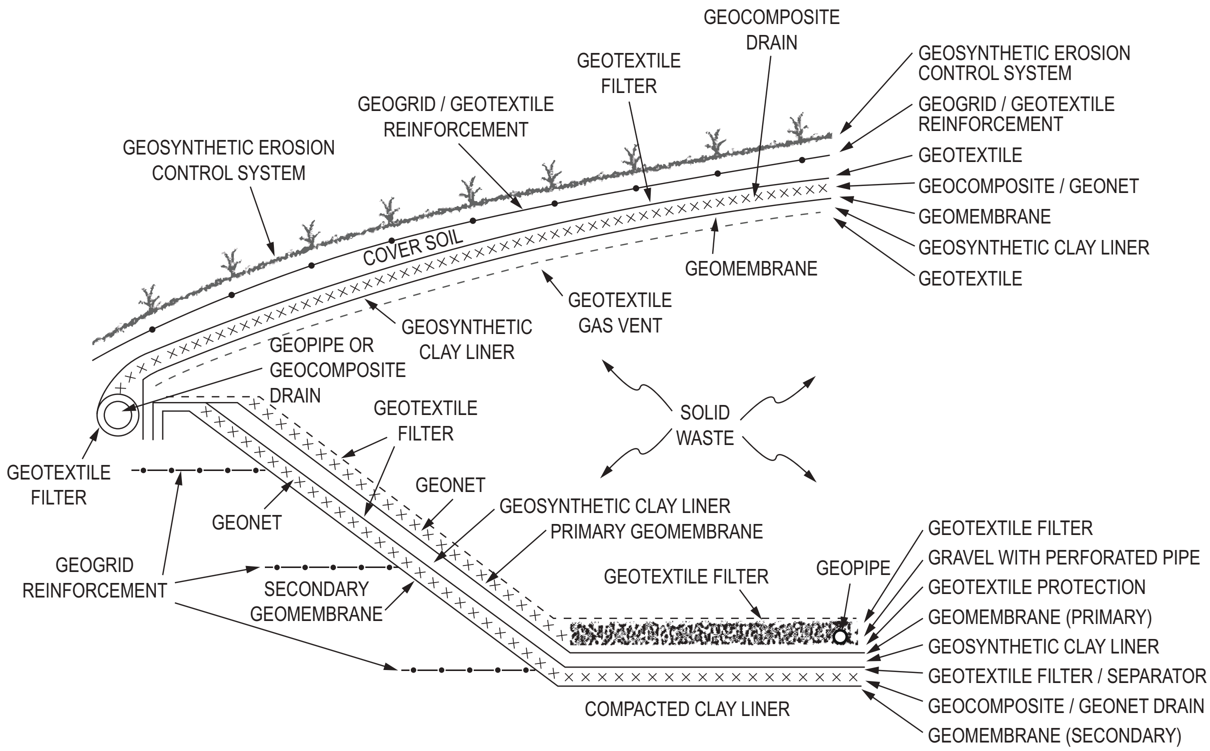
Source: Bureau of Reclamation. *Reclamation: Managing Water in the West. Design Standards No. 13: Embankment Dams*. Chapter 8: Seepage. Phase 4 (Final). DS-13(8)-4.1. Washington, DC: U.S. Department of Interior, January 2014. Figure 8.4.5-1, p. 8-53. https://www.usbr.gov/tsc/techreferences/designstandards-datacollectionguides/finalds-pdfs/DS13-8.pdf.



Seepage Through Embankment and Foundation

Source: Bureau of Reclamation. *Reclamation: Managing Water in the West. Design Standards No. 13: Embankment Dams*. Chapter 8: Seepage. Phase 4 (Final). DS-13(8)-4.1. Washington, DC: U.S. Department of Interior, January 2014. Figure 8.4.5-3, p. 8-55. https://www.usbr.gov/tsc/techreferences/designstandards-datacollectionguides/finalds-pdfs/DS13-8.pdf.

3.15 Landfills



Solid-Waste Containment System with High Geosynthetic Utilization

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 2. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 5.35, p. 663.
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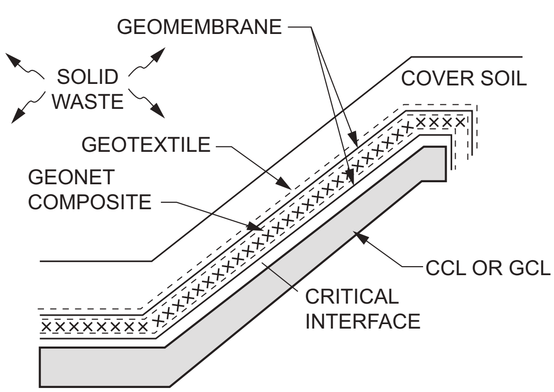
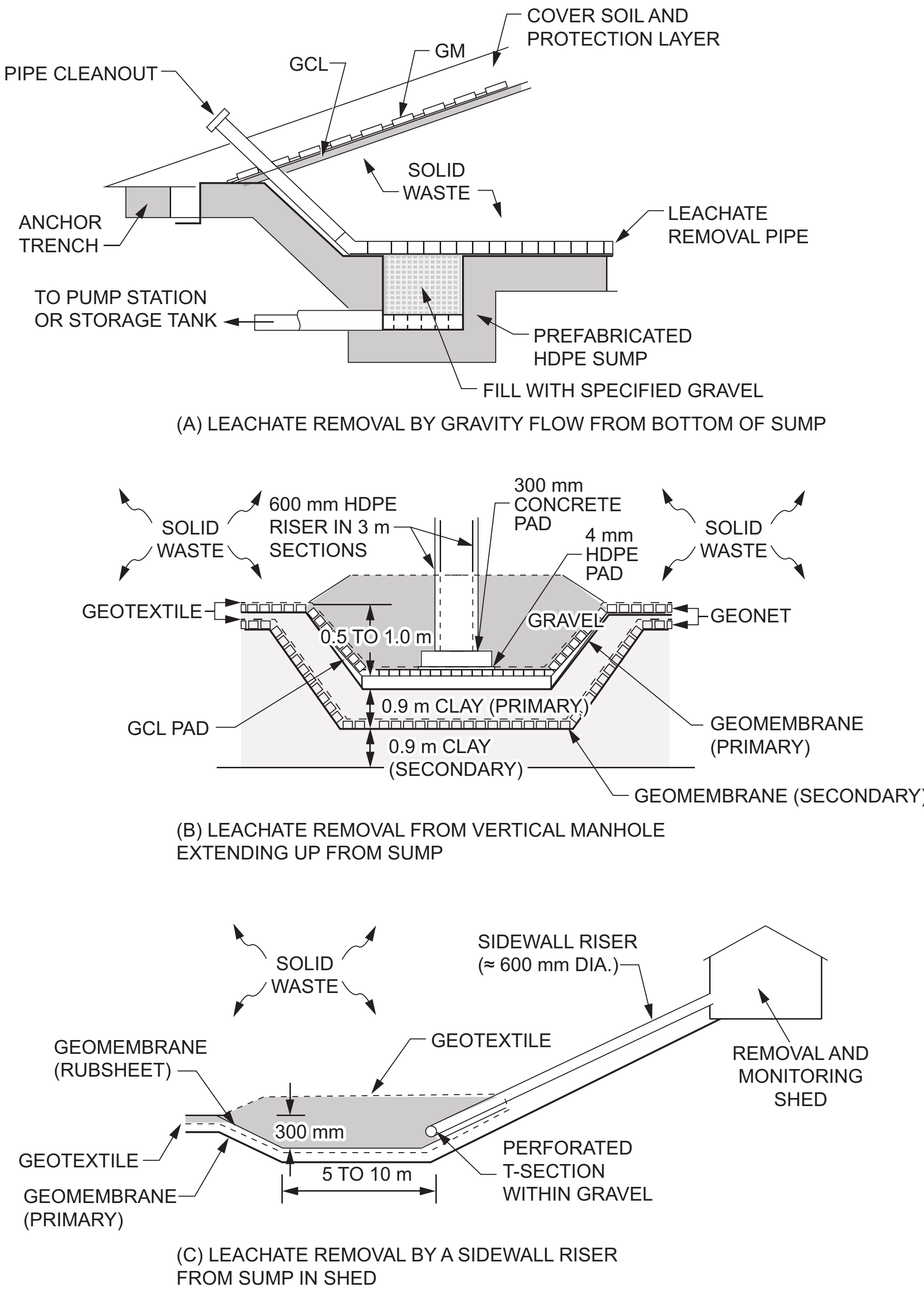


Geotextile/Geomembrane/Geonet Composite/Geomembrane Above a CCL or GCL

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 2. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 5.31, p. 650.
Used with permission of the Geosynthetic Institute.



Various Leachate Removal Designs for Primary Leachate Collection Systems

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 2. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 5.29, p. 639.
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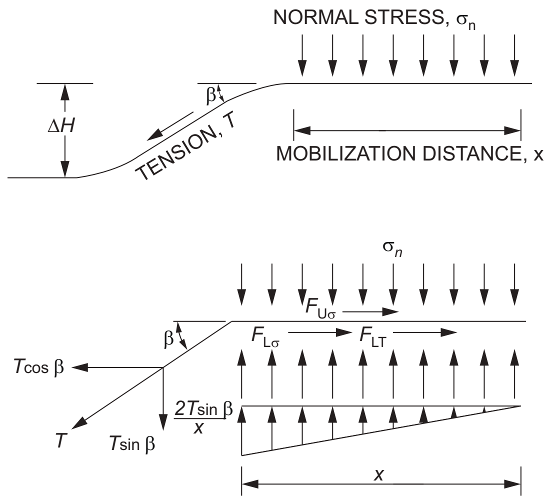


Design Model and Related Forces Used to Calculate Geomembrane Thickness

Source: Koerner, Robert M. *Designing with Geosynthetics*. Vol. 2. 6th ed. Bloomington, IN: Xlibris, 2012, Fig. 5.14, p. 576.
Used with permission of the Geosynthetic Institute.

$$T = \sigma_{allow} t$$

$$t = \frac{\sigma_n x \left(\tan \delta_U + \tan \delta_L\right)}{\sigma_{allow}\left(\cos\beta - \sin\beta \tan\delta_L\right)}$$

where

T = tension mobilized in the geomembrane

t = thickness of the geomembrane

σ_{allow} = allowable geomembrane stress (determined from laboratory tests)

β = settlement angle mobilizing the geomembrane tension

$F_{U\sigma}$ = shear force above geomembrane due to applied soil pressure (does not occur for liquid or thin soil covers)

$F_{L\sigma}$ = shear force below geomembrane due to the overlying liquid pressure (and soil if applicable)

F_{LT} = shear force below geomembrane due to vertical component of T

x = distance of mobilized geomembrane deformation

σ_n = applied stress from reservoir contents

δ = angle of shearing resistance between geomembrane and the adjacent material (i.e., soil or geotextile)

δ_U = 0° for liquid containment and 10° to 40° for landfill containment (determined from laboratory tests)

δ_L = 10° to 40° (determined from laboratory tests)

3.16 Groundwater and Seepage

3.16.1 Darcy's Law

$Q = kiA$

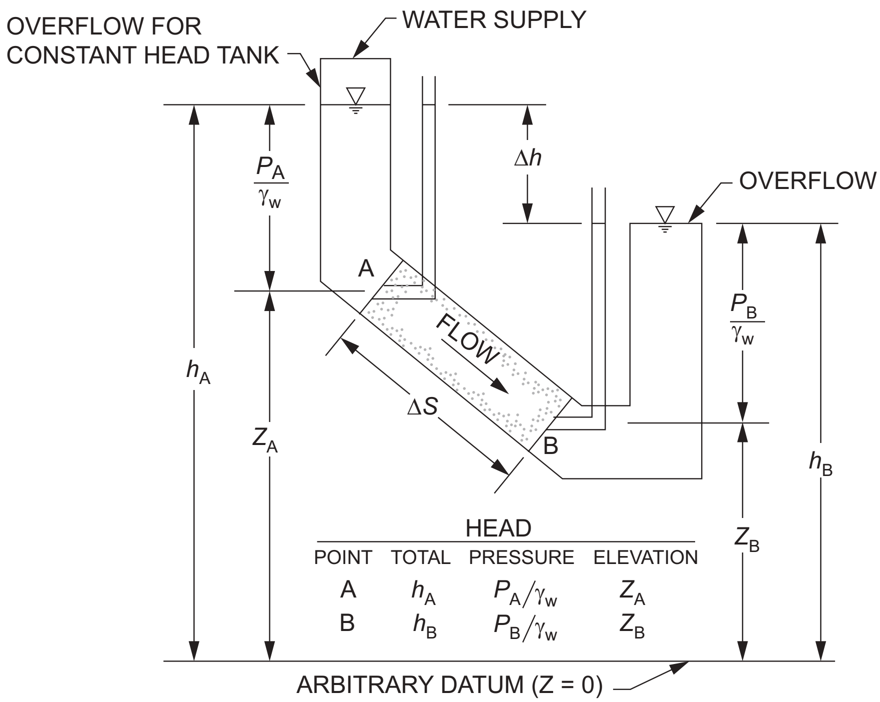
where

Q = rate of flow

k = permeability

A = cross-sectional area

i = hydraulic gradient = $\Delta h/\Delta S$



Darcy's Law for Flow Through Inclined Soil Column (prepared by WES, modified by NCEES)

Source: U.S. Army Corps of Engineers. *Engineering and Design: Seepage Analysis and Control for Dams*. EM 1110-2-1901. Washington, DC: U.S. Department of the Army, 30 September 1986 (Original), 30 April 1993 (Change 1), Fig. 2-1, p. 2-2. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1901.pdf?ver=2013-09-04-072923-387.

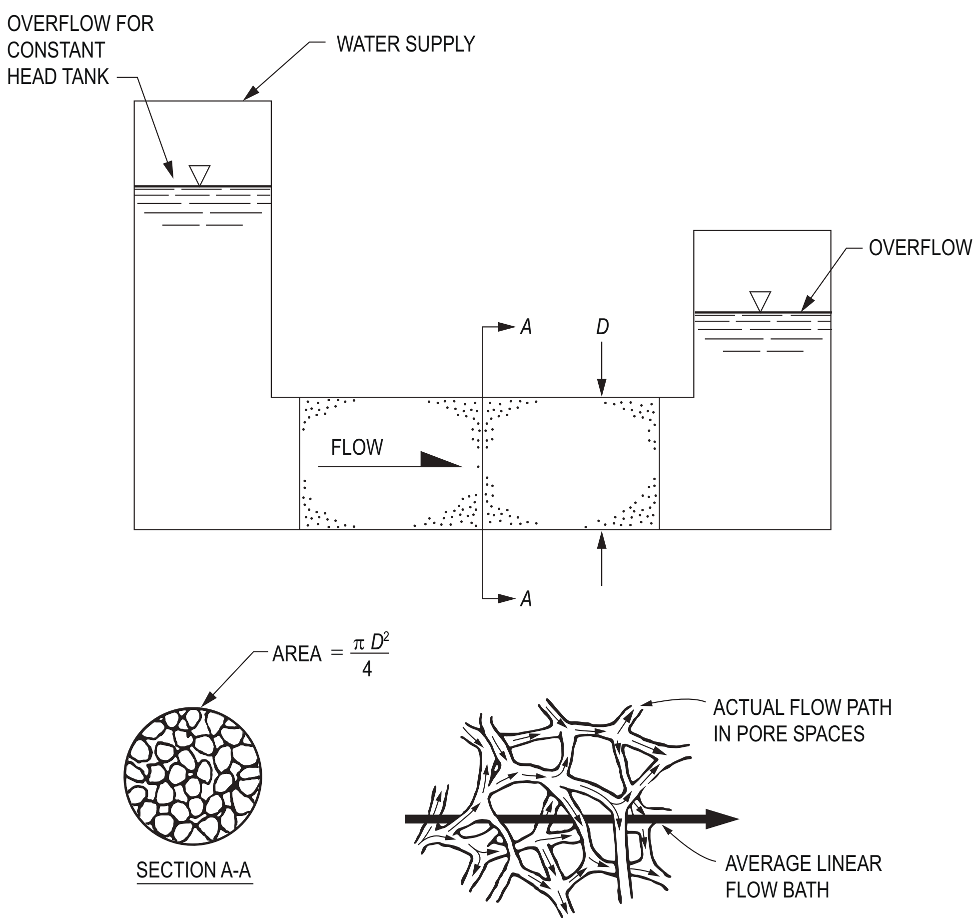
3.16.2 Permeability of Various Sands

Approximate Coefficient of Permeability for Various Sands

Type of Sand (Unified Soil Classification System)	Coefficient of Permeability k	
	$\times 10^{-4}$ cm/sec	$\times 10^{-4}$ ft/min
Sandy silt	5–20	10–40
Silty sand	20–50	40–100
Very fine sand	50–200	100–400
Fine sand	200–500	400–1,000
Fine to medium sand	500–1,000	1,000–2,000
Medium sand	1,000–1,500	2,000–3,000
Medium to coarse sand	1,500–2,000	3,000–4,000
Coarse sand and gravel	2,000–5,000	4,000–10,000

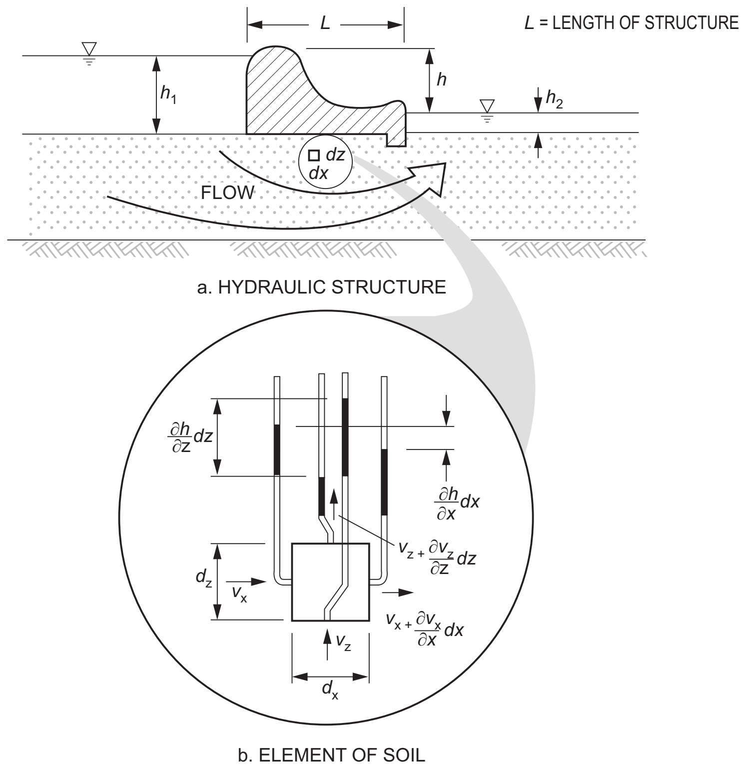
Source: Unified Facilities Criteria (UFC). Dewatering and Groundwater Control. UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, Table 3-4, p. 3-6.
www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf.

3.16.3 Flow Through Soil



Concepts of Flow Paths Through a Soil Column (prepared by WES)

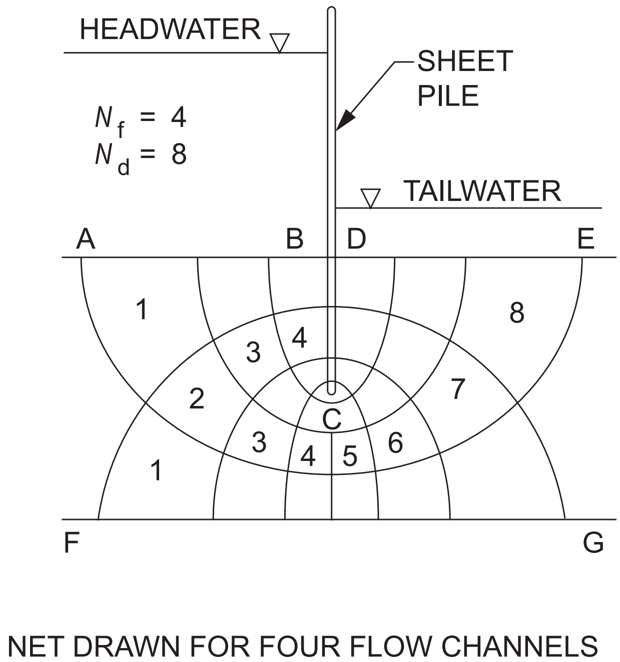
Source: U.S. Army Corps of Engineers. *Engineering and Design: Seepage Analysis and Control for Dams*. EM 1110-2-1901. Washington, DC: U.S. Department of the Army, 30 September 1986 (Original), 30 April 1993 (Change 1), Fig. 2-2, p. 2-3.
www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1901.pdf?ver=2013-09-04-072923-387.



Flow of Water Through Saturated Pervious Soil Beneath a Hydraulic Structure (courtesy of John Wiley and Sons)

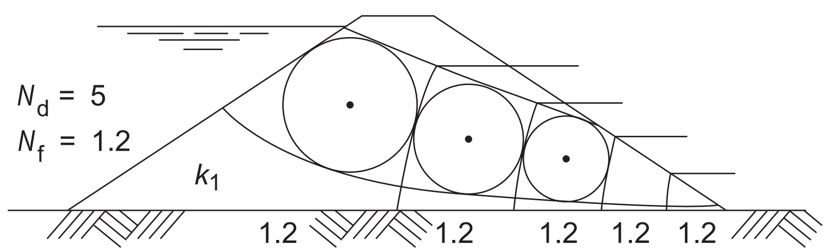
Source: U.S. Army Corps of Engineers. *Engineering and Design: Seepage Analysis and Control for Dams*. EM 1110-2-1901. Washington, DC: U.S. Department of the Army, 30 September 1986 (Original), 30 April 1993 (Change 1), Fig. 4-2, p. 4-5. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1901.pdf?ver=2013-09-04-072923-387.

3.16.4 Flow Nets



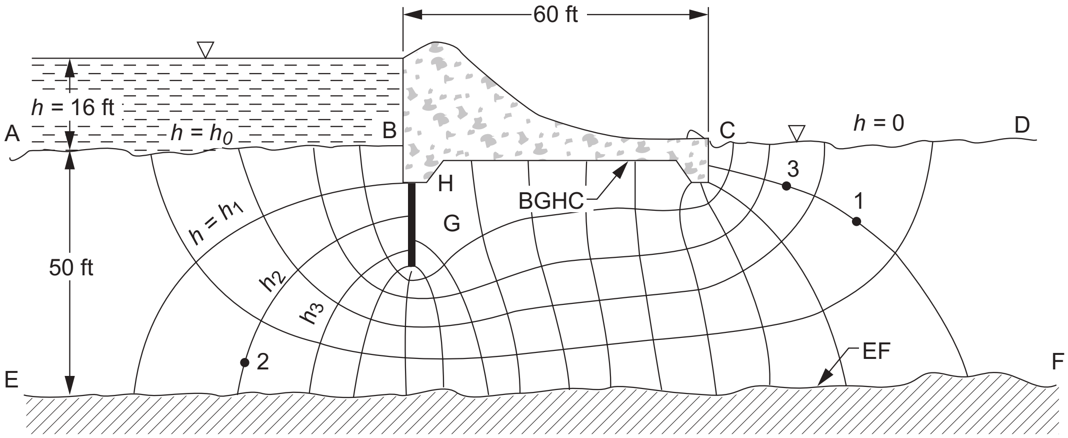
Flow Net Concepts

Source: U.S. Army Corps of Engineers. *Engineering and Design: Seepage Analysis and Control for Dams*. EM 1110-2-1901. Washington, DC: U.S. Department of the Army, 30 September 1986 (Original), 30 April 1993 (Change 1), Fig. 4-5, p. 4-10. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1901.pdf?ver=2013-09-04-072923-387.



Seepage Through an Embankment Underlaid by an Impermeable Foundation

Source: Cedergren, H. R. *Seepage, Drainage and Flow Nets*. 2nd ed. John Wiley and Sons, 1977. As found in U.S. Army Corps of Engineers. *Engineering and Design: Seepage Analysis and Control for Dams*. EM 1110-2-1901. Washington, DC: U.S. Department of the Army, 30 September 1986 (Original), 30 April 1993 (Change 1), Fig. 4-6, p. 4-11. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1901.pdf?ver=2013-09-04-072923-392.



Gravity Dam on Pervious Foundation of Finite Depth (courtesy McGraw-Hill Book Company)

Source: Harr, M. E. Groundwater and Seepage. McGraw-Hill, 1962. As found in U.S. Army Corps of Engineers. *Engineering and Design: Seepage Analysis and Control for Dams*. EM 1110-2-1901. Washington, DC: U.S. Department of the Army, 30 September 1986 (Original), 30 April 1993 (Change 1), Fig. 5-2, p. 5-3. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1901.pdf?ver=2013-09-04-072923-392.

Flow below gravity dam over permeable foundation of finite depth:

$$Q = kh\left(\frac{N_f}{N_d}\right)$$

where

Q = rate of flow

k = permeability

h = total hydraulic head differential

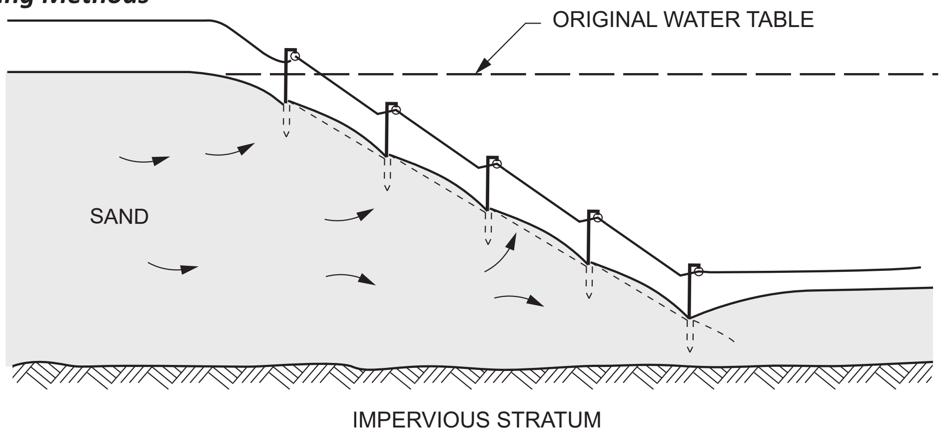
N_f = number of flow channels

N_d = number of equipotential drops

Source: Unified Facilities Criteria (UFC). *Dewatering and Groundwater Control.* UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, p. 4-31. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf.

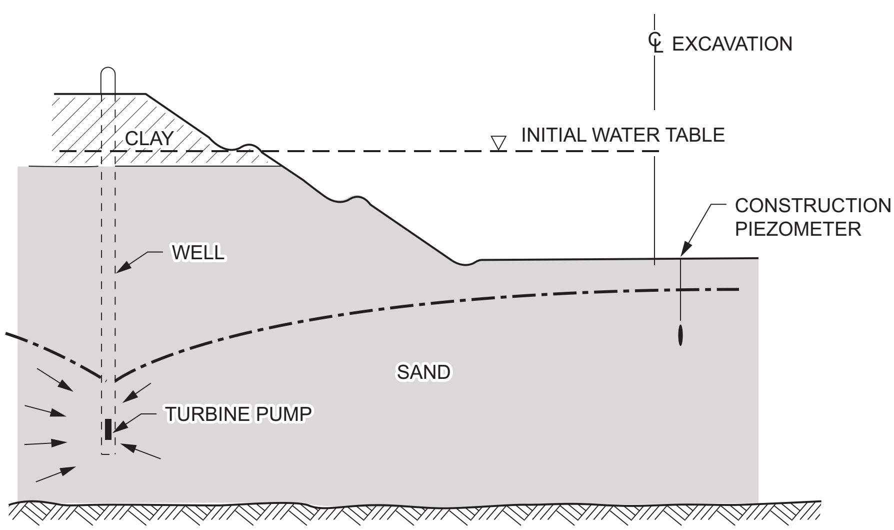
3.16.5 Construction Dewatering

3.16.5.1 Dewatering Methods



Drainage of an Open Deep Cut by Means of a Multistage Well Point System

Source: Terzaghi, K., and R. B. Peck. *Soil Mechanics in Engineering Practice*. John Wiley and Sons, 1948. As found in Unified Facilities Criteria (UFC). *Dewatering and Groundwater Control.* UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, Fig. 2-4, p. 2-4. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf.



Deep-Well System for Dewatering an Excavation in Sand

Source: Unified Facilities Criteria (UFC). *Dewatering and Groundwater Control*. UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, Fig. 2-7, p. 2-7. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf.

3.16.5.2 Single Dewatering Well

Refer to Groundwater and Wells—Unconfined Aquifers

Partially Penetrating Well

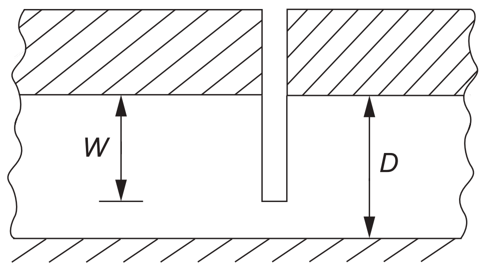
Q_{wp} = flow from a partially penetrating well (ft^3/sec)

$$Q_{wp} = \frac{2\pi k D (H - h_w) G}{\ln(R/r_w)} = Q_{w-100\%} \times G$$

where

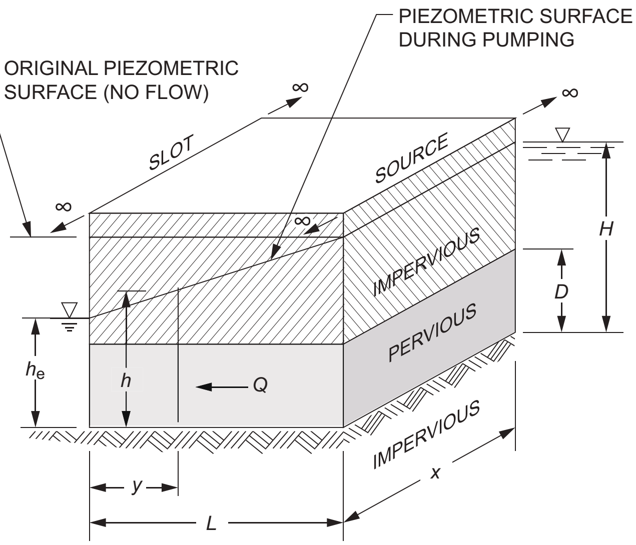
G is equal to the ratio of flow from a partially penetrating well, Q_{wp}, to that for a fully penetrating well for the same drawdown, $H - h_w$, at the periphery of the wells. Approximate values of G can be computed from the formula:

$$G = \frac{W}{D}\left(1 + 7\sqrt{r_w/2W}\ \cos\frac{\pi W/D}{2}\right)$$



Source: Unified Facilities Criteria (UFC). *Dewatering and Groundwater Control.* UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, Fig. 4-10, p. 4-11. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf.

3.16.5.3 Artesian Flow

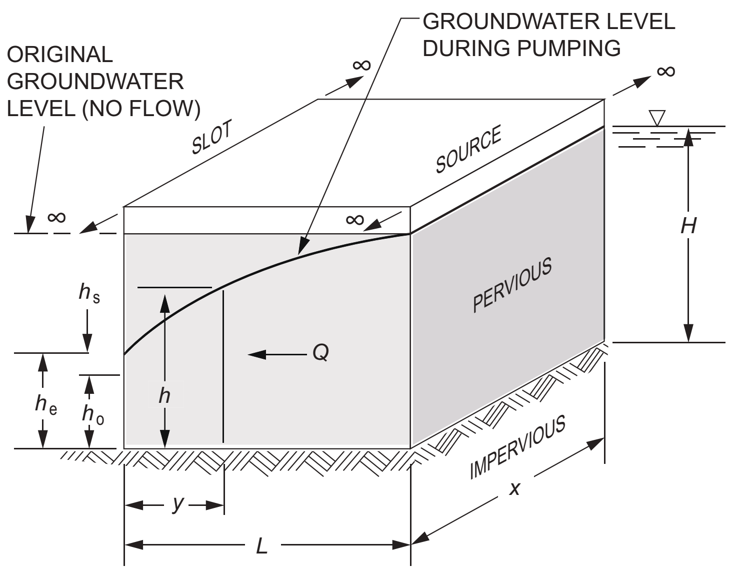


Artesian Flow: $Q = \frac{kDx}{L}\left(H - h_e\right)$

Drawdown: At any distance y from slot

$$H - h = \frac{Q}{kDx}\left(L - y\right) = \frac{L - y}{L}\left(H - h_e\right)$$

3.16.5.4 Gravity Flow



Gravity Flow: $Q = \frac{kx}{2L}\left(H^2 - h_0^2\right)$

Drawdown: At any distance y from slot

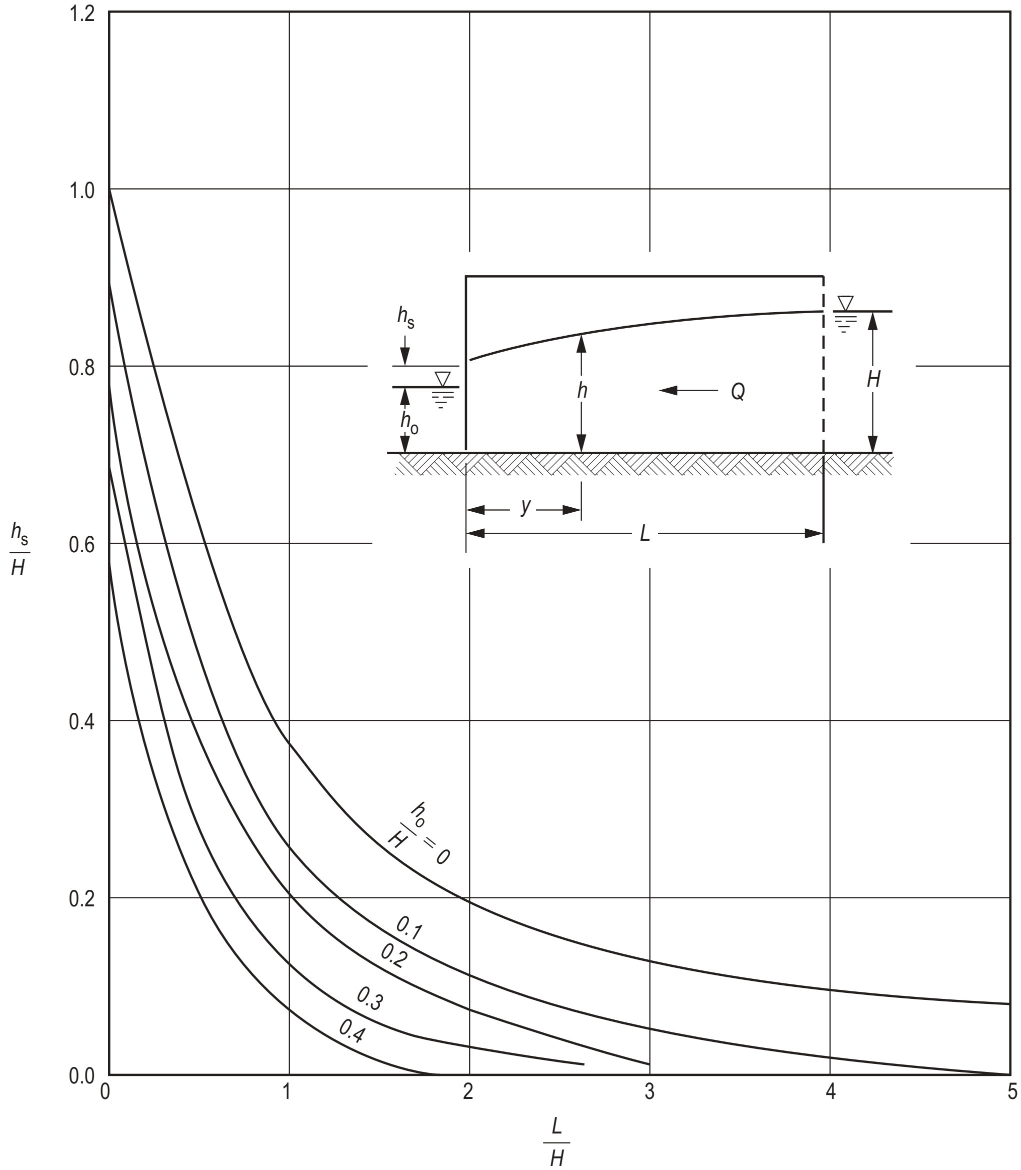
$$H^2 - h^2 = \frac{L - y}{L}\left(H^2 - h_e^2\right)$$

where

$h_e = h_0 + h_s$

(h_s is obtained from the following figure, "Height of Free Discharge Surface h; Gravity Flow")

Source for above figures and equations: Unified Facilities Criteria (UFC). *Dewatering and Groundwater Control*. UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, Fig. 4-1, p. 4-2. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf.



Height of Free Discharge Surface *h*; Gravity Flow

Source: Modified from Leonards, G.A., ed. *Foundation Engineering*. McGraw-Hill, 1962. As found in Unified Facilities Criteria (UFC). *Dewatering and Groundwater Control.* UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, Fig. 4-2, p. 4-3. www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf.

3.16.5.5 Filters

Criteria for Filters

Base Soil Category	Base Soil Description and Percent Finer Than the No. 200 (0.075 mm) Sieve[a, b]	
1	Fine silts and clays; more than 85 percent finer	$D_{15} \le 9 \times d_{85}$ [c]
2	Sands, silts, clays, and silty and clayey sands; 40 to 85 percent finer	$D_{15} \le 0.7$ mm
3	Silty and clayey sands and gravels; 15 to 39 percent finer	$D_{15} \le \left(\frac{40 - A}{40 - 15}\right)\left[\left(4 \times d_{85}\right) - 0.7 \text{ mm}\right] + 0.7 \text{ mm}$ [d,e]
4	Sands and gravels; less than 15 percent finer	$D_{15} \le 4$ to $5 \times d_{85}$ [f]

[a] Category designation for soil containing particles larger than 4.75 mm is determined from a gradation curve of the base soil, which has been adjusted to 100 percent passing the No. 4 (4.75 mm) sieve.
[b] Filters are to have a maximum particle size of 75 mm (3 in.) and a maximum of 5 percent passing the No. 200 (0.075 mm) sieve with the plasticity index (*PI*) of the fines equal to zero. *PI* is determined on the material passing the No. 40 (0.425 mm) sieve in accordance with EM 1110-2-1906, "Laboratory Soils Testing." To ensure sufficient permeability, filters are to have a D_{15} size equal to or greater than $4 \times d_{15}$ but no smaller than 0.1 mm.
[c] When $9 \times d_{85}$ is less than 0.2 mm, use 0.2 mm.
[d] A = percent passing the No. 200 (0.075 mm) sieve after any regrading.
[e] When $4 \times d_{85}$ is less than 0.7 mm, use 0.7 mm.
[f] In category 4, the $D_{15} \le 4 \times d_{85}$ criterion should be used in the case of filters beneath riprap subject to wave action and drains, which may be subject to violent surging and/or vibration.

Source: U.S. Army Corps of Engineers. *Engineering and Design: Design and Construction of Levees*. EM 1110-2-1913. Washington, DC: U.S. Department of the Army, April 2000, Table D-2, p. D-3. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1913.pdf.

D_{10} and D_{90} Limits for Preventing Segregation

Minimum D_{10} (mm)	Maximum D_{90} (mm)
< 0.5	20
0.5–1.0	25
1.0–2.0	30
2.0–5.0	40
5.0–10	50
10–50	60

Source: U.S. Army Corps of Engineers. *Engineering and Design: Design and Construction of Levees*. EM 1110-2-1913. Washington, DC: U.S. Department of the Army, April 2000, Table D-3, p. D-3. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1913.pdf.

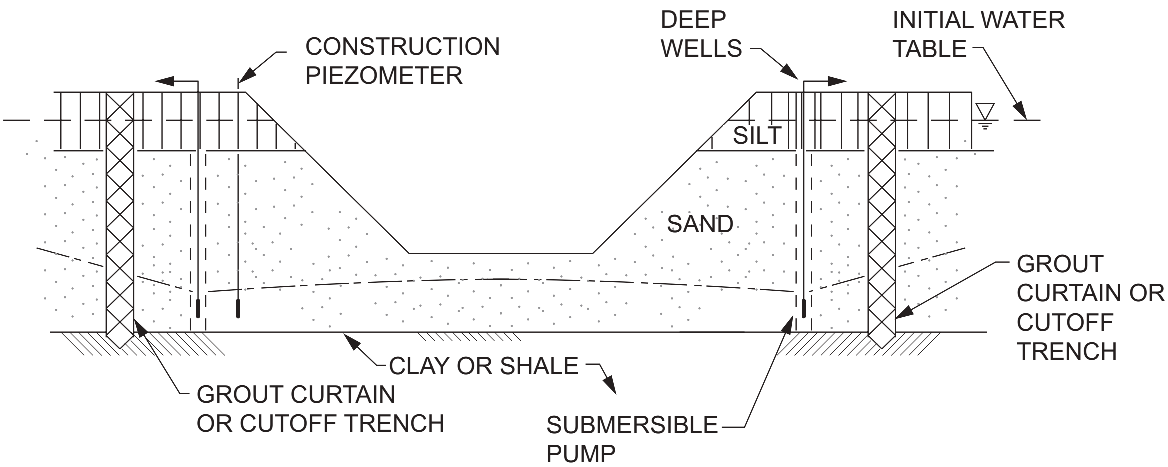
3.16.5.6 Perforated Pipe

Criteria for preventing infiltration of filter material into perforated pipe, screens, etc.:

$$\frac{\text{Minimum 50 percent size of filter material}}{\text{Hole diameter or slot width}} \ge 1.0$$

Source: U.S. Army Corps of Engineers. *Engineering and Design: Design and Construction of Levees*. EM 1110-2-1913. Washington, DC: U.S. Department of the Army, April 2000, p. D-4. www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1913.pdf.

3.16.5.7 Grouting and Other Methods to Reduce Seepage



Grout Curtain or Cutoff Trench Around an Excavation

Source: Unified Facilities Criteria (UFC). *Dewatering and Groundwater Control*. UFC 3-220-05. Washington, DC: U.S. Department of Defense, January 2004, Fig. 2-11, p. 2-10.
www.wbdg.org/FFC/DOD/UFC/ufc_3_220_05_2004.pdf

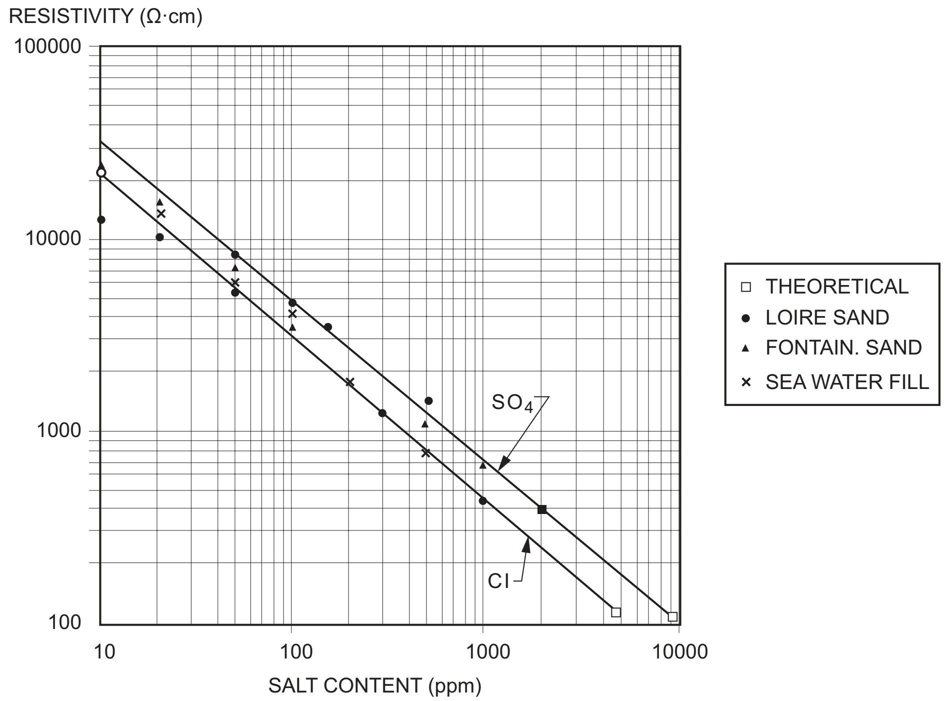
3.17 Problematic Soil and Rock Conditions

3.17.1 Reactive and Corrosive Soils

Effect of Resistivity on Corrosion (NCHRP, 1978)

Aggressiveness	Resistivity (ohm•cm)
Very corrosive	< 700
Corrosive	700–2,000
Moderately corrosive	2,000–5,000
Mildly corrosive	5,000–10,000
Noncorrosive	> 10,000

Source: Federal Highway Administration. National Highway Institute. *Corrosion/Degradation of Soil Reinforcements for Mechanically Stabilized Earth Walls and Reinforced Soil Slopes*. FHWA-NHI-09-087. Washington, DC: U.S. Department of Transportation, November 2009, Table 2-3, p. 2-14.
www.fhwa.dot.gov/engineering/geotech/pubs/nhi09087/nhi09087.pdf.



Resistivity vs. Soluble Salts

Source: Federal Highway Administration. *Durability/Corrosion of Soil Reinforced Structures*. FHWA-RD-89-186. Washington, DC: U.S. Department of Transportation, December 1990, Fig. 12, p. 39. https://vulcanhammernet.files.wordpress.com/2017/01/fhwa-rd-89-186.pdf.

3.17.2 Corrosion of Buried Steel

$$x = Kt^n$$

where

x = loss of thickness or pit depth at time t (microns, μm)

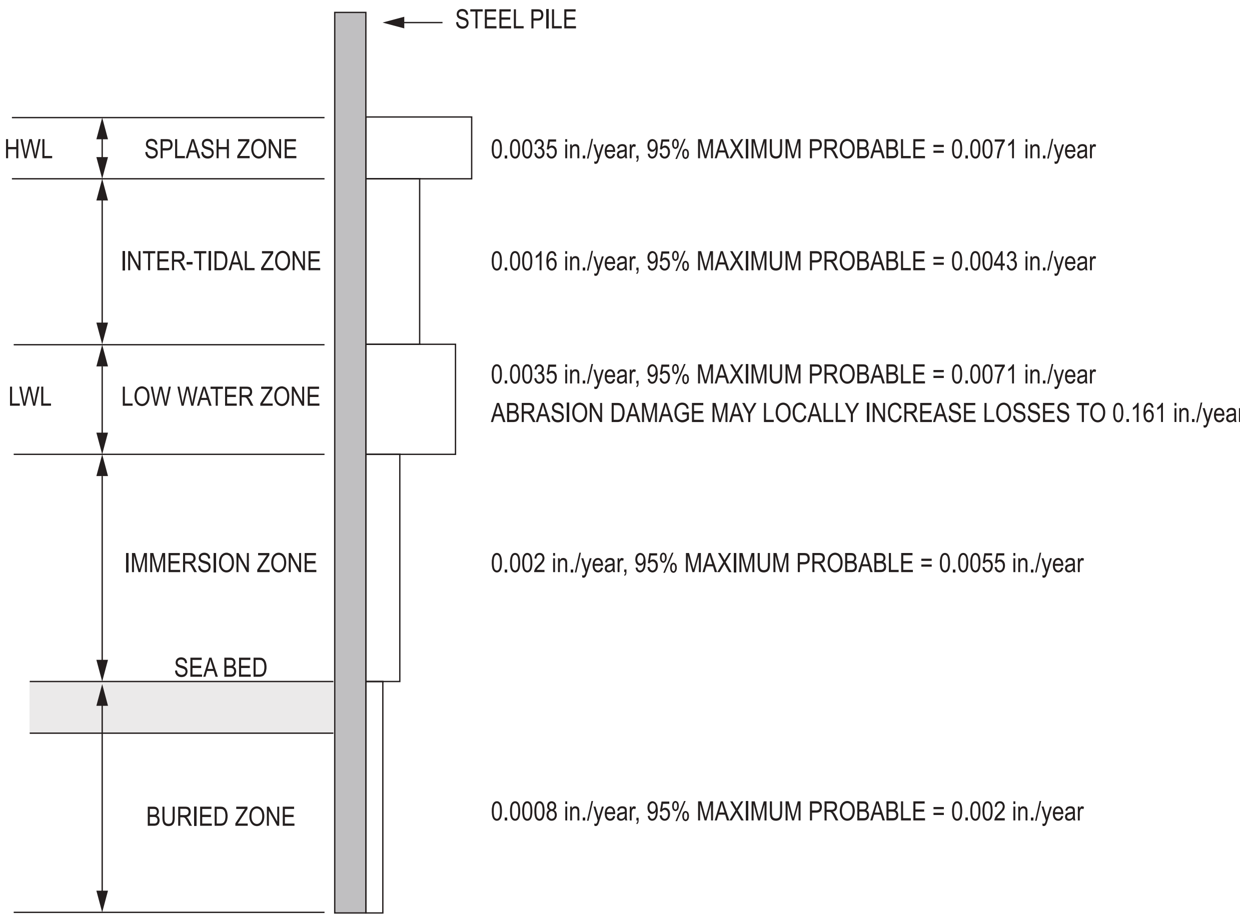
n = 0.65 for galvanized steel
0.80 for carbon steel

K = 25 (average) and 50 (max.) for galvanized steel
40 (average) and 80 (max.) for carbon steel

t = time (years)

1,000 microns = 1 mm = 0.039 in. = 39 mils

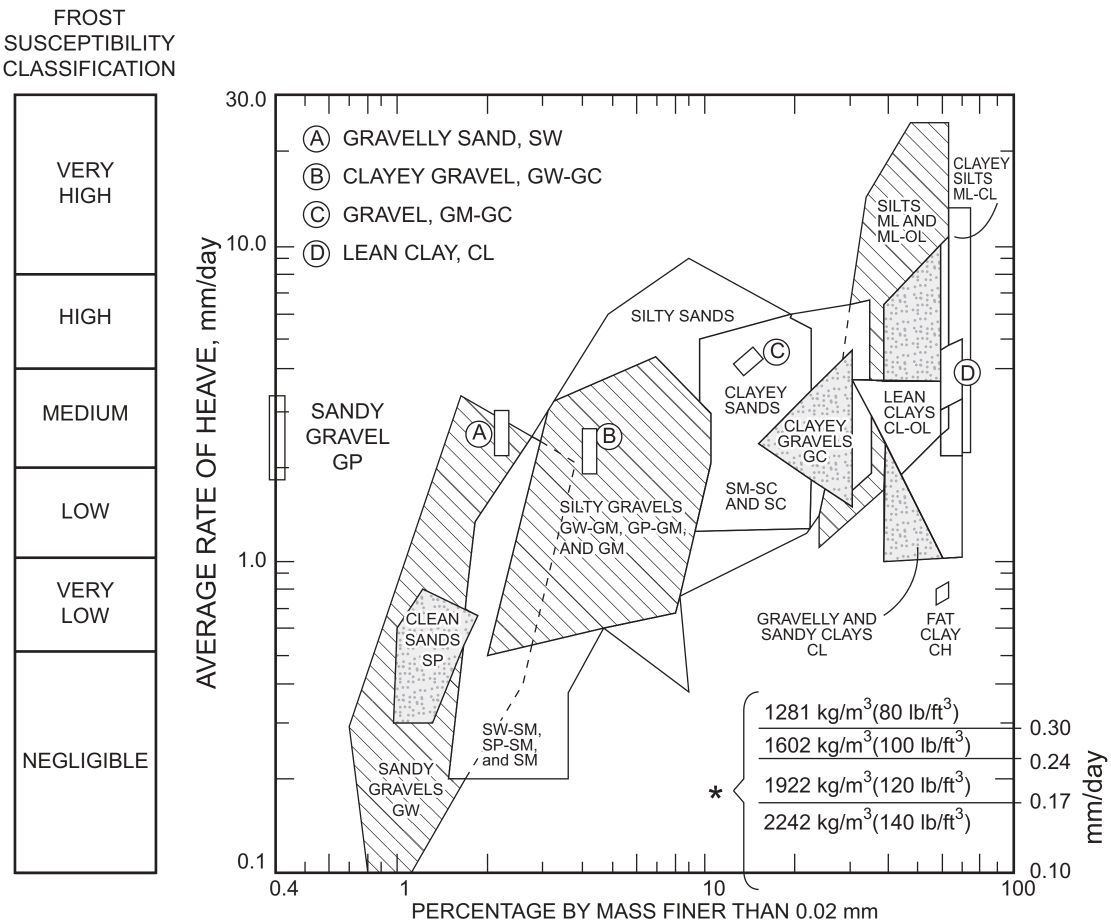
Source: Federal Highway Administration, National Highway Institute. *Corrosion/Degradations of Soil Reinforcements for Mechanically Stabilized Earth Walls and Reinforced Soil Slopes.* FHWA-NHI-09-087. Washington, DC: U.S. Department of Transportation, November 2009, pp. 2-21 and 2-22. www.fhwa.dot.gov/engineering/geotech/pubs/nhi09087/nhi09087.pdf.



Loss of Thickness by Corrosion for Steel Piles in Seawater (after Morley and Bruce 1983)

Source: Federal Highway Administration, National Highway Institute. *Design and Construction of Driven Pile Foundations,* Volume I. FHWA-NHI-16-009. FHWA GEC 012 - Volume I. Washington, DC: U.S. Department of Transportation, July 2016, Figure 6-29, p. 193. www.fhwa.dot.gov/engineering/geotech/pubs/gec12/nhi16009_v1.pdf.

3.17.3 Frost Susceptibility



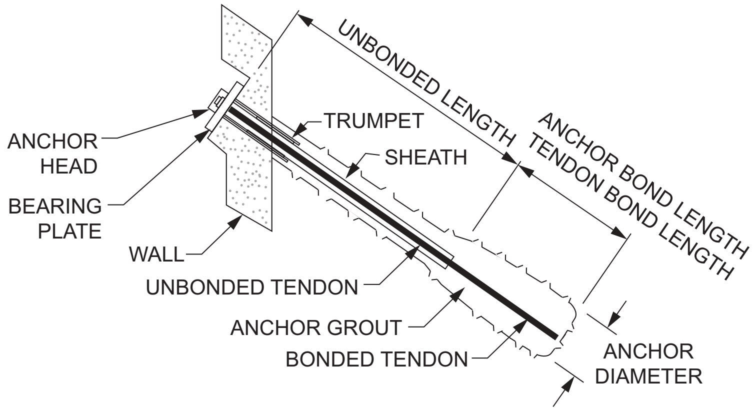
* Indicated heave rate due to expansion in volume if all original water in 100-percent-saturated specimen was frozen, with rate of frost penetration 6.35 mm (0.25 in.) per day

Frost Susceptibility Classification by Percentage of Mass Finer Than 0.02 mm and Average Rate of Heave Versus % Fines for Natural Soil Gradations

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 7-20, p. 7-60. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

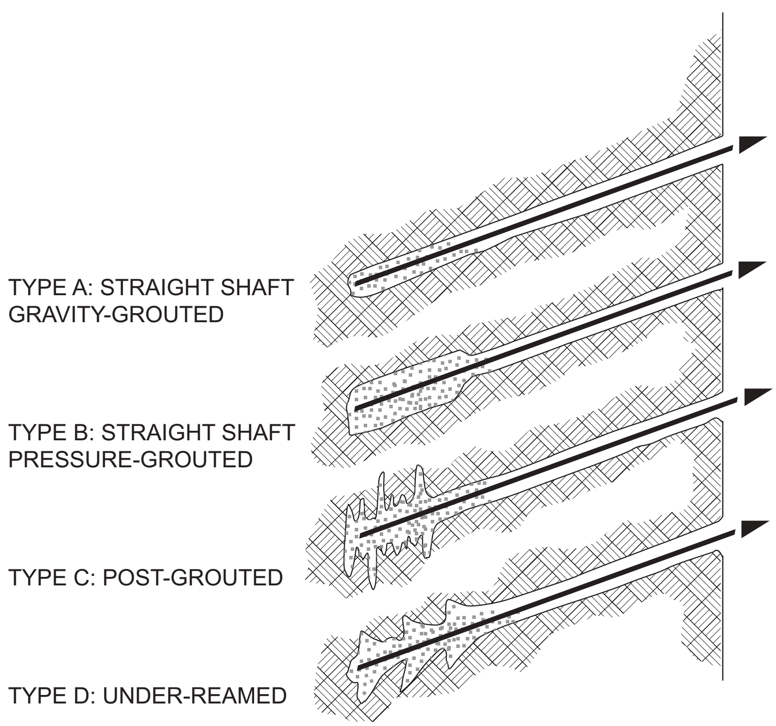
3.18 Earth Retention—Anchored Walls

3.18.1 Ground Anchor Components and Types



Components of a Ground Anchor

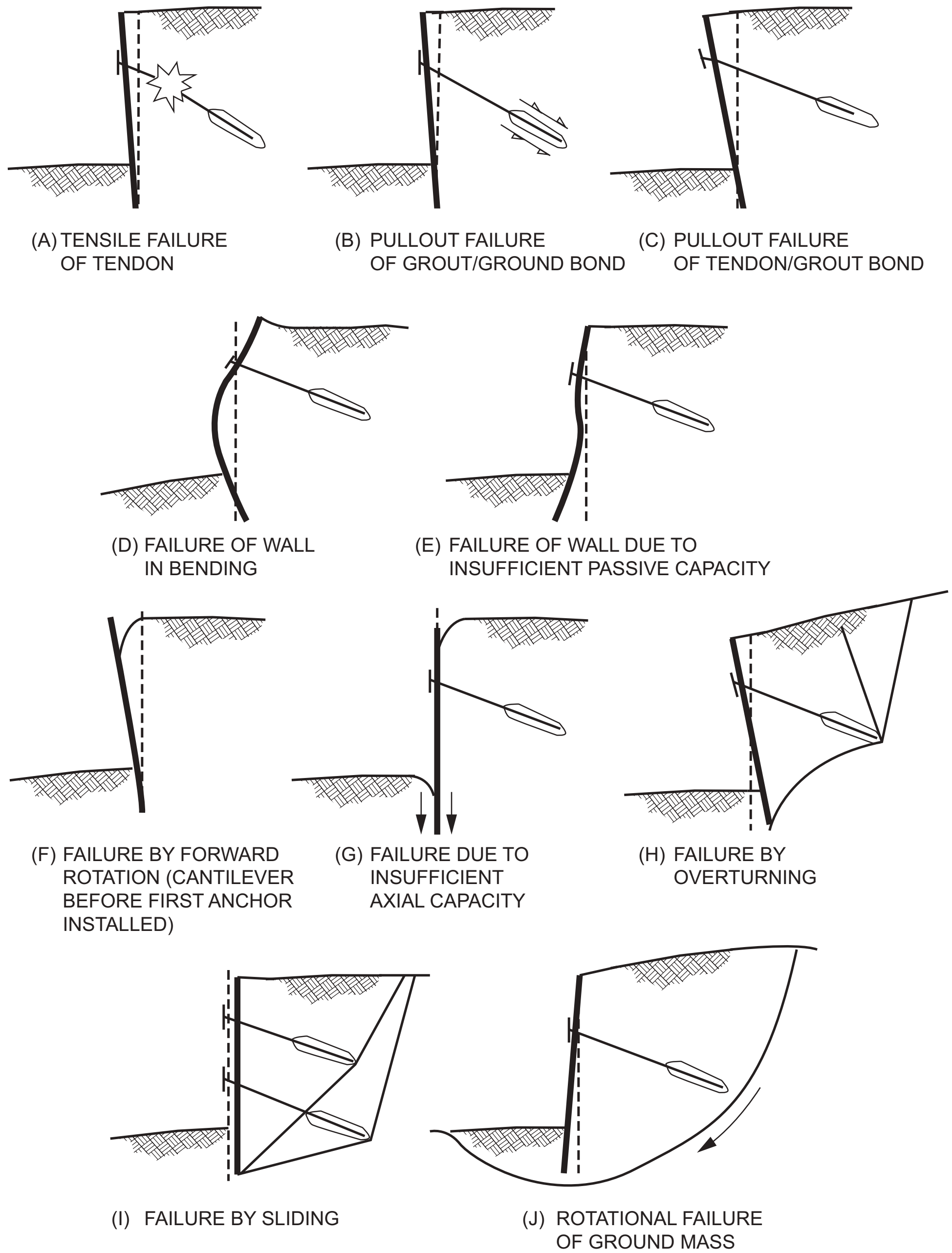
Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 1, p. 4. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



Main Types of Grouted Ground Anchors

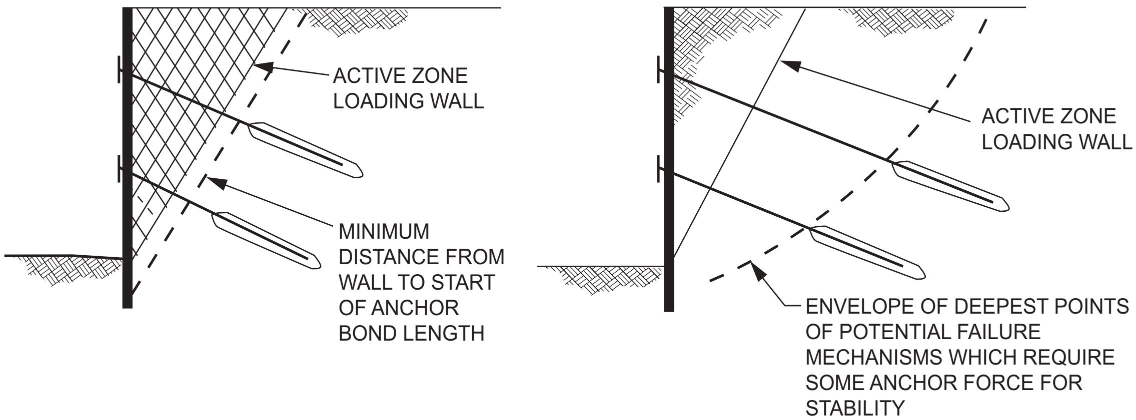
Source: Modified after Littlejohn. "Ground Anchorage Practice". *Design and Performance of Earth Retaining Structures*. Geotechnical Special Publication No. 25. ASCE, 1990. As found in Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 4, p. 7. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

3.18.2 Potential Modes of Failure



Potential Failure Conditions to be Considered in Design of Anchored Walls

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 11, p. 27. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



Contribution of Ground Anchors to Wall Stability

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 12, p. 28. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

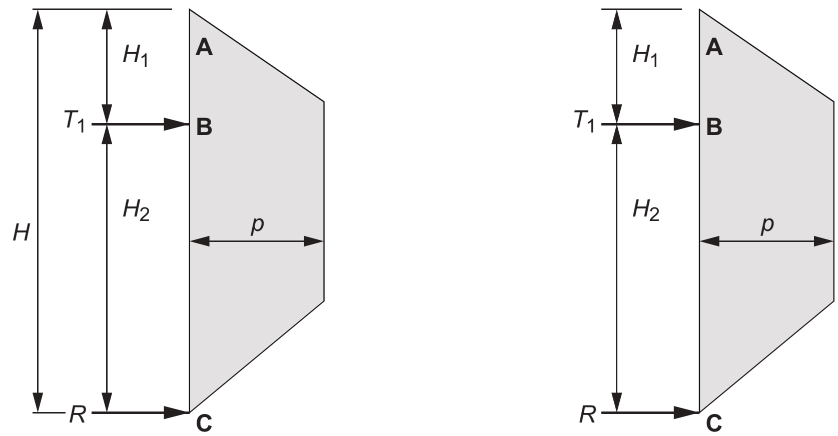
Typical Factors Influencing Bond Stress Transfer for Small Diameter Ground Anchors

Factor	Soil Type	
	Cohesionless	Cohesive
Soil Properties	Friction angle and grain size distribution	Adhesion and plasticity index
Drilling Method	Driven casing increases normal stress and friction	Drilling without casing or with fluids decreases capacity
Bond Length	Steady increase in anchor capacity to 6 m with moderating increases to 12 m	Steady increase in anchor capacity for soils with undrained strength less than 96 kPa
Hole Diameter	Slight increase in anchor capacity to 100 mm	Anchor capacity increases to 300 mm.
Grout Pressure	Anchor capacity increases with increasing pressure	Anchor capacity increases only with stage grouting. High initial pressures should be avoided.

Note: To ensure ground-grout bond, the drill hole should be cleaned and the grout placed as quickly as possible after the hole has been drilled.

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Table 3, p. 30. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

3.18.3 Anchor Loads



TRIBUTARY AREA METHOD

T_1 = Load over length $H_1 + \dfrac{H_2}{2}$

R = Load over length $\dfrac{H_2}{2}$

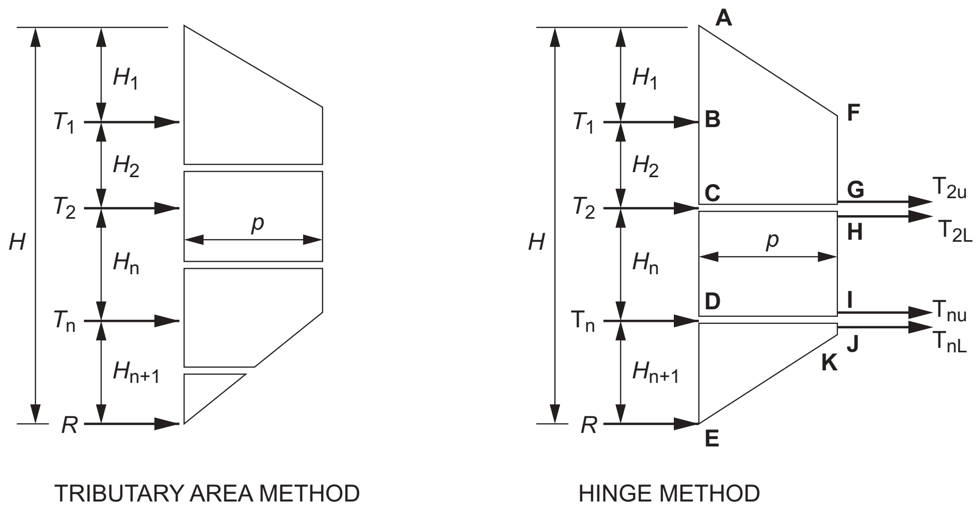
HINGE METHOD

T_1 calculated from $\sum M_C = 0$

R = Total earth pressure $- T_1$

Calculation of Anchor Loads from One-Level Wall

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 33, p. 66. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



T_1 = Load over length $H_1 + \dfrac{H_2}{2}$

T_2 = Load over length $\dfrac{H_2}{2} + \dfrac{H_n}{2}$

T_n = Load over length $\dfrac{H_n}{2} + \dfrac{H_{n+1}}{2}$

R = Load over length $\dfrac{H_{n+1}}{2}$

T_1 calculated from $\sum M_C = 0$

T_{2u} = Total earth pressure (ABCGF) − T_1

T_{2L} calculated from $\sum M_D = 0$

T_{nu} = Total earth pressure (CDIH) − T_{2L}

T_{nL} calculated from $\sum M_E = 0$

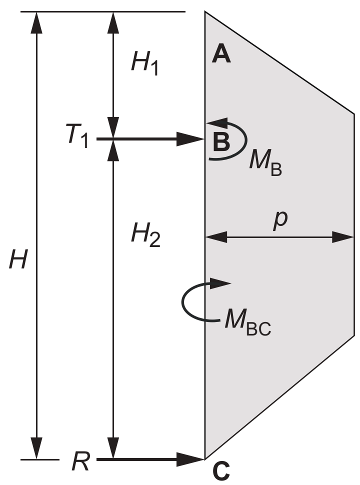
R = Total earth pressure − $T_1 - T_2 - T_n$

$T_2 = T_{2u} + T_{2L}$

$T_n = T_{nu} + T_{nL}$

Calculation of Anchor Loads for Multilevel Wall

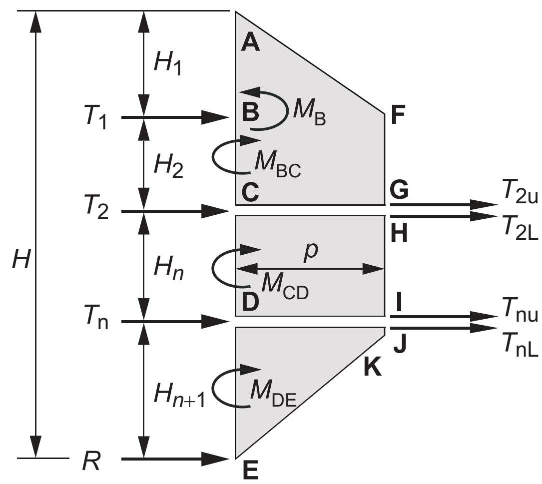
Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 34, p. 67. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



$M_B = \Sigma M_B$

M_{BC} = Maximum moment between B and C; located at point where shear = 0

(a) Walls with one level of ground anchors



$M_B = \Sigma M_B$

$M_C = M_D = M_E = 0$

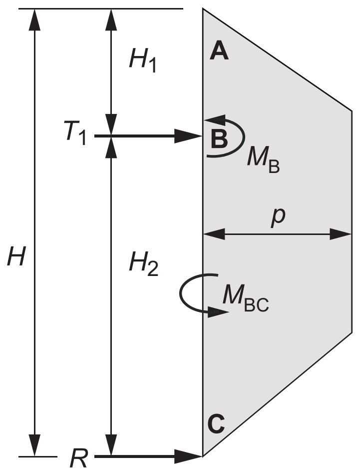
M_{BC} = Maximum moment between B and C; located at point where shear = 0

M_{CD} ; M_{DE} ; Calculated as for M_{BC}

(b) Walls with multiple levels of ground anchors

Calculation of Wall Bending Moments Using Hinge Method

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 38, p. 79. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



$$M_B = \frac{13}{54} H_1^2 p$$

$$T_1 = \frac{\left(23H^2 - 10HH_1\right)}{54\left(H - H_1\right)} p$$

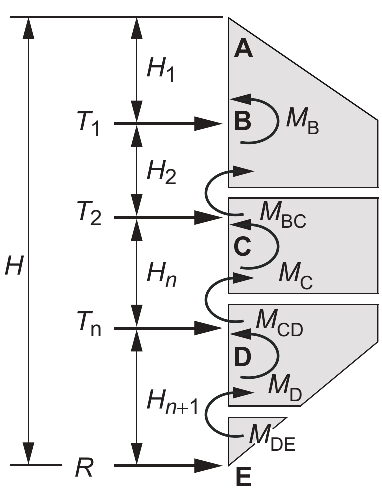
$$R = \frac{2}{3} Hp - T_1$$

Solve for point of zero shear

$$x = \frac{1}{9}\sqrt{\left(26H^2 - 52HH_1\right)}$$

$$M_{BC} = Rx - \frac{px^3}{4\left(H - H_1\right)}$$

(a) Walls with single level of ground anchors



$$M_B = \frac{13}{54} H_1^2 \ p$$

$$T_1 = \left(\frac{2}{3} H_1 + \frac{H_2}{2}\right) p$$

$$T_2 = \left(\frac{H_2}{2} + \frac{H_n}{2}\right) p$$

$$T_n = \left(\frac{H_n}{2} + \frac{23H_{n+1}}{48}\right) p$$

$$R = \left(\frac{3}{16} \ H_{n+1}\right) p$$

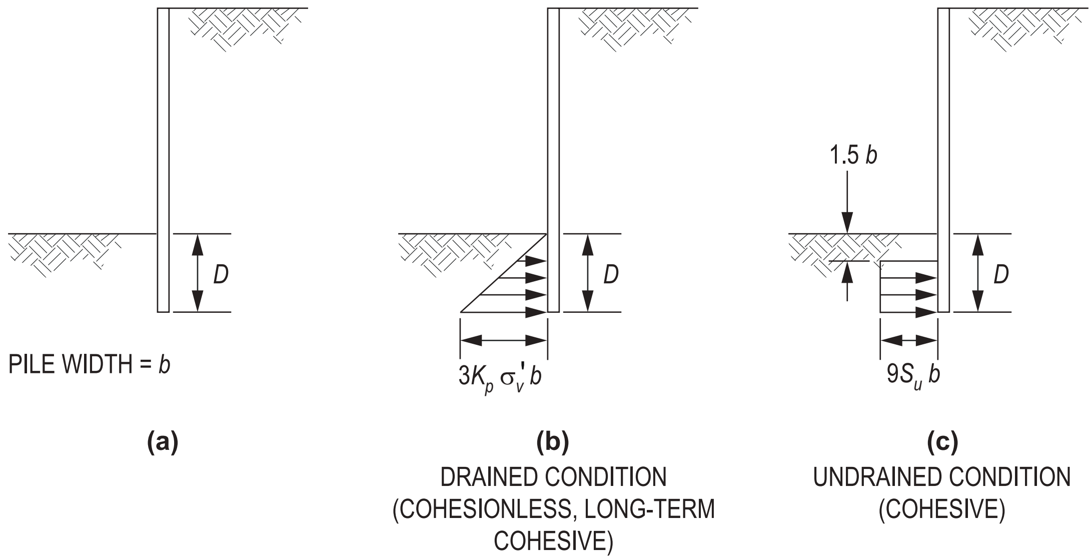
Maximum moment below $B = pL^2/10$

where L is the larger of H_2, H_n, H_{n+1}

(b) Walls with multiple levels of ground anchors

Calculation of Wall Bending Moments Using Tributary Area Method

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 39, p. 80. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



Broms Method for Evaluating Ultimate Passive Resistance

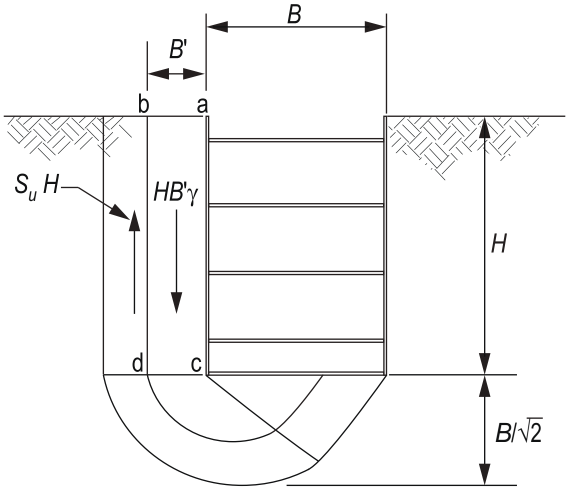
Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 41, p. 86. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

Recommended Factors for Safety for Axial Capacity of Driven and Drilled-in Soldier Beams

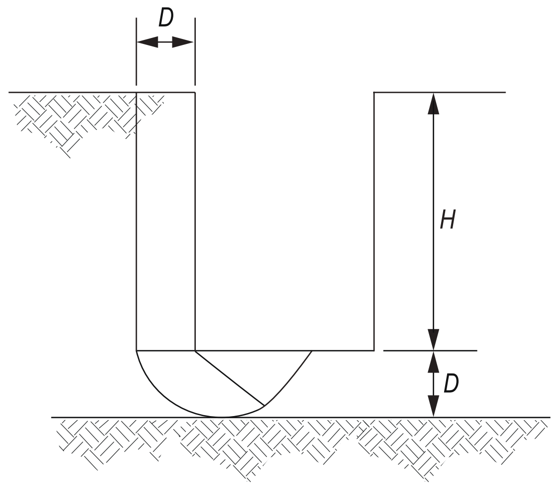
Soil Type	Factor of Safety on Skin Friction	Factor of Safety on End Bearing
Clays	2.5	2.5
Sands	2.0	2.5

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Table 14, p. 90. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

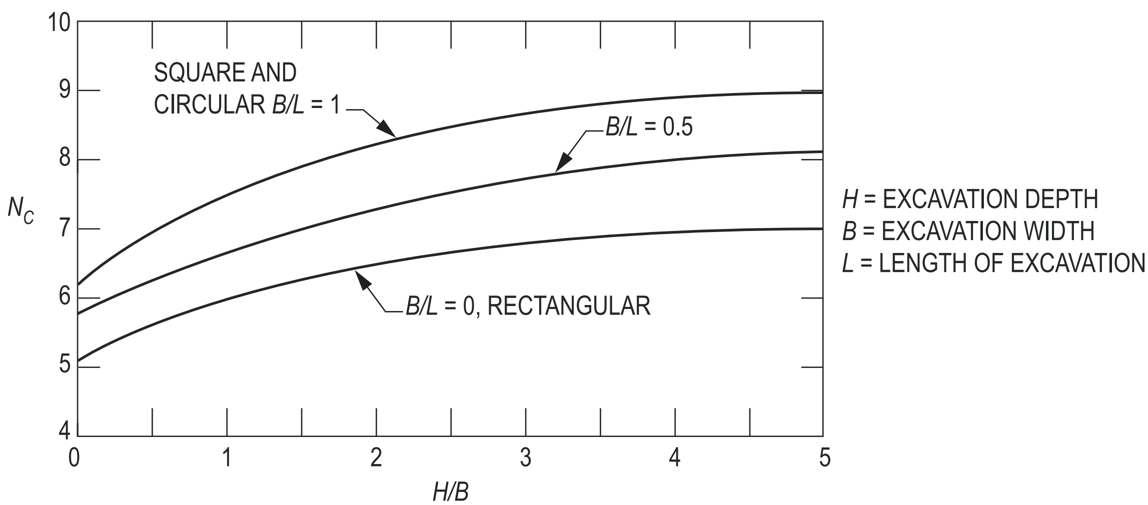
Evaluation of Bottom Heave Potential in Soft to Medium Clays



(a) FAILURE PLANES, DEEP DEPOSITS OF WEAK CLAY



(b) FAILURE PLANE, STIFF LAYER BELOW BOTTOM OF EXCAVATION



(c) BEARING CAPACITY FACTOR, N_C

Analysis of Basal Stability
(modified after Terzaghi et al., 1996, Soil Mechanics in Engineering Practice, Reprinted with permission of John Wiley & Sons, Inc.)

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, Fig. 51, p. 106. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

For deep clay deposits:

$$FS = \frac{N_c\, S_u}{H\left(\gamma - \frac{S_u \sqrt{2}}{B}\right)}$$

For cuts with stiff layer below bottom excavation:

$$FS = \frac{N_c\, S_u}{H\left(\gamma - \frac{S_u}{D}\right)}$$

S_u = shear strength

γ = soil density

N_c = bearing capacity factor

H = excavation depth

B = excavation width

$B' = B/\sqrt{2}$

L = length of excavation

D = depth to firm layer

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, p. 106. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

3.18.4 Anchor Capacity

Presumptive Ultimate Values of Load Transfer for Preliminary Design of Small-Diameter Straight-Shaft Gravity-Grouted Ground Anchors in Soil

Soil Type	Relative Density/Consistency (SPT Range) [1]	Estimated Ultimate Transfer Load (kip/ft)
Sand and gravel	Loose (4–10) Medium dense (11–30) Dense (31–50)	9.9 15 19.9
Sand	Loose (4–10) Medium dense (11–30) Dense (31–50)	6.9 9.9 13
Sand and silt	Loose (4–10) Medium dense (11–30) Dense (31–50)	4.8 6.9 8.9
Silt-clay mixture with low plasticity or fine micaceous sand or silt mixtures	Stiff (10–20) Hard (21–40)	2 4.1

[1] SPT values are corrected for overburden pressure.

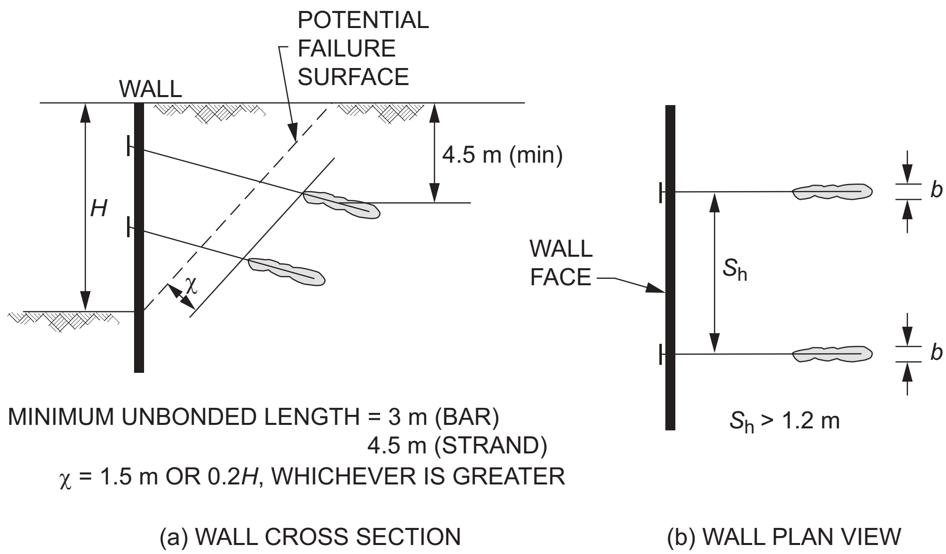
Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Table 6, p. 71. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

Presumptive Average Ultimate Bond Stress for Ground/Grout Interface Along Anchor Bond Zone (after PTI, 1966)

Rock		Cohesive Soil		Cohesionless Soil	
Rock Type	Average Ultimate Bond Stress (psi)	Anchor Type	Average Ultimate Bond Stress (psi)	Anchor Type	Average Ultimate Bond Stress (psi)
Granite and basalt	250–450	Gravity-grouted anchors (straight shaft)	5–10	Gravity-grouted anchors (straight shaft)	10–20
Dolomitic limestone	200–300	Pressure-grouted anchors (straight shaft)			
Soft limestone	150–200	Soft silty clay	5–10	Fine to med. sand, med. dense to dense	12–55
Slates and hard shales	120–200	Silty clay	5–10	Med. to coarse sand (w/gravel), med. dense	16–95
Soft shales	30–120	Stiff clay, med. to high plasticity	5–15	Med. to coarse sand (w/gravel), dense to very dense	35–140
Sandstones	120–250	Very stiff clay, med. to high plasticity	10–25	Silty sands	25–60
Weathered sandstones	100–120	Stiff clay, med. plasticity	15–25	Dense glacial till	43–75
Chalk	30–155	Very stiff clay, med. plasticity	20–50	Sandy gravel, med. dense to dense	31–200
Weathered marl	25–35	Very stiff sandy silt, med. plasticity	40–55	Sandy gravel, dense to very dense	40–200
Concrete	200–400				

Note: Actual values for pressure-grouted anchors depend on the ability to develop pressures in each soil type.

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Table 7, p. 73. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



Vertical and Horizontal Spacing Requirements for Ground Anchors

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 37, p. 76. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

Properties of Prestressing Steel Bars (ASTM A722)

Steel Grade	Nominal Diameter	Ultimate Stress, f_{pu}	Nominal Cross-Section Area, A_{ps}	Ultimate Strength, $f_{pu} A_{ps}$	Prestressing Force		
					$0.8 f_{pu} A_{ps}$	$0.7 f_{pu} A_{ps}$	$0.6 f_{pu} A_{ps}$
(ksi)	(in.)	(ksi)	(in^2)	(kips)	(kips)	(kips)	(kips)
150	1	150	0.85	127.5	102.0	89.3	76.5
	1 1/4	150	1.25	187.5	150.0	131.3	112.5
	1 3/8	150	1.58	237.0	189.6	165.9	142.2
	1 3/4	150	2.66	400.0	320.0	280.0	240.0
	2 1/2	150	5.19	778.0	622.4	435.7	466.8
160	1	160	0.85	136.0	108.8	95.2	81.6
	1 1/4	160	1.25	200.0	160.0	140.0	120.0
	1 3/8	160	1.58	252.8	202.3	177.0	151.7
(ksi)	(mm)	(N/mm^2)	(mm^2)	(kN)	(kN)	(kN)	(kN)
150	26	1,035	548	568	454	398	341
	32	1,035	806	835	668	585	501
	36	1,035	1,019	1,055	844	739	633
	45	1,035	1,716	1,779	1,423	1,246	1,068
	64	1,035	3,348	3,461	2,769	2,423	2,077
160	26	1,104	548	605	484	424	363
	32	1,104	806	890	712	623	534
	36	1,104	1,019	1,125	900	788	675

Source: Reproduced with permission from ASTM. A722 *Standard Specification for High-Strength Steel Bars for Prestressed Concrete*. Copyright ASTM International, www.astm.org. As found in Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U. S. Department of Transportation, May 2006, Table 9, p. 77. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf

Properties of 15-mm Diameter Prestressing Steel Strands [ASTM A416, Grade 270 (metric 1860)]

Number of 15-mm-Diameter Strands	Cross-Section Area		Ultimate Strength		Prestressing Force					
					$0.8\,f_{pu}\,A_{ps}$		$0.7\,f_{pu}\,A_{ps}$		$0.6\,f_{pu}\,A_{ps}$	
	(in^2)	(mm^2)	(kips)	(kN)	(kips)	(kN)	(kips)	(kN)	(kips)	(kN)
1	0.217	140	58.6	260.7	46.9	209	41.0	182	35.2	156
3	0.651	420	175.8	782.1	140.6	626	123.1	547	105.5	469
4	0.868	560	234.4	1,043	187.5	834	164.1	730	140.6	626
5	1.085	700	293.0	1,304	234.4	1,043	205.1	912	175.8	782
7	1.519	980	410.2	1,825	328.2	1,460	287.1	1,277	246.1	1,095
9	1.953	1,260	527.4	2,346	421.9	1,877	369.2	1,642	316.4	1,408
12	2.604	1,680	703.2	3,128	562.6	2,503	492.2	2,190	421.9	1,877
15	3.255	2,100	879.0	3,911	703.2	3,128	615.3	2,737	527.4	2,346
19	4.123	2,660	1,113.4	4,953	890.7	3,963	779.4	3,467	668.0	2,972

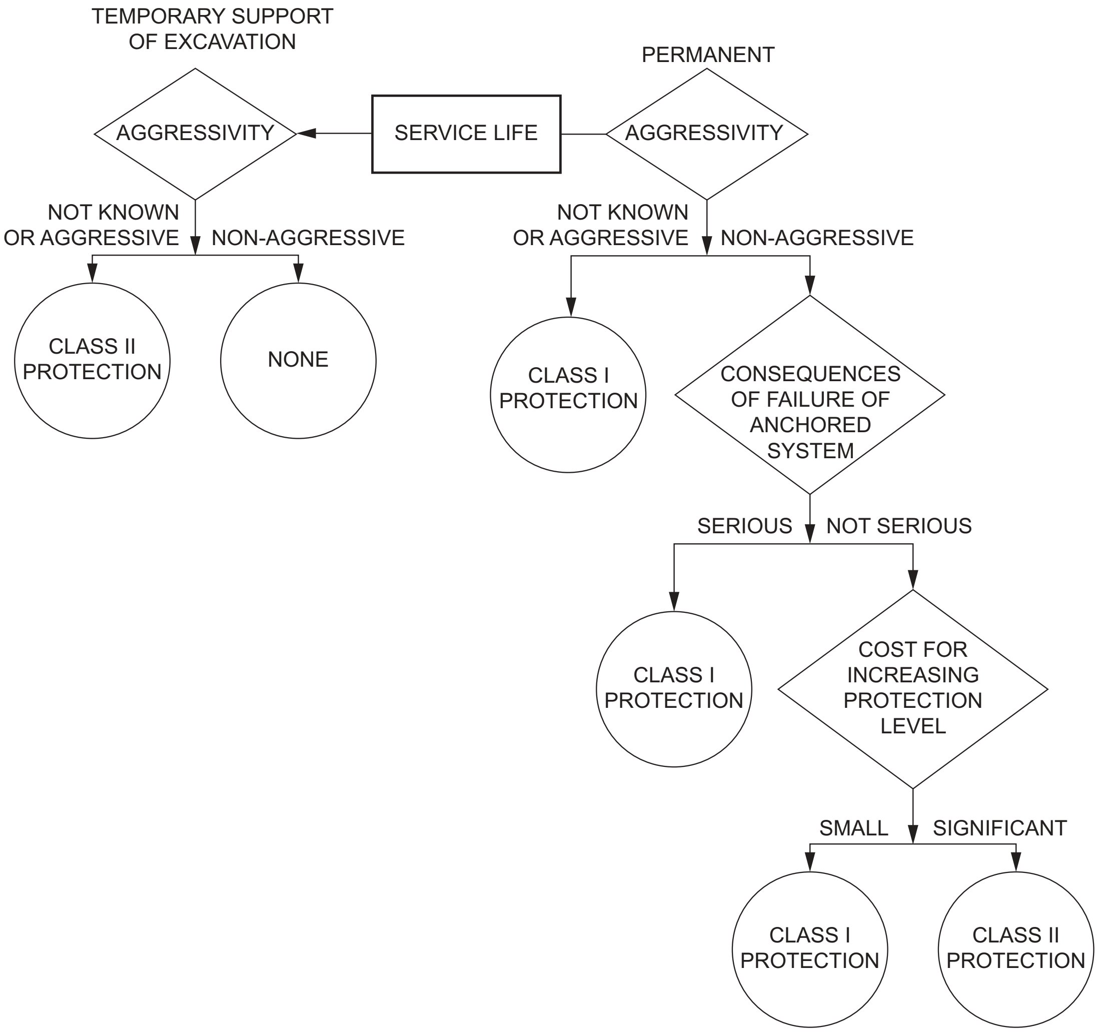
Source: Reproduced with permission from ASTM. A416 *Standard Specification for Low-Relaxation, Seven-Wire Steel Strand for Prestressed Concrete*. Copyright ASTM International, www.astm.org. As found in Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Table 10, p. 78. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

3.18.5 Corrosion Protection

Corrosion Protection Requirements (modified after PTI, 1996)

Class	Protection Requirements		
	Anchorage	Unbonded Length	Tendon Bond Length
I (Encapsulated Tendon)	1. Trumpet 2. Cover if exposed	1. Encapsulate tendons composed of individual grease-filled extruded strand sheaths with a common smooth sheath. 2. Encapsulate tendons composed of individual grease-filled strand sheaths with a grout-filled smooth sheath. 3. Use smooth bondbreaker over a grout-filled bar sheath.	1. Grout-filled encapsulation or 2. Fusion-bonded epoxy
II (Grout-protected tendon)	1. Trumpet 2. Cover if exposed	1. Grease-filled sheath or 2. Heat-shrink sleeve	Grout

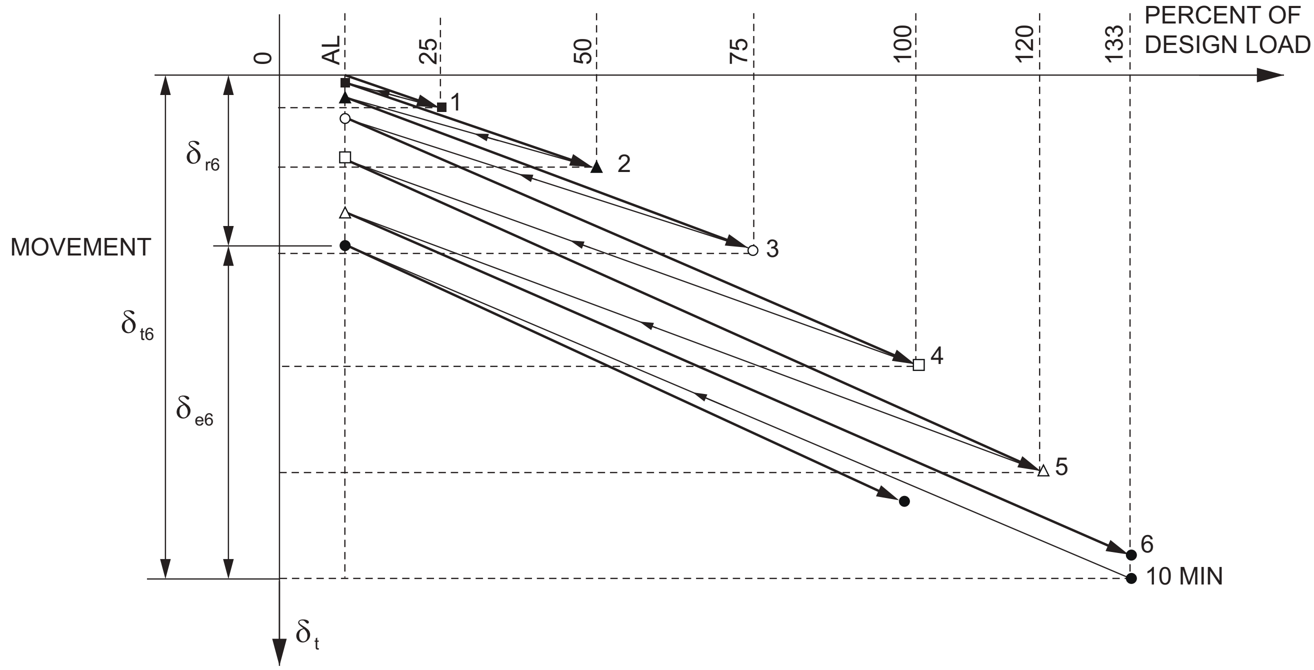
Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Table 20, p. 131. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



Decision Tree for Selection of Corrosion Protection Level (modified after PTI, 1996)

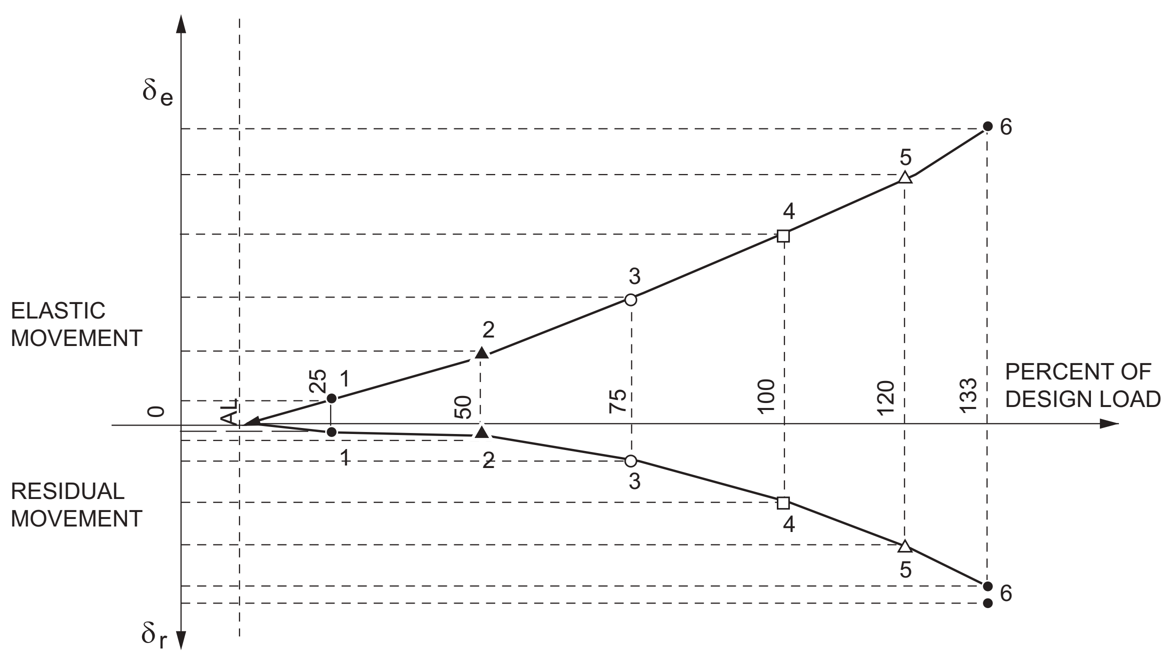
Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 63, p. 134. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

3.18.6 Load Testing



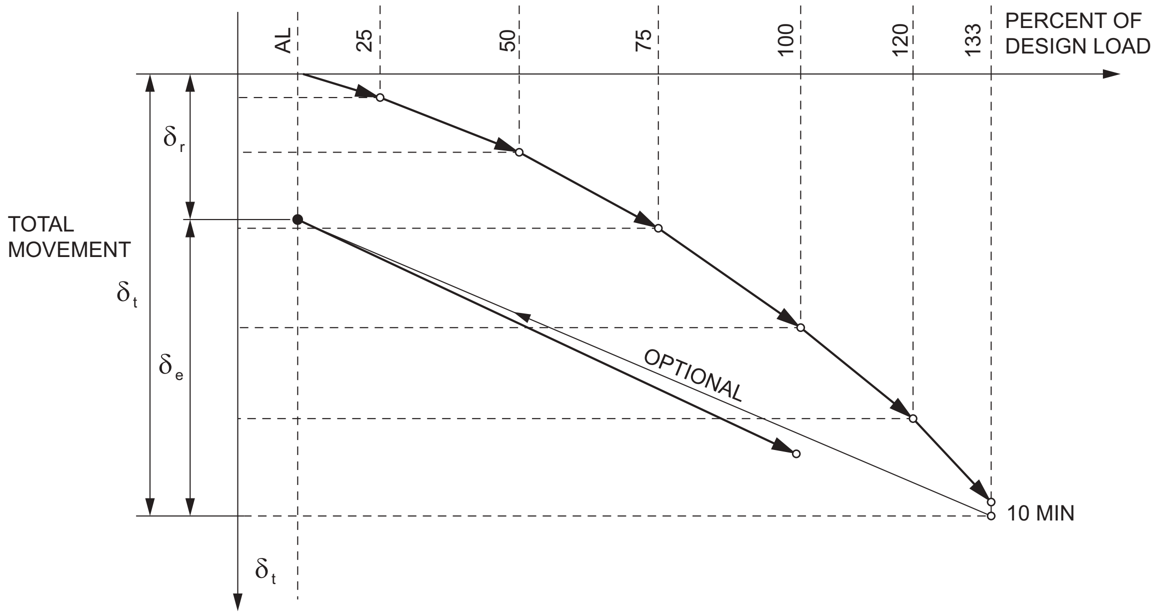
Plotting Performance Test Data (after PTI, 1996)

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 69, p. 145. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



Plotting Elastic and Residual Movement for a Performance Test (after PTI, 1996)

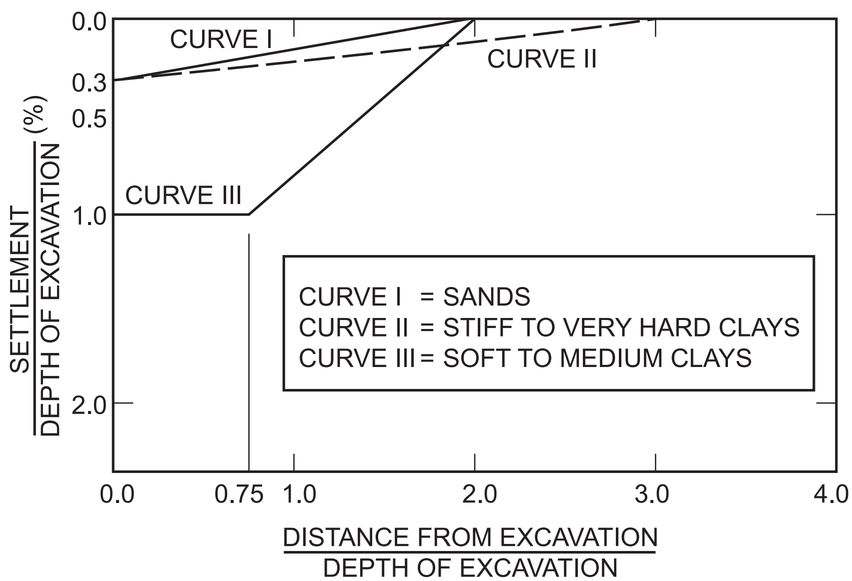
Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems*. FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 70, p. 146. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.



Plotting of Proof Test Data (after PTI, 1996)

Source: Federal Highway Administration. *Geotechnical Engineering Circular No. 4: Ground Anchors and Anchored Systems.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 71, p. 148. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

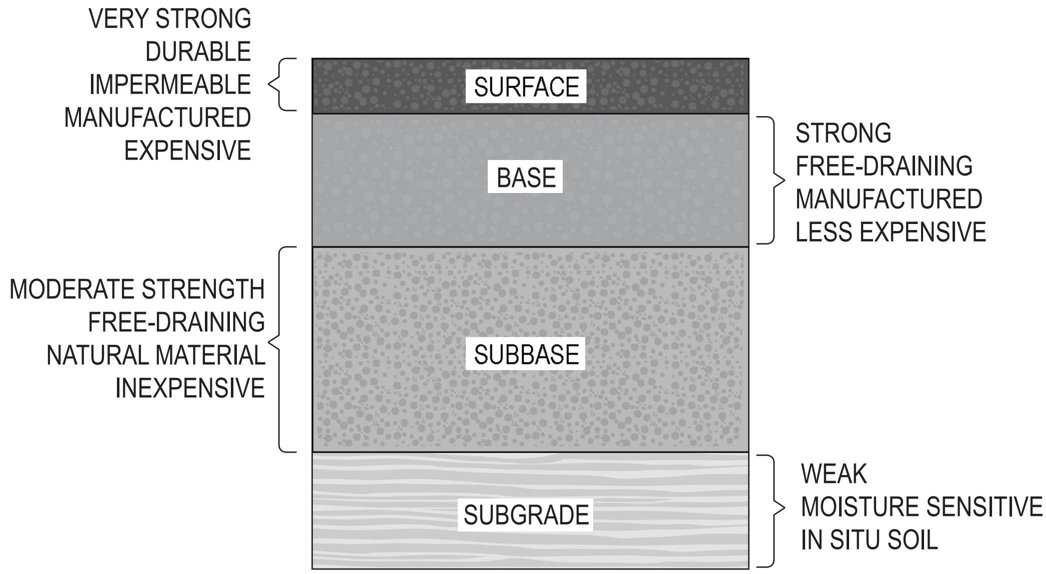
3.18.7 Settlement



Settlement Profile Behind Braced and Anchored Walls

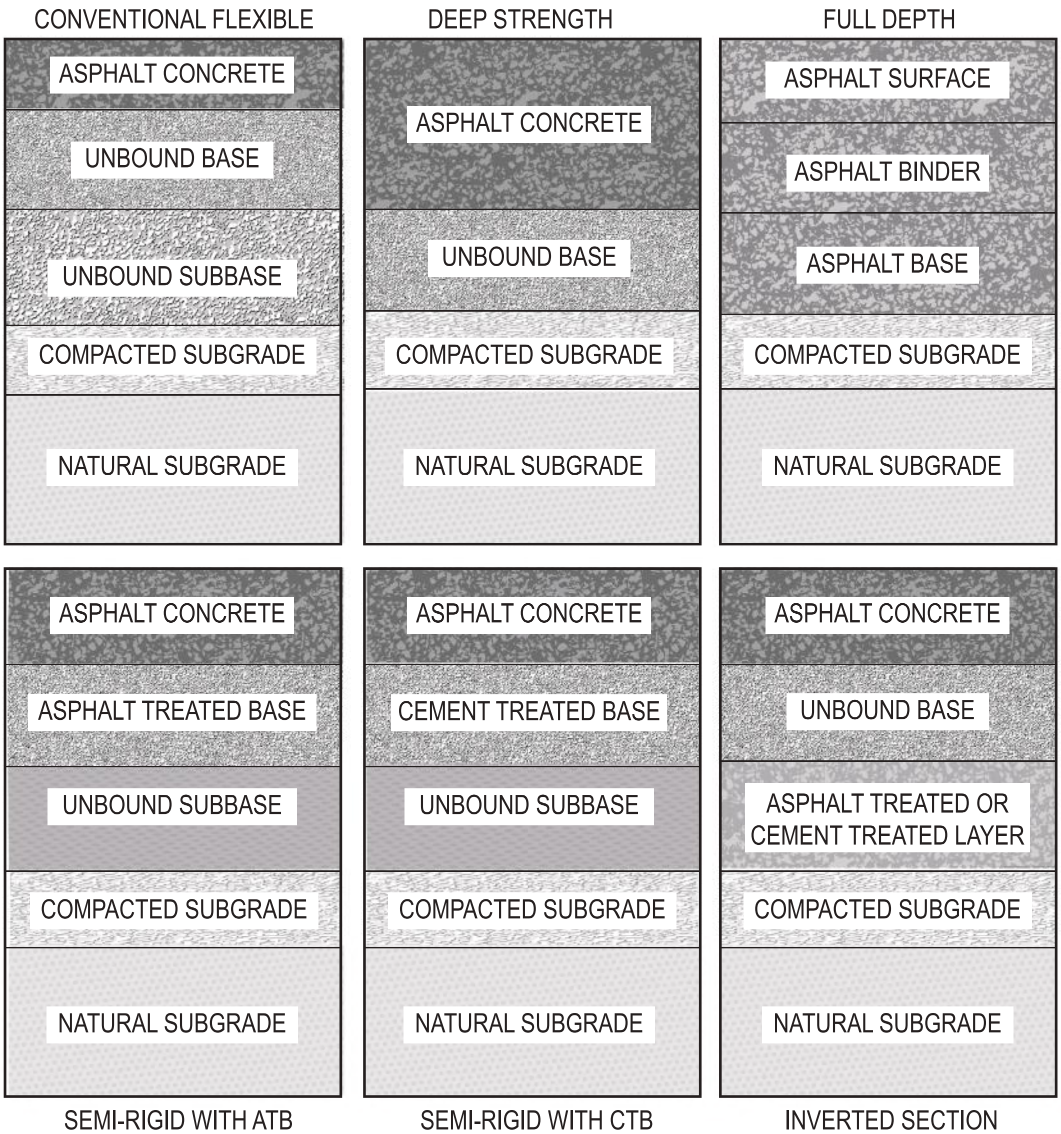
Source: Federal Highway Administration. *Ground Anchors and Anchored Systems: Geotechnical Engineering Circular No. 4.* FHWA-IF-99-015. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 59, p. 120. https://www.fhwa.dot.gov/engineering/geotech/pubs/if99015.pdf.

3.19 Pavements



Basic Pavement Structure

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements.* FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 3-2, p. 3-3. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.



Common Pavement Systems

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 1-3, p. 1-11. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

Geotechnical Influences on Major Distresses in Flexible Pavements

	Insufficient Base Stiffness/ Strength	Insufficient Subgrade Stiffness/ Strength	Moisture/ Drainage Problems	Freeze/ Thaw	Swelling	Contamination	Erosion	Spatial Variability
Fatigue Cracking	X	X	X	X		X		
Rutting	X	X	X	X		X		
Corrugations	X							
Bumps				X	X			X
Depressions	X		X	X		X		X
Potholes			X	X				X
Roughness	X	X	X	X	X	X		X

Geotechnical Influences on Major Distresses in Rigid Pavements

	Insufficient Base Stiffness/ Strength	Insufficient Subgrade Stiffness/ Strength	Moisture/ Drainage Problems	Freeze/ Thaw	Swelling	Contamination	Erosion	Spatial Variability
Fatigue Cracking	X	X	X	X		X	X	
Punchouts (CRCP)	X	X	X	X		X	X	
Pumping			X				X	
Faulting	X		X	X	X	X	X	
Roughness	X		X	X	X	X	X	X

Geotechnical Influences on Pavements

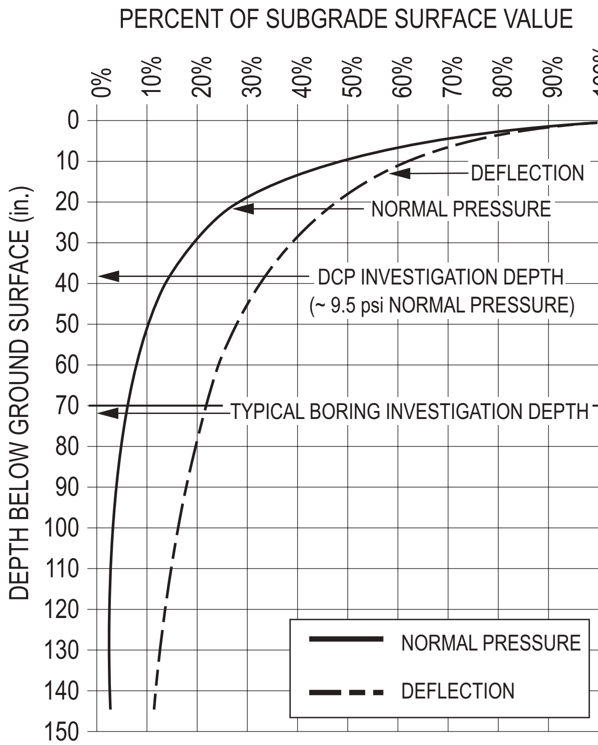
Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Tables 1-1 & 1-2, p. 1-18. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

In Situ Tests for Subsurface Exploration in Pavement Design and Construction

Type of Test	Best Suited For	Not Applicable	Properties That Can be Determined for Pavement Design and Construction	Remarks
Standard Penetration Test (SPT)* AASHTO T 206 and ASTM D1586	Sand and silt	Gravel, questionable results in saturated silt	Crude estimate of modulus in sand. Disturbed samples for identification and classification. Evaluation of density for classification.	Test best suited for sands. Estimated clay shear strengths are crude and should not be used for design.
Dynamic Cone Test (DCP)* ASTM D6951	Sand, gravel, and clay	Clay with varying gravel content	Qualitative correlation to CBR. Identify spatial variation in subgrade soil and stratification.	
Static Piezocone Test (CPT)* ASTM D3441	Sand, silt, clay		Undrained shear strength and correlation to CBR in clays, density and strength of sand and gravel. Evaluation of subgrade soil type, vertical strata limits, and groundwater level.	Use piezocone for pore pressure data. Tests in clay are reliable only when used in conjunction with other calibration tests (e.g., vane tests).
Field CBR	Sand, gravel, silt, clay	Granular soils (lab and field correlations erratic)	Load-deflection test providing direct evaluation of CBR and can be correlated with subgrade modulus k-value	Slow, and field moisture may not represent worst-case condition.
Plate Load Test AASHTO T222 and ASTM D1196	Sand, gravel, silt, clay		Subgrade modulus k-value	Slow and labor intensive
Vane Shear Test (VST) AASHTO T-223	Clay	Silt, sand, gravel	Undrained shear strength, C_u with correlation to CBR	Test should be used with care, particularly in fissured, varved, and highly plastic clays.
Permeability Test ASTM D51216 and ASTM D6391	Sand, gravel	Clay	Evaluation of coefficient of permeability in base and subbase for rehabilitation projects	Variable head tests in boreholes have limited accuracy.
Pressuremeter Test (PMT) ASTM D4719	Soft rock, sand, silt, clay		Subgrade modulus k-value and undrained shear strength with correlation to CBR	Requires highly skilled field personnel.
Dilatometer Test (DMT)	Sand, clay		Soil stiffness can be related to subgrade modulus k and compressibility.	Limited database and requires highly skilled field personnel.

* These tests can be used in pavement design to qualitatively evaluate subgrade stratification and determine optimum undisturbed sample locations required to obtain design property values.

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Table 4-7, p. 4-36. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.



Typical Zone of Influence for an Asphalt Pavement Section (Vandre et al., 1998)

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 4-3, p. 4-17. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

Other Tests for Aggregate Quality and Durability

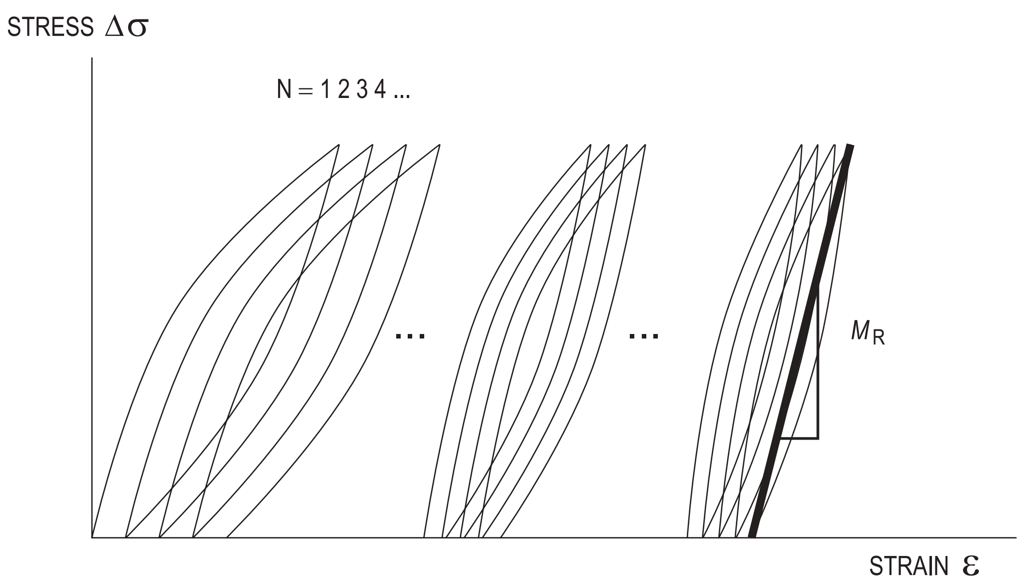
Property	Use	AASHTO Specification	ASTM Specification
Fine Aggregate Quality			
Sand equivalent	Measure of the relative proportion of plastic fines and dust to sand size particles in material passing the No. 4 sieve	T 176	D 2419
Fine aggregate angularity (also termed uncompacted air voids)	Index property for fine aggregate internal friction in Superpave asphalt mix design method	T 304	C 1252
Coarse Aggregate Quality			
Coarse aggregate angularity	Index property for coarse aggregate internal friction in Superpave asphalt mix design method		D 5821
Flat, elongated particles	Index property for particle shape in Superpave asphalt mix design method		D 4791
General Aggregate Quality			
Absorption	Percentage of water absorbed into permeable voids	T 84/T 85	C 127/C 128
Particle index	Index test for particle shape		D 3398
Los Angeles degradation	Measure of coarse aggregate resistance to degradation by abrasion and impact	T 96	C 131 or C 535
Soundness	Measure of aggregate resistance to weathering in concrete and other applications	T 104	C 88
Durability	Index of aggregate durability	T 210	D 3744
Expansion	Index of aggregate suitability		D 4792
Deleterious materials	Describes presence of contaminants like shale, clay lumps, wood, and organic material	T 112	C 142

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Table 5-26, p. 5-34. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

Typical CBR Values (after U.S. Army Corps of Engineers, 1953)

USCS Soil Class	Field CBR
GW	60–80
GP	35–60
GM	40–80
GC	20–40
SW	20–40
SP	15–25
SM	20–40
SC	10–20
ML	5–15
CL	5–15
OL	4–8
MH	4–8
CH	3–5
OH	3–5

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Table 5-28, p. 5-39. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.



Resilient Modulus under Cyclic Loading

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 5-14, p. 5-47. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

Correlations between M_R and other soil properties include the following:

AASHTO 1993 Guide
Granular base and subbase layers

θ (psi)	M_R (psi)
100	$740 \times CBR$
	$1{,}000 + 780 \times R$
30	$440 \times CBR$
	$1{,}000 + 450 \times R$
20	$340 \times CBR$
	$1{,}000 + 350 \times R$
10	$250 \times CBR$
	$1{,}000 + 250 \times R$

Subgrade (roadbed) soils

M_R (psi) = $1{,}500 \times CBR$ for $CBR \le 10$ (Heukelom and Klomp, 1962)

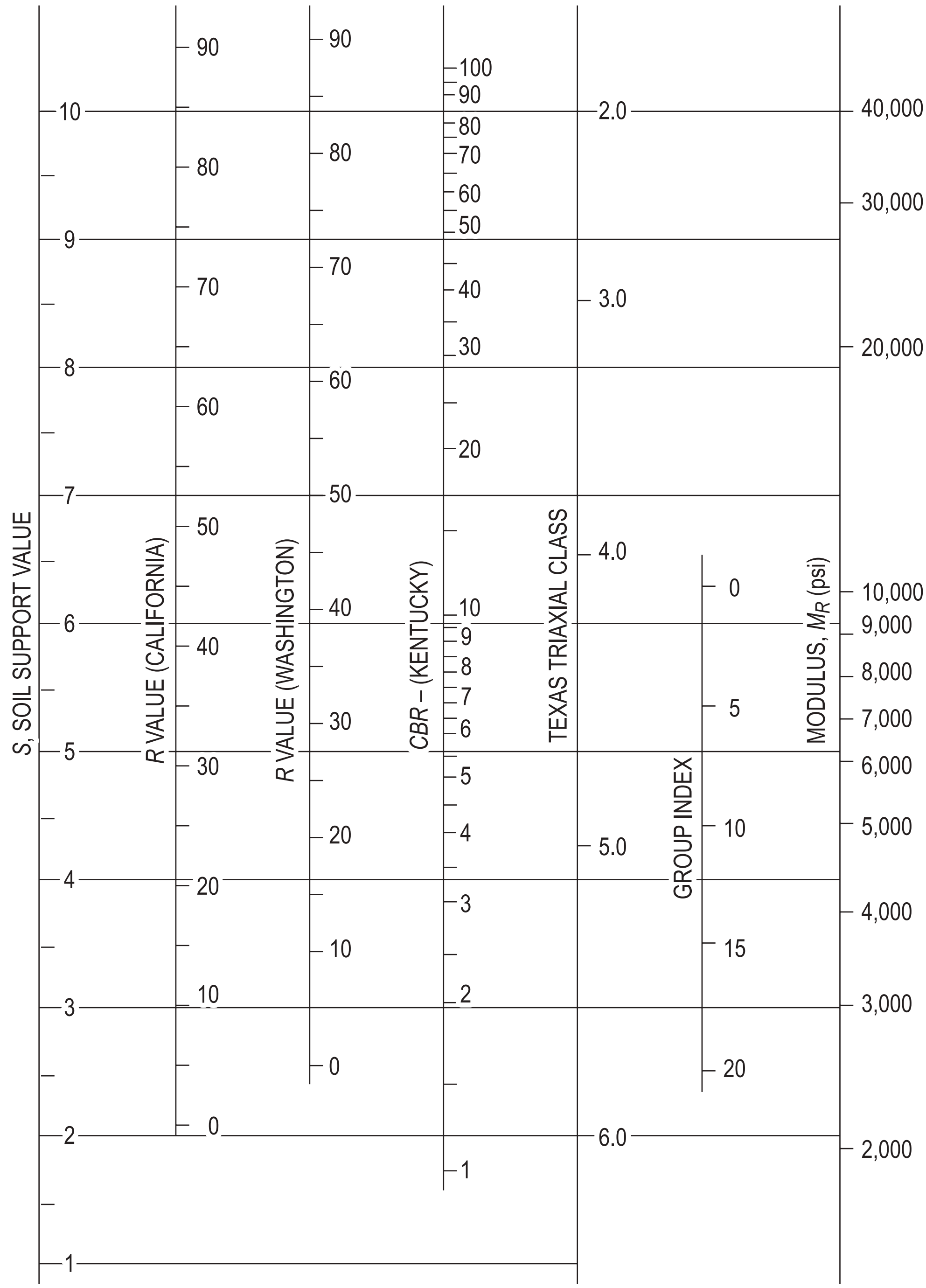
M_R (psi) = $A + B \times$ (R-value)

with A = 772 to 1,155; B = 369 to 555 (Asphalt Institute, 1982)

M_R (psi) = $1{,}000 + 555 \times R$-value (recommended values)

Additional useful correlations for subgrade M_R are provided in the following figure (Subgrade Resilient Modulus).

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Table 5-31, p. 5-49. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.



Subgrade Resilient Modulus

Correlations between subgrade resilient modulus and other soil properties
(1 psi = 6.9 kPa; from Huang, 1993, after Van Til *et al.*, 1972)

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 5-17, p. 5-52. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

Correlations between Subgrade Resilient Modulus and Other Soil Properties (1 psi = 6.9 kPa) (from Huang, 1993, after Van Til et al., 1972)

S, Soil Support Value	Modulus, M_R (psi)	*R* Value (California)	Modulus, M_R (psi)	*R* Value (Washington)	Modulus, M_R (psi)	*CBR* (Kentucky)	Modulus, M_R (psi)	Texas Triaxial Class	Modulus, M_R (psi)	Group Index	Modulus, M_R (psi)
10.0	40,000	90	47,000	90	47,500	100	44,000	2.0	40,000	0	10,000
9.5	34,000	85	40,000	85	41,000	95	42,500	(2.5)	30,000	(2.5)	8,800
9.0	27,000	80	34,000	80	35,800	90	41,000	3.0	23,000	5	7,200
8.5	22,000	75	28,000	75	29,100	85	40,000	(3.5)	16,000	(7.5)	6,000
8.0	19,000	70	27,500	70	25,000	80	39,000	4.0	11,300	10	5,000
7.5	16,000	65	20,000	65	21,200	75	37,000	(4.5)	7,400	(12.5)	4,300
7.0	13,500	60	17,000	60	18,000	70	36,000	5.0	4,800	15	3,700
6.5	11,300	55	14,400	55	15,500	65	33,800	(5.5)	3,200	(17.5)	3,200
6.0	9,400	50	12,200	50	13,400	60	31,200	6.0	2,080	20	2,500
5.5	7,900	45	10,100	45	11,500	55	29,200				
5.0	6,400	40	8,800	40	9,600	50	27,900				
4.5	5,200	35	7,400	35	8,200	45	25,400				
4.0	4,400	30	6,000	30	6,800	40	23,900				
3.5	3,700	25	5,000	25	4,750	35	21,500				
3.0	3,000	20	4,600	20	4,900	30	19,800				
2.5	2,700	15	3,600	15	4,200	25	17,000				
2.0	2,080	10	3,100	10	3,600	20	15,000				
1.5	1,700	5	2,550	5	3,000	15	12,000				
1.0	1,460	0	2,150	0	2,580	10	9,600				
						9.5	9,200				
						9	9,000				
						8.5	8,600				
						8	8,200				
						7.5	7,800				
						7	7,450				
						6.5	7,000				
						6	6,600				
						5.5	6,200				
						5	5,800				
						4.5	5,200				
						4	4,900				
						3.5	4,500				
						3	4,100				
						2.5	3,500				
						2	3,050				
						1.5	2,400				
						1	1,890				

(X.X) = Estimated Value

NOTE: Soil properties values are approximate and estimated based on the log scale of Subgrade Resilient Modulus, M_R, from the original figure included in the FHWA 05-037, May 2006, NHI Course No. 132040, Geotechnical Aspects of Pavements Reference Manual/Participant Workbook

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 5-17, p. 5-52. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

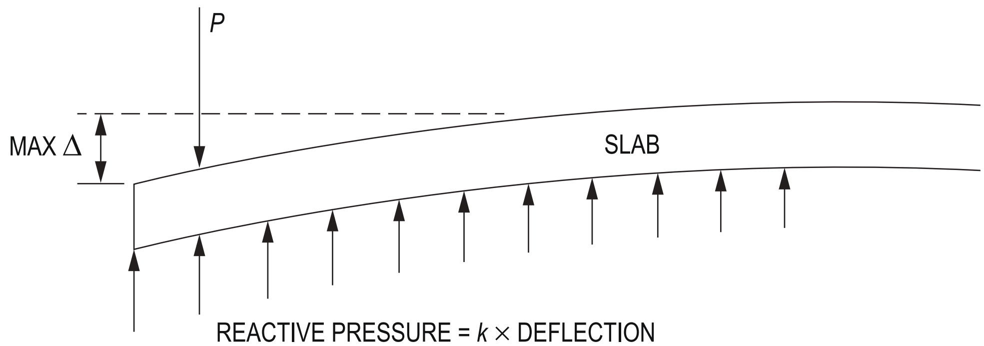
Default M_R Values for Unbound Granular and Subgrade Materials at Unsoaked Optimum Moisture Content and Density Conditions (NCHRP 1-37A, 2004)

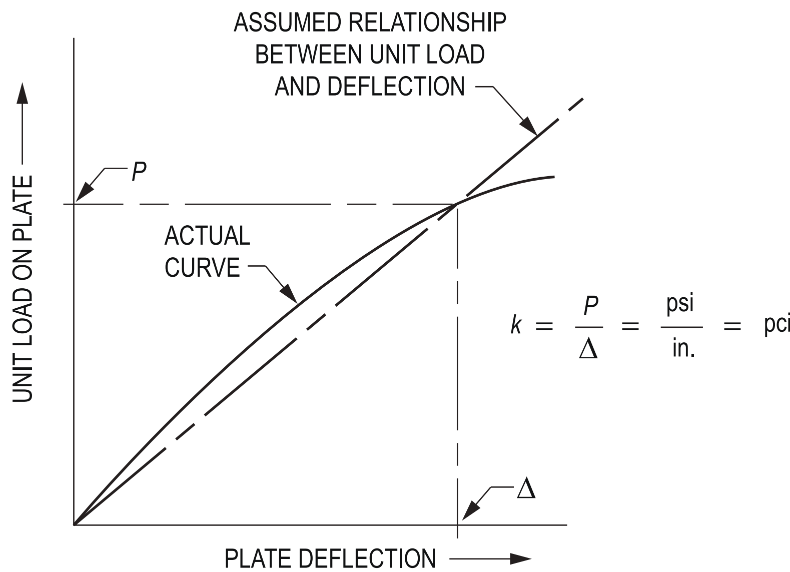
Material Classification	M_R Range (psi)*	Typical M_R (psi)*
AASHTO Soil Class		
A-1-a	38,500–42,000	40,000
A-1-b	35,500–40,000	38,000
A-2-4	28,000–37,500	32,000
A-2-5	24,000–33,000	28,000
A-2-6	21,500–31,000	26,000
A-2-7	21,500–28,000	24,000
A-3	24,500–35,500	29,000
A-4	21,500–29,000	24,000
A-5	17,000–25,500	20,000
A-6	13,500–24,000	17,000
A-7-5	8,000–17,500	12,000
A-7-6	5,000–13,500	8,000
USCS Soil Class		
GW	39,500–42,000	41,000
GP	35,500–40,000	38,000
GM	33,000–42,000	38,500
GC	24,000–37,500	31,000
GW-GM	35,500–40,500	38,500
GP-GM	31,000–40,000	36,000
GW-GC	28,000–40,000	34,500
GP-GC	28,000–39,000	34,000
SW	28,000–37,500	32,000
SP	24,000–33,000	28,000
SM	28,000–37,500	32,000
SC	21,500–28,000	24,000
SW-SM	24,000–33,000	28,000
SP-SM	24,000–33,000	28,000
SW-SC	21,500–31,000	25,500
SP-SC	21,500–31,000	25,500
ML	17,000–25,500	20,000
CL	13,500–24,000	17,000
MH	8,000–17,500	11,500
CH	5,000–13,500	8,000

*Multiply by 0.069 to convert to MPa.

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Table 5-35, p. 5-54. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

Modulus of Subgrade Reaction





Coefficient of subgrade reaction *k* (Yoder and Witczak, 1975).

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Fig. 5-23, p. 5-68. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

Subgrade Improvement Methods

Roadway Design Conditions		Geosynthetic Type					
Subgrade	Base/Subbase Thickness[1] (mm)	Geotextile		Geogrid[2]		GG-GT Composite	
		Nonwoven	Woven	Extruded	Knitted or Woven	Open-Graded Base[3]	Well-Graded Base
Soft ($CBR < 3$) ($M_R < 30$ MPa)	150–300	❹	●	●	❏	●	⑤
	> 300	❹	❹	◗	◗	◗	⑤
Firm–Vy. Stiff ($3 \le CBR \le 8$) ($30 \le MR \le 80$)	150–300	❍	◗	●	❏	●	⑤
	> 300	❍	❍	❍	❍	❍	❍

KEY:

●—usually applicable ◗—applicable for some conditions

❍—usually not applicable ❏—insufficient information at this time ⑤—see note

NOTES:

1. Total base or subbase thickness with geosynthetic reinforcement. Reinforcement may be placed at bottom of base or subbase, or within base for thicker (usually > 300 mm (12 in.)) thicknesses. Thicknesses less than 150 mm (6 in.) not recommended for construction over soft subgrade. Placement of less than 150 mm (6 in.) over a geosynthetic not recommended.
2. For open-graded base or thin bases over wet, fine grained subgrades, a separation geotextile should be considered with geogrid reinforcement.
3. Potential assumes base placed directly on subgrade. A subbase also may provide filtration.

❹ Reinforcement usually applicable, but typically addressed as a subgrade stabilization.

⑤ Geotextile component of composite likely is not required for filtration with a well-graded base course; therefore, composite reinforcement usually not applicable.

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Table 7-11, p. 7-38. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

Guide for Selection of Admixture Stabilization Method(s) (Austroads, 1998)

	MORE THAN 25% PASSING 75 μm			LESS THAN 25% PASSING 75 μm		
PLASTICITY INDEX	$PI \le 10$	$10 < PI < 20$	$PI \ge 20$	$PI \le 6$ PI x % PASSING 75 μm ≤ 60	$PI \le 10$	$PI > 10$
FORM OF STABILIZATION						
CEMENT AND CEMENTITIOUS BLENDS	Usually suitable	Doubtful	Usually not suitable	Usually suitable	Usually suitable	Usually suitable
LIME	Doubtful	Usually suitable	Usually suitable	Usually not suitable	Doubtful	Usually suitable
BITUMEN	Doubtful	Doubtful	Usually not suitable	Usually suitable	Usually suitable	Doubtful
BITUMEN/ CEMENT BLENDS	Usually suitable	Doubtful	Usually not suitable	Usually suitable	Usually suitable	Doubtful
GRANULAR	Usually suitable	Usually not suitable	Usually not suitable	Usually suitable	Usually suitable	Doubtful
MISCELLANEOUS CHEMICALS*	Usually not suitable	Usually suitable	Usually suitable	Usually not suitable	Doubtful	Usually suitable

KEY: USUALLY SUITABLE (dark shading); DOUBTFUL (vertical hatching); USUALLY NOT SUITABLE (blank)

*Should be taken as a broad guideline only. Refer to trade literature for further information.

Note: The above forms of stabilization may be used in combination, e.g. lime stabilization to dry out materials and reduce their plasticity, making them suitable for other methods of stabilization.

Source: Federal Highway Administration. National Highway Institute. *Geotechnical Aspects of Pavements*. FHWA-NHI-05-037. Washington, DC: U.S. Department of Transportation, May 2006, Table 7-16, p. 7-78. https://www.fhwa.dot.gov/engineering/geotech/pubs/05037/03a.cfm.

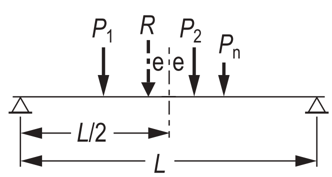
4 STRUCTURAL

4.1 Structural Analysis

4.1.1 Influence Lines for Beams and Trusses

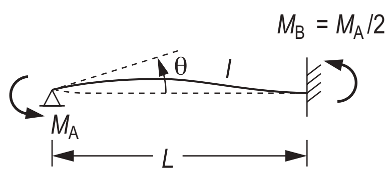
An influence line shows the variation of an effect (reaction, shear, and moment in beams, bar force in a truss) caused by moving a unit load across the structure. An influence line is used to determine the position of a moveable set of loads that causes the maximum value of the effect.

4.1.2 Moving Concentrated Load Sets



The **absolute maximum moment** produced in a beam by a set of *n* moving loads occurs when the resultant *R* of the load set and an adjacent load are equidistant from the centerline of the beam. In general, two possible load set positions must be considered, one for each adjacent load.

4.1.3 Beam Stiffness and Moment Carryover



$$\theta = \frac{ML}{4EI} \rightarrow M = \left(\frac{4EI}{L}\right)\theta = k_{AB}\,\theta$$

where

k_{AB} = stiffness

$M_B = \dfrac{M_A}{2}$ = carryover

4.1.4 Truss Deflection by Unit Load Method

The displacement of a truss joint caused by external effects (truss loads, member temperature change, member misfit) is found by applying a unit load at the point that corresponds to the desired displacement.

$$\Delta_{\text{joint}} = \sum_{i=1}^{\text{members}} f_i (\Delta L)_i$$

where

Δ_{joint} = joint displacement at point of application of unit load (+ in direction of unit load)

f_i = force in member i caused by unit load (+ tension)

$(\Delta L)_i$ = change in length caused by external effect (+ for increase in member length):

$= \left(\frac{FL}{AE}\right)_i$ for bar force F caused by external load

$= \alpha L_i (\Delta T)_i$ for temperature change in member

= member misfit

where

α = coefficient of thermal expansion

L = member length

A = member cross-sectional area

E = member elastic modulus

4.1.5 Frame Deflection by Unit Load Method

The displacement of any point on a frame caused by external loads is found by applying a unit load at that point that corresponds to the desired displacement:

$$\Delta = \sum_{i=1}^{\text{members}} \int_{x=0}^{x=L_i} \frac{m_i M_i}{EI_i} dx$$

where

Δ = displacement at point of application of unit load (+ in direction of unit load)

m_i = moment equation in member i caused by the unit load

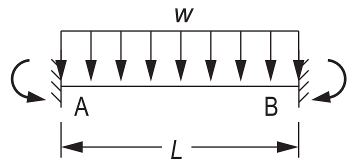
M_i = moment equation in member i caused by loads applied to frame

L_i = length of member i

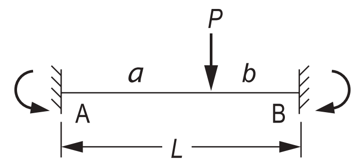
I_i = moment of inertia of member i

If either the real loads or the unit load cause no moment in a member, that member can be omitted from the summation.

4.1.6 Member Fixed-End Moments (Magnitudes)



$$FEM_{AB} = FEM_{BA} = \frac{wL^2}{12}$$



$$FEM_{AB} = \frac{Pab^2}{L^2} \qquad FEM_{BA} = \frac{Pa^2 b}{L^2}$$

4.1.7 Moment, Shear, and Deflection Diagrams

Source: Adapted from American Institute of Steel Construction, *Steel Construction Manual*, 15th ed, 2017, Table 3-23. pp. 3-208 through 3-223.

E = modulus of elasticity of steel = 29,000 ksi

I = moment of inertia of beam (in^4)

L = total length of beam between reaction points (ft)

M_{max} = maximum moment (in.-kips)

M_1 = maximum moment in left section of beam (in.-kips)

M_2 = maximum moment in right section of beam (in.-kips)

M_3 = maximum positive moment in beam with combined end moment conditions (in.-kips)

M_x = moment at distance x from end of beam (in.-kips)

P = concentrated load (kips)

P_1 = concentrated load nearest left reaction (kips)

P_2 = concentrated load nearest right reaction, and of different magnitude than P_1 (kips)

R = end beam reaction for any condition of symmetrical loading (kips)

R_1 = left end beam reaction (kips)

R_2 = right end or intermediate beam reaction (kips)

R_3 = right end beam reaction (kips)

V = maximum vertical shear for any condition of symmetrical loading (kips)

V_1 = maximum vertical shear in left section of beam (kips)

V_2 = vertical shear at right reaction point, or to left of intermediate reaction point of beam (kips)

V_3 = vertical shear at right reaction point, or to right of intermediate reaction point of beam (kips)

V_x = vertical shear at distance x from end of beam (kips)

W = total load on beam (kips)

a = measured distance along beam (in.)

b = measured distance along beam, which may be greater or less than a (in.)

l = total length of beam between reaction points (in.)

w = uniformly distributed load per unit of length (kip/in.)

w_1 = uniformly distributed load per unit of length nearest left reaction (kip/in.)

w_2 = uniformly distributed load per unit of length nearest right reaction and of different magnitude than w_1 (kip/in.)

x = any distance measured along beam from left reaction (in.)

x_1 = any distance measured along overhang section of beam from nearest reaction point (in.)

Δ_{max} = maximum deflection (in.)

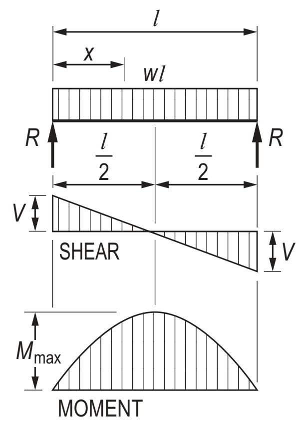
Δ_a = deflection at point of load (in.)

Δ_x = deflection at any point x distance from left reaction (in.)

Δ_{x1} = deflection of overhang section of beam at any distance from nearest reaction point (in.)

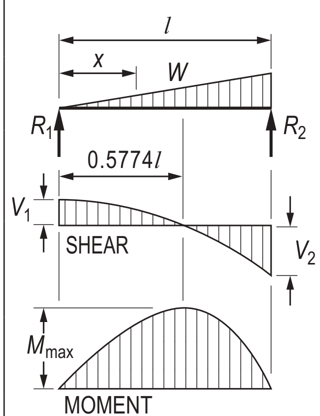
Shears, Moments, and Deflections

1. Simple Beam—Uniformly Distributed Load



Total Equiv. Uniform Load	=	wl
$R = V$	=	$\frac{wl}{2}$
V_x	=	$w\left(\frac{l}{2} - x\right)$
M_{max} (at center)	=	$\frac{wl^2}{8}$
M_x	=	$\frac{wx}{2}(l - x)$
Δ_{max} (at center)	=	$\frac{5wl^4}{384EI}$
Δ_x	=	$\frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

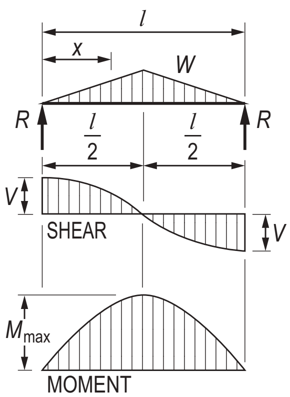
2. Simple Beam—Load Increasing Uniformly to One End



Total Equiv. Uniform Load	=	$\frac{16W}{9\sqrt{3}} = 1.03W$
$R_1 = V_1$	=	$\frac{W}{3}$
$R_2 = V_2 = V_{max}$	=	$\frac{2W}{3}$
V_x	=	$\frac{W}{3} - \frac{Wx^2}{l^2}$
$M_{max}\left(\text{at } x = \frac{l}{\sqrt{3}} = 0.577l\right)$	=	$\frac{2Wl}{9\sqrt{3}} = 0.128Wl$
M_x	=	$\frac{Wx}{3l^2}\left(l^2 - x^2\right)$
$\Delta_{max}\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = 0.519l\right)$	=	$0.0130\frac{Wl^3}{EI}$
Δ_x	=	$\frac{Wx}{180EIl^2}\left(3x^4 - 10l^2x^2 + 7l^4\right)$

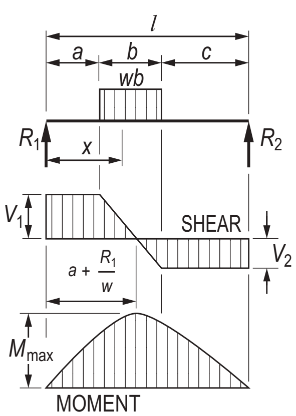
Shears, Moments, and Deflections (cont'd)

3. Simple Beam—Load Increasing Uniformly to Center



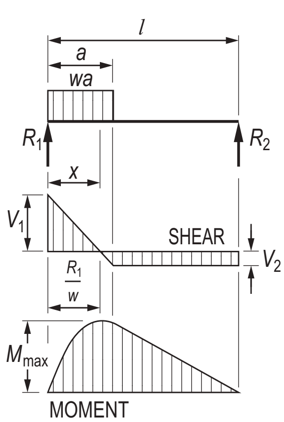
Total Equiv. Uniform Load	=	$\frac{4W}{3}$
$R = V$	=	$\frac{W}{2}$
$V_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{W}{2l^2}\left(l^2 - 4x^2\right)$
M_{max} (at center)	=	$\frac{Wl}{6}$
$M_x \left(\text{when } x < \frac{l}{2}\right)$	=	$Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$
Δ_{max} (at center)	=	$\frac{Wl^3}{60EI}$
$\Delta_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{Wx}{480EIl^2}\left(5l^2 - 4x^2\right)^2$

4. Simple Beam—Uniform Load Partially Distributed



$R_1 = V_1$ (max. when $a < c$)	=	$\frac{wb}{2l}\left(2c + b\right)$
$R_2 = V_2$ (max. when $a > c$)	=	$\frac{wb}{2l}\left(2a + b\right)$
V_x [when $x > a$ and $< (a + b)$]	=	$R_1 - w(x - a)$
$M_{max} \left(\text{at } x = a + \frac{R_1}{w}\right)$	=	$R_1\left(a + \frac{R_1}{2w}\right)$
M_x (when $x < a$)	=	$R_1 x$
M_x [when $x > a$ and $< (a + b)$]	=	$R_1 x - \frac{w}{2}\left(x - a\right)^2$
M_x [when $x > (a + b)$]	=	$R_2(l - x)$

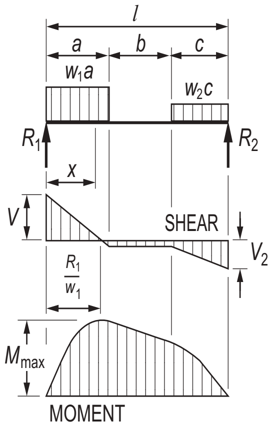
5. Simple Beam—Uniform Load Partially Distributed at One End



$R_1 = V_1 = V_{max}$	=	$\frac{wa}{2l}\left(2l - a\right)$
$R_2 = V_2$	=	$\frac{wa^2}{2l}$
V_x (when $x < a$)	=	$R_1 - wx$
$M_{max} \left(\text{at } x = \frac{R_1}{w}\right)$	=	$\frac{R_1^2}{2w}$
M_x (when $x < a$)	=	$R_1 x - \frac{wx^2}{2}$
M_x (when $x > a$)	=	$R_2(l - x)$
Δ_x (when $x < a$)	=	$\frac{wx}{24EIl}\left[a^2\left(2l - a\right)^2 - 2ax^2\left(2l - a\right) + lx^3\right]$
Δ_x (when $x > a$)	=	$\frac{wa^2\left(l - x\right)}{24EIl}\left(4xl - 2x^2 - a^2\right)$

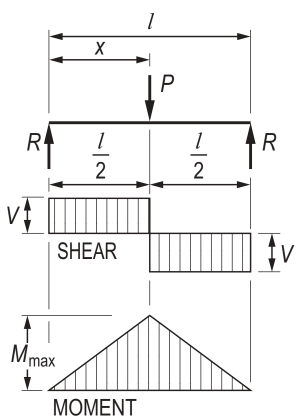
Shears, Moments, and Deflections (cont'd)

6. Simple Beam—Uniform Load Partially Distributed at Each End



$R_1 = V_1$	=	$\dfrac{w_1 a(2l-a)+w_2 c^2}{2l}$
$R_2 = V_2$	=	$\dfrac{w_2 c(2l-c)+w_1 a^2}{2l}$
V_x (when $x < a$)	=	$R_1 - w_1 x$
V_x [when $a < x < (a+b)$]	=	$R_1 - w_1 a$
V_x [when $x > (a+b)$]	=	$R_2 - w_2(l-x)$
$M_{max}\left(\text{at } x = \dfrac{R_1}{w_1}, \text{ when } R_1 < w_1 a\right)$	=	$\dfrac{R_1^2}{2w_1}$
$M_{max}\left(\text{at } x = l - \dfrac{R_2}{w_2}, \text{ when } R_2 < w_2 c\right)$	=	$\dfrac{R_2^2}{2w_2}$
M_x (when $x < a$)	=	$R_1 x - \dfrac{w_1 x^2}{2}$
M_x [when $a < x < (a+b)$]	=	$R_1 x - \dfrac{w_1 a^2}{2}(2x-a)$
M_x [when $x > (a+b)$]	=	$R_2(l-x) - \dfrac{w_2(l-x)^2}{2}$

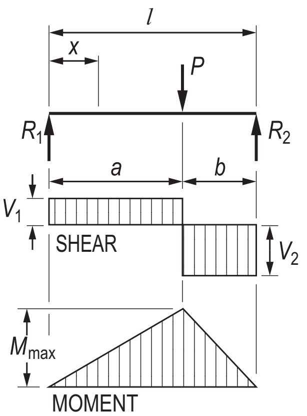
7. Simple Beam—Concentrated Load at Center



Total Equiv. Uniform Load	=	$2P$
$R = V$	=	$\dfrac{P}{2}$
M_{max} (at point of load)	=	$\dfrac{Pl}{4}$
$M_x\left(\text{when } x < \dfrac{l}{2}\right)$	=	$\dfrac{Px}{2}$
Δ_{max} (at point of load)	=	$\dfrac{Pl^3}{48EI}$
$\Delta_x\left(\text{when } x < \dfrac{l}{2}\right)$	=	$\dfrac{Px}{48EI}(3l^2 - 4x^2)$

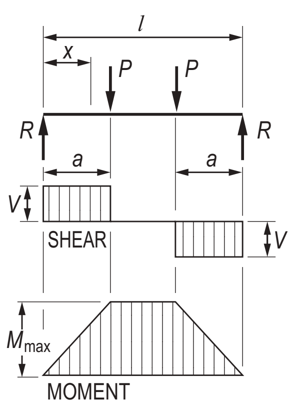
Shears, Moments, and Deflections (cont'd)

8. Simple Beam—Concentrated Load at Any Point



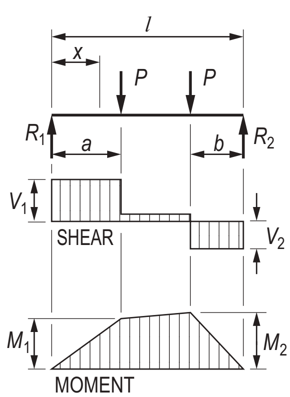
Total Equiv. Uniform Load	=	$\frac{8Pab}{l^2}$
$R_1 = V_1$ (= V_{max} when $a < b$)	=	$\frac{Pb}{l}$
$R_2 = V_2$ (= V_{max} when $a > b$)	=	$\frac{Pa}{l}$
M_{max} (at point of load)	=	$\frac{Pab}{l}$
M_x (when $x < a$)	=	$\frac{Pbx}{l}$
$\Delta_{max}\left[\text{at } x = \sqrt{\frac{a(a+2b)}{3}}\text{, when } a > b\right]$	=	$\frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$
Δ_a (at point of load)	=	$\frac{Pa^2b^2}{3EIl}$
Δ_x (when $x < a$)	=	$\frac{Pbx}{6EIl}\left(l^2 - b^2 - x^2\right)$

9. Simple Beam—Two Equal Concentrated Loads Symmetrically Placed



Total Equiv. Uniform Load	=	$\frac{8Pa}{l}$
$R = V$	=	P
M_{max} (between loads)	=	Pa
M_x (when $x < a$)	=	Px
Δ_{max} (at center)	=	$\frac{Pa}{24EI}\left(3l^2 - 4a^2\right)$
$\Delta_{max}\left(\text{at } a = \frac{l}{3}\right)$	=	$\frac{23Pl^3}{648EI}$
Δ_x (when $x < a$)	=	$\frac{Px}{6EI}\left(3la - 3a^2 - x^2\right)$
Δ_x [when $a < x < (l - a)$]	=	$\frac{Pa}{6EI}\left(3lx - 3x^2 - a^2\right)$

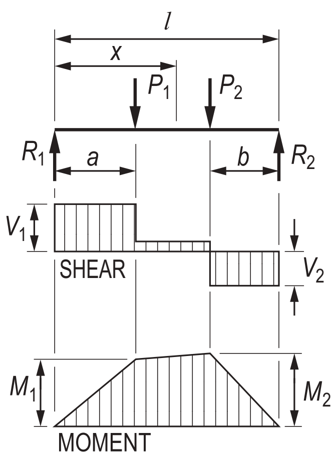
10. Simple Beam—Two Equal Concentrated Loads Unsymmetrically Placed



$R_1 = V_1$ (=V_{max} when $a < b$)	=	$\frac{P}{l}(l - a + b)$
$R_2 = V_2$ (= V_{max} when $a > b$)	=	$\frac{P}{l}(l - b + a)$
V_x [when $a < x < (l - b)$]	=	$\frac{P}{l}(b - a)$
M_1 (= M_{max} when $a > b$)	=	R_1a
M_2 (= M_{max} when $a < b$)	=	R_2b
M_x (when $x < a$)	=	R_1x
M_x [when $a < x < (l - b)$]	=	$R_1x - P(x - a)$

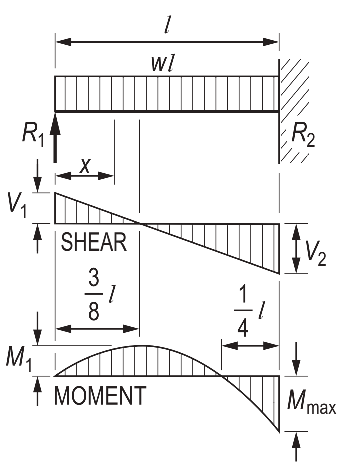
Shears, Moments, and Deflections (cont'd)

11. Simple Beam—Two Unequal Concentrated Loads Unsymmetrically Placed



$R_1 = V_1$	=	$\dfrac{P_1(l-a)+P_2 b}{l}$
$R_2 = V_2$	=	$\dfrac{P_1 a+P_2(l-b)}{l}$
V_x [when $a < x < (l - b)$]	=	$R_1 - P_1$
M_1 (= M_{max} when $R_1 < P_1$)	=	$R_1 a$
M_2 (= M_{max} when $R_2 < P_2$)	=	$R_2 b$
M_x (when $x < a$)	=	$R_1 x$
M_x [when $a < x < (l - b)$]	=	$R_1 x - P_1(x - a)$

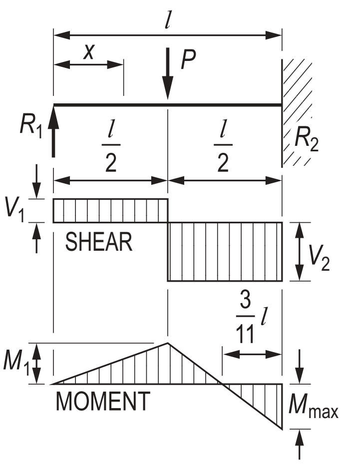
12. Beam Fixed At One End, Supported At Other—Uniformly Distributed Load



Total Equiv. Uniform Load	=	wl
$R_1 = V_1$	=	$\dfrac{3wl}{8}$
$R_2 = V_2 = V_{max}$	=	$\dfrac{5wl}{8}$
V_x	=	$R_1 - wx$
M_{max}	=	$\dfrac{wl^2}{8}$
$M_1 \left(\text{at } x = \dfrac{3}{8}l\right)$	=	$\dfrac{9}{128}wl^2$
M_x	=	$R_1 x - \dfrac{wx^2}{2}$
$\Delta_{max}\left[\text{at } x = \dfrac{l}{16}\left(1+\sqrt{33}\right) = 0.422l\right]$	=	$\dfrac{wl^4}{185EI}$
Δ_x	=	$\dfrac{wx}{48EI}\left(l^3 - 3lx^2 + 2x^3\right)$

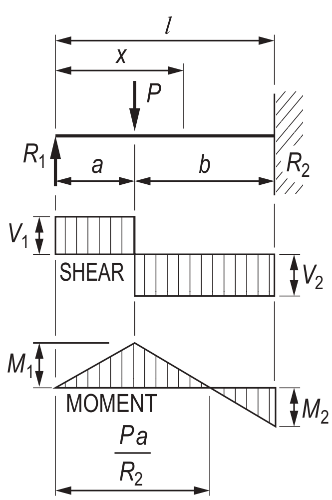
Shears, Moments, and Deflections (cont'd)

13. Beam Fixed at One End, Supported at Other—Concentrated Load at Center



Total Equiv. Uniform Load	=	$\frac{3P}{2}$
$R_1 = V_1$	=	$\frac{5P}{16}$
$R_2 = V_2 = V_{max}$	=	$\frac{11P}{16}$
M_{max} (at fixed end)	=	$\frac{3Pl}{16}$
M_1 (at point of load)	=	$\frac{5Pl}{32}$
$M_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{5Px}{16}$
$M_x \left(\text{when } x > \frac{l}{2}\right)$	=	$P\left(\frac{l}{2} - \frac{11x}{16}\right)$
$\Delta_{max} \left(\text{at } x = \frac{l}{\sqrt{5}} = 0.447l\right)$	=	$\frac{Pl^3}{48EI\sqrt{5}} = 0.00932\frac{Pl^3}{EI}$
Δ_x (at point of load)	=	$\frac{7Pl^3}{768EI}$
$\Delta_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{Px}{96EI}\left(3l^2 - 5x^2\right)$
$\Delta_x \left(\text{when } x > \frac{l}{2}\right)$	=	$\frac{P}{96EI}(x - l)^2 (11x - 2l)$

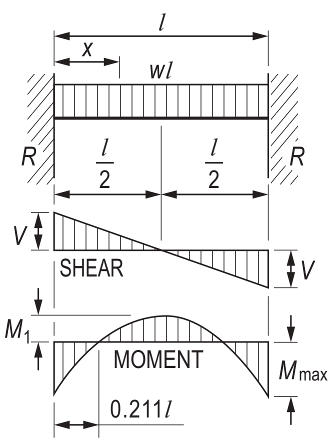
14. Beam Fixed at One End, Supported at Other—Concentrated Load at Any Point



$R_1 = V_1$	=	$\frac{Pb^2}{2l^3}(a + 2l)$
$R_2 = V_2$	=	$\frac{Pa}{2l^3}\left(3l^2 - a^2\right)$
M_1 (at point of load)	=	$R_1 a$
M_2 (at fixed end)	=	$\frac{Pab}{2l^2}(a + l)$
M_x (when $x < a$)	=	$R_1 x$
M_x (when $x > a$)	=	$R_1 x - P(x - a)$
$\Delta_{max} \left[\text{when } a < 0.414l \text{ at } x = l\frac{\left(l^2 + a^2\right)}{\left(3l^2 - a^2\right)}\right]$	=	$\frac{Pa}{3EI}\frac{\left(l^2 - a^2\right)^3}{\left(3l^2 - a^2\right)^2}$
$\Delta_{max} \left(\text{when } a > 0.414l \text{ at } x = l\sqrt{\frac{a}{2l + a}}\right)$	=	$\frac{Pab^2}{6EI}\sqrt{\frac{a}{2l + a}}$
Δ_a (at point of load)	=	$\frac{Pa^2 b^3}{12EIl^3}(3l + a)$
Δ_x (when $x < a$)	=	$\frac{Pb^2 x}{12EIl^3}\left(3al^2 - 2lx^2 - ax^2\right)$
Δ_x (when $x > a$)	=	$\frac{Pa}{12EIl^3}(l - x)^2\left(3l^2 x - a^2 x - 2a^2 l\right)$

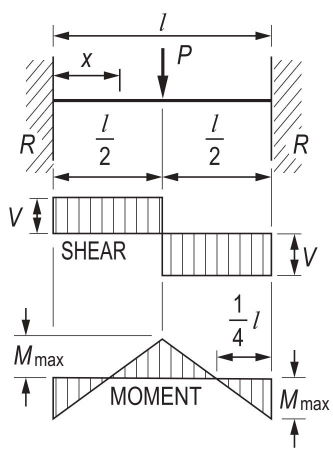
Shears, Moments, and Deflections (cont'd)

15. Beam Fixed at Both Ends—Uniformly Distributed Loads



Total Equiv. Uniform Load	=	$\frac{2wl}{3}$
$R = V$	=	$\frac{wl}{2}$
V_x	=	$w\left(\frac{l}{2} - x\right)$
M_{max} (at ends)	=	$\frac{wl^2}{12}$
M_1 (at center)	=	$\frac{wl^2}{24}$
M_x	=	$\frac{w}{12}\left(6lx - l^2 - 6x^2\right)$
Δ_{max} (at center)	=	$\frac{wl^4}{384EI}$
Δ_x	=	$\frac{wx^2}{24EI}(l - x)^2$

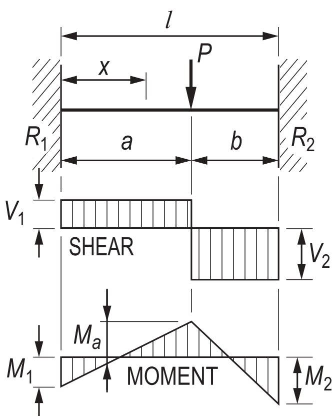
16. Beam Fixed at Both Ends—Concentrated Load at Center



Total Equiv. Uniform Load	=	P
$R = V$	=	$\frac{P}{2}$
M_{max} (at center and ends)	=	$\frac{Pl}{8}$
$M_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{P}{8}(4x - l)$
Δ_{max} (at center)	=	$\frac{Pl^3}{192EI}$
$\Delta_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{Px^2}{48EI}(3l - 4x)$

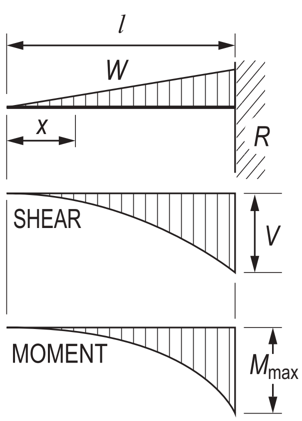
Shears, Moments, and Deflections (cont'd)

17. Beam Fixed at Both Ends—Concentrated Load at Any Point



$R_1 = V_1$ (= V_{max} when $a < b$)	=	$\frac{Pb^2}{l^3}(3a+b)$
$R_2 = V_2$ (= V_{max} when $a > b$)	=	$\frac{Pa^2}{l^3}(a+3b)$
M_1 (= M_{max} when $a < b$)	=	$\frac{Pab^2}{l^2}$
M_2 (= M_{max} when $a > b$)	=	$\frac{Pa^2 b}{l^2}$
M_a (at point of load)	=	$\frac{2Pa^2 b^2}{l^3}$
M_x (when $x < a$)	=	$R_1 x - \frac{Pab^2}{l^2}$
$\Delta_{max}\left(\text{when } a > b \text{ at } x = \frac{2al}{3a+b}\right)$	=	$\frac{2Pa^3 b^2}{3EI(3a+b)^2}$
Δ_a (at point of load)	=	$\frac{Pa^3 b^3}{3EIl^3}$
Δ_x (when $x < a$)	=	$\frac{Pb^2 x^2}{6EIl^3}(3al - 3ax - bx)$

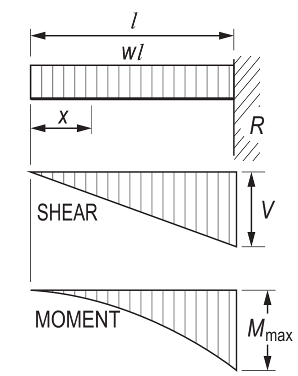
18. Cantilevered Beam—Load Increasing Uniformly to Fixed End



Total Equiv. Uniform Load	=	$\frac{8}{3}W$
$R = V$	=	W
V_x	=	$W\frac{x^2}{l^2}$
M_{max} (at fixed end)	=	$\frac{Wl}{3}$
M_x	=	$\frac{Wx^3}{3l^2}$
Δ_{max} (at free end)	=	$\frac{Wl^3}{15EI}$
Δ_x	=	$\frac{W}{60EIl^2}(x^5 - 5l^4 x + 4l^5)$

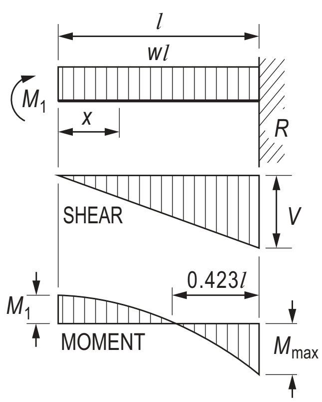
Shears, Moments, and Deflections (cont'd)

19. Cantilevered Beam—Uniformly Distributed Load



Total Equiv. Uniform Load	=	$4wl$
$R = V$	=	wl
V_x	=	wx
M_{max} (at fixed end)	=	$\frac{wl^2}{2}$
M_x	=	$\frac{wx^2}{2}$
Δ_{max} (at free end)	=	$\frac{wl^4}{8EI}$
Δ_x	=	$\frac{w}{24EI}\left(x^4 - 4l^3x + 3l^4\right)$

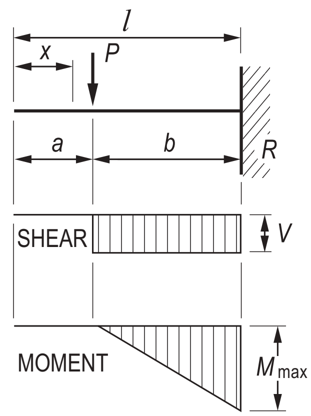
20. Beam Fixed at One End, Free to Deflect Vertically but Not Rotate at Other—Uniformly Distributed Load



Total Equiv. Uniform Load	=	$\frac{8}{3}wl$
$R = V$	=	wl
V_x	=	wx
M_1 (at deflected end)	=	$\frac{wl^2}{6}$
M_{max} (at fixed end)	=	$\frac{wl^2}{3}$
M_x	=	$\frac{w}{6}\left(l^2 - 3x^2\right)$
Δ_{max} (at deflected end)	=	$\frac{wl^4}{24EI}$
Δ_x	=	$\frac{w\left(l^2 - x^2\right)^2}{24EI}$

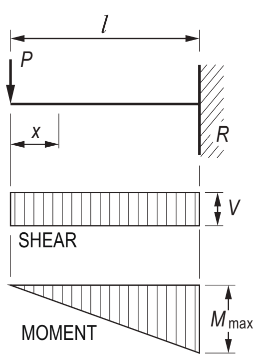
Shears, Moments, and Deflections (cont'd)

21. Cantilevered Beam—Concentrated Load at Any Point



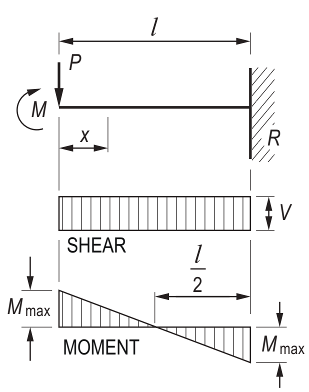
Total Equiv. Uniform Load	=	$\frac{8Pb}{l}$
$R = V$	=	P
M_{max} (at fixed end)	=	Pb
M_x (when $x > a$)	=	$P(x - a)$
Δ_{max} (at free end)	=	$\frac{Pb^2}{6EI}(3l - b)$
Δ_a (at point of load)	=	$\frac{Pb^3}{3EI}$
Δ_x (when $x < a$)	=	$\frac{Pb^2}{6EI}(3l - 3x - b)$
Δ_x (when $x > a$)	=	$\frac{P(l-x)^2}{6EI}(3b - l + x)$

22. Cantilevered Beam—Concentrated Load at Free End



Total Equiv. Uniform Load	=	$8P$
$R = V$	=	P
M_{max} (at fixed end)	=	Pl
M_x	=	Px
Δ_{max} (at free end)	=	$\frac{Pl^3}{3EI}$
Δ_x	=	$\frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

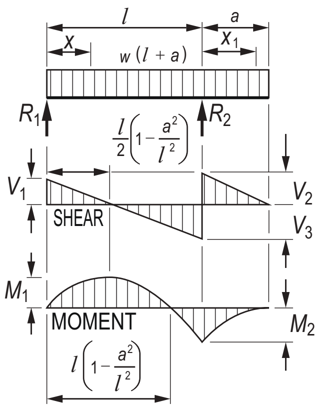
23. Beam Fixed at One End, Free to Deflect Vertically but Not Rotate at Other—Concentrated Load at Deflected End



Total Equiv. Uniform Load	=	$4P$
$R = V$	=	P
M_{max} (at both ends)	=	$\frac{Pl}{2}$
M_x	=	$P\left(\frac{l}{2} - x\right)$
Δ_{max} (at deflected end)	=	$\frac{Pl^3}{12EI}$
Δ_x	=	$\frac{P(l-x)^2}{12EI}(l + 2x)$

Shears, Moments, and Deflections (cont'd)

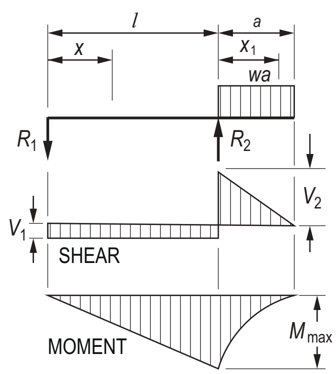
24. Beam Overhanging One Support—Uniformly Distributed Load



$R_1 = V_1$	=	$\frac{w}{2l}\left(l^2 - a^2\right)$
$R_2 = V_2 + V_3$	=	$\frac{w}{2l}\left(l + a\right)^2$
V_2	=	wa
V_3	=	$\frac{w}{2l}\left(l^2 + a^2\right)$
V_x (between supports)	=	$R_1 - wx$
V_{x_1} (for overhang)	=	$w\left(a - x_1\right)$
$M_1 \left[\text{at } x = \frac{l}{2}\left(1 - \frac{a^2}{l^2}\right)\right]$	=	$\frac{w}{8l^2}\left(l + a\right)^2\left(l - a\right)^2$
M_2 (at R_2)	=	$\frac{wa^2}{2}$
M_x (between supports)	=	$\frac{wx}{2l}\left(l^2 - a^2 - xl\right)$
M_{x_1} (for overhang)	=	$\frac{w}{2}\left(a - x_1\right)^2$
Δ_x (between supports)	=	$\frac{wx}{24EIl}\left(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2\right)$
Δ_{x_1} (for overhang)	=	$\frac{wx_1}{24EI}\left(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3\right)$

Note: For a negative value of Δ_x, deflection is upward.

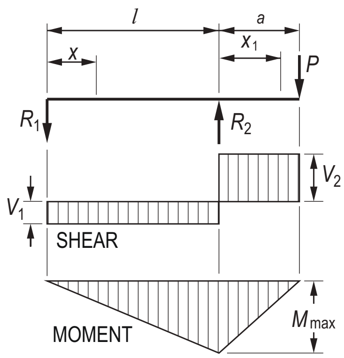
25. Beam Overhanging One Support—Uniformly Distributed Load on Overhang



$R_1 = V_1$	=	$\frac{wa^2}{2l}$
$R_2 = V_1 + V_2$	=	$\frac{wa}{2l}\left(2l + a\right)$
V_2	=	wa
V_{x_1} (for overhang)	=	$w\left(a - x_1\right)$
M_{max} (at R_2)	=	$\frac{wa^2}{2}$
M_x (between supports)	=	$\frac{wa^2x}{2l}$
M_{x_1} (for overhang)	=	$\frac{w}{2}\left(a - x_1\right)^2$
$\Delta_{max}\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right)$	=	$\frac{wa^2l^2}{18\sqrt{3}\,EI} = 0.0321\frac{wa^2l^2}{EI}$
Δ_{max} (for overhang at $x_1 = a$)	=	$\frac{wa^3}{24EI}\left(4l + 3a\right)$
Δ_x (between supports)	=	$\frac{wa^2x}{12EIl}\left(l^2 - x^2\right)$
Δ_{x_1} (for overhang)	=	$\frac{wx_1}{24EI}\left(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3\right)$

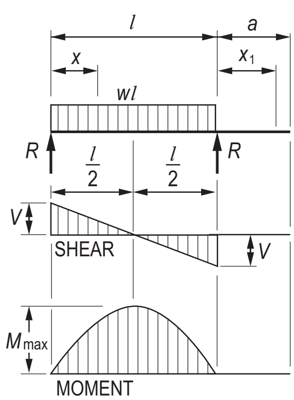
Shears, Moments, and Deflections (cont'd)

26. Beam Overhanging One Support—Concentrated Load at End of Overhang



$R_1 = V_1$	=	$\frac{Pa}{l}$
$R_2 = V_1 + V_2$	=	$\frac{P}{l}(l+a)$
V_2	=	P
M_{max} (at R_2)	=	Pa
M_x (between supports)	=	$\frac{Pax}{l}$
M_{x_1} (for overhang)	=	$P(a - x_1)$
Δ_{max} $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right)$	=	$\frac{Pal^2}{9\sqrt{3}\,EI} = 0.0642\frac{Pal^2}{EI}$
Δ_{max} (for overhang at $x_1 = a$)	=	$\frac{Pa^2}{3EI}(l+a)$
Δ_x (between supports)	=	$\frac{Pax}{6EIl}(l^2 - x^2)$
Δ_{x_1} (for overhang)	=	$\frac{Px_1}{6EI}(2al + 3ax_1 - x_1^2)$

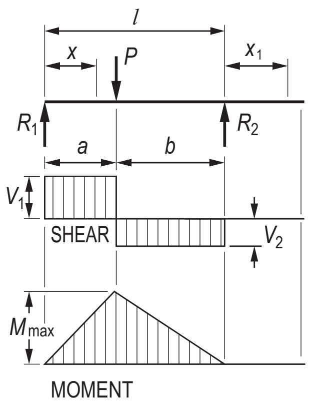
27. Beam Overhanging One Support—Uniformly Distributed Load Between Supports



Total Equiv. Uniform Load	=	wl
$R = V$	=	$\frac{wl}{2}$
V_x	=	$w\left(\frac{l}{2} - x\right)$
M_{max} (at center)	=	$\frac{wl^2}{8}$
M_x	=	$\frac{wx}{2}(l - x)$
Δ_{max} (at center)	=	$\frac{5wl^4}{384EI}$
Δ_x	=	$\frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$
Δ_{x_1}	=	$\frac{wl^3 x_1}{24EI}$

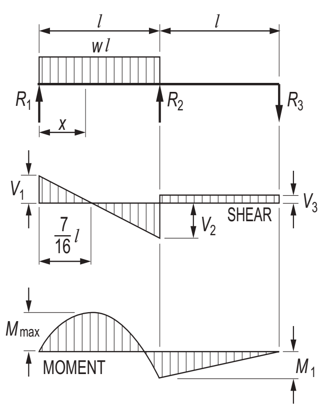
Shears, Moments, and Deflections (cont'd)

28. Beam Overhanging One Support—Concentrated Load at Any Point Between Supports



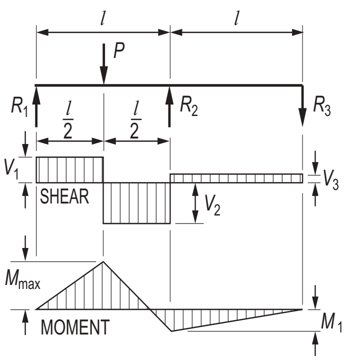
Total Equiv. Uniform Load	=	$\frac{8Pab}{l^2}$
$R_1 = V_1$ (= V_{max} when $a < b$)	=	$\frac{Pb}{l}$
$R_2 = V_2$ (= V_{max} when $a > b$)	=	$\frac{Pa}{l}$
M_{max} (at point of load)	=	$\frac{Pab}{l}$
M_x (when $x < a$)	=	$\frac{Pbx}{l}$
$\Delta_{max}\left[\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right]$	=	$\frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$
Δ_a (at point of load)	=	$\frac{Pa^2b^2}{3EIl}$
Δ_x (when $x < a$)	=	$\frac{Pbx}{6EIl}\left(l^2 - b^2 - x^2\right)$
Δ_x (when $x > a$)	=	$\frac{Pa(l-x)}{6EIl}\left(2lx - x^2 - a^2\right)$
Δ_{x_1}	=	$\frac{Pabx_1}{6EIl}(l+a)$

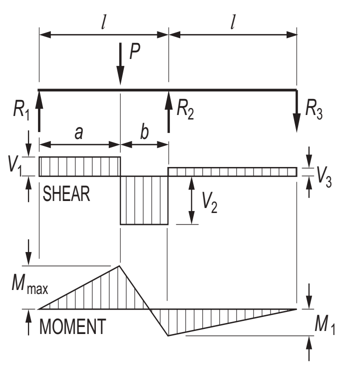
29. Continuous Beam—Two Equal Spans—Uniform Load on One Span



Total Equiv. Uniform Load	=	$\frac{49}{64}wl$
$R_1 = V_1$	=	$\frac{7}{16}wl$
$R_2 = V_2 + V_3$	=	$\frac{5}{8}wl$
$R_3 = V_3$	=	$-\frac{1}{16}wl$
V_2	=	$\frac{9}{16}wl$
$M_{max}\left(\text{at } x = \frac{7}{16}l\right)$	=	$\frac{49}{512}wl^2$
M_1 (at support R_2)	=	$\frac{1}{16}wl^2$
M_x (when $x < l$)	=	$\frac{wx}{16}(7l - 8x)$
Δ_{max} (at 0.472 l from R_1)	=	$\frac{0.0092wl^4}{EI}$

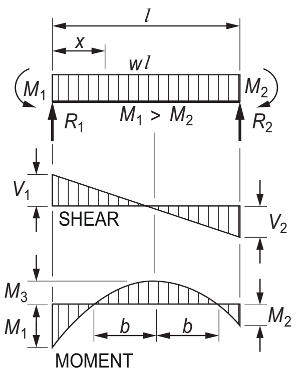
Shears, Moments, and Deflections (cont'd)

30. Continuous Beam—Two Equal Spans—Concentrated Load at Center of One Span		
	Total Equiv. Uniform Load	$= \frac{13}{8}P$
	$R_1 = V_1$	$= \frac{13}{32}P$
	$R_2 = V_2 + V_3$	$= \frac{11}{16}P$
	$R_3 = V_3$	$= \frac{3}{32}P$
	V_2	$= \frac{19}{32}P$
	M_{max} (at point of load)	$= \frac{13}{64}Pl$
	M_1 (at support R_2)	$= \frac{3}{32}Pl$
	Δ_{max} (at 0.480 l from R_1)	$= \frac{0.015Pl^3}{EI}$

31. Continuous Beam—Two Equal Spans—Concentrated Load at Any Point		
	$R_1 = V_1$	$= \frac{Pb}{4l^3}\left[4l^2 - a\left(l+a\right)\right]$
	$R_2 = V_2 + V_3$	$= \frac{Pa}{2l^3}\left[2l^2 + b\left(l+a\right)\right]$
	$R_3 = V_3$	$= \frac{Pab}{4l^3}\left(l+a\right)$
	V_2	$= \frac{Pa}{4l^3}\left[4l^2 + b\left(l+a\right)\right]$
	M_{max} (at point of load)	$= \frac{Pab}{4l^3}\left[4l^2 - a\left(l+a\right)\right]$
	M_1 (at support R_2)	$= \frac{Pab}{4l^2}\left(l+a\right)$

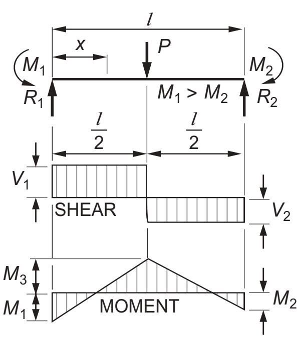
Shears, Moments, and Deflections (cont'd)

32. Beam—Uniformly Distributed Load and Variable End Moments



$R_1 = V_1$	=	$\dfrac{wl}{2} + \dfrac{M_1 - M_2}{l}$
$R_2 = V_2$	=	$\dfrac{wl}{2} - \dfrac{M_1 - M_2}{l}$
V_x	=	$w\left(\dfrac{l}{2} - x\right) + \dfrac{M_1 - M_2}{l}$
$M_3 \left(\text{at } x = \dfrac{l}{2} + \dfrac{M_1 - M_2}{wl}\right)$	=	$\dfrac{wl^2}{8} - \dfrac{M_1 + M_2}{2} + \dfrac{\left(M_1 - M_2\right)^2}{2wl^2}$
M_x	=	$\dfrac{wx}{2}(l - x) + \left(\dfrac{M_1 - M_2}{l}\right)x - M_1$
b (to locate inflection points)	=	$\sqrt{\dfrac{l^2}{4} - \left(\dfrac{M_1 + M_2}{w}\right) + \left(\dfrac{M_1 - M_2}{wl}\right)^2}$
Δ_x	=	$\dfrac{wx}{24EI}\left[x^3 - \left(2l + \dfrac{4M_1}{wl} - \dfrac{4M_2}{wl}\right)x^2 + \dfrac{12M_1}{w}x + l^3 - \dfrac{8M_1 l}{w} - \dfrac{4M_2 l}{w}\right]$

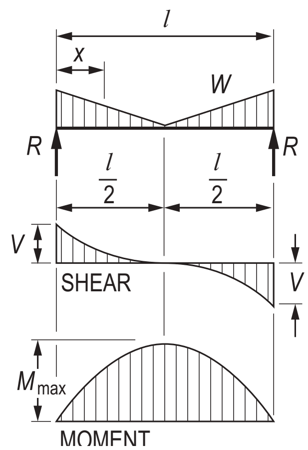
33. Beam—Concentrated Load at Center and Variable End Moments



$R_1 = V_1$	=	$\dfrac{P}{2} + \dfrac{M_1 - M_2}{l}$
$R_2 = V_2$	=	$\dfrac{P}{2} - \dfrac{M_1 - M_2}{l}$
M_3 (at center)	=	$\dfrac{Pl}{4} - \dfrac{M_1 - M_2}{2}$
$M_x \left(\text{when } x < \dfrac{l}{2}\right)$	=	$x\left(\dfrac{P}{2} + \dfrac{M_1 - M_2}{l}\right) - M_1$
$M_x \left(\text{when } x > \dfrac{l}{2}\right)$	=	$\dfrac{P}{2}(l - x) + \dfrac{x\left(M_1 - M_2\right)}{l} - M_1$
$\Delta_x \left(\text{when } x < \dfrac{l}{2}\right)$	=	$\dfrac{Px}{48EI}\left\{3l^2 - 4x^2 - \dfrac{8(l - x)}{Pl}\left[M_1(2l - x) + M_2(l + x)\right]\right\}$

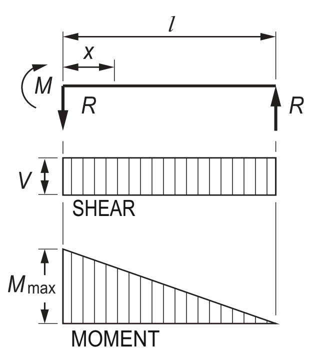
Shears, Moments, and Deflections (cont'd)

34. Simple Beam—Load Increasing Uniformly From Center



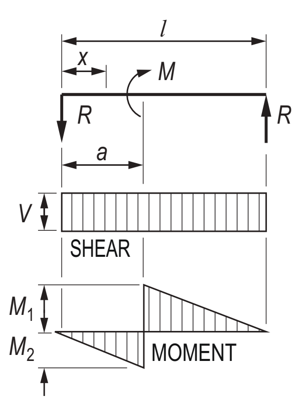
Total Equiv. Uniform Load	=	$\frac{2W}{3}$
$R = V$	=	$\frac{W}{2}$
$V_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{W}{2}\left(\frac{l-2x}{l}\right)^2$
M_{max} (at center)	=	$\frac{Wl}{12}$
$M_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{W}{2}\left(x - \frac{2x^2}{l} + \frac{4x^3}{3l^2}\right)$
Δ_{max} (at center)	=	$\frac{3Wl^3}{320EI}$
$\Delta_x \left(\text{when } x < \frac{l}{2}\right)$	=	$\frac{W}{12EI}\left(x^3 - \frac{x^4}{l} + \frac{2x^5}{5l^2} - \frac{3l^2 x}{8}\right)$

35. Simple Beam—Concentrated Moment at End



Total Equiv. Uniform Load	=	$\frac{8M}{l}$
$R = V$	=	$\frac{M}{l}$
M_{max}	=	M
M_x	=	$M\left(1 - \frac{x}{l}\right)$
Δ_{max} (at $x = 0.423\, l$)	=	$0.0642\, \frac{Ml^2}{EI}$
Δ_x	=	$\frac{M}{6EI}\left(3x^2 - \frac{x^3}{l} - 2lx\right)$

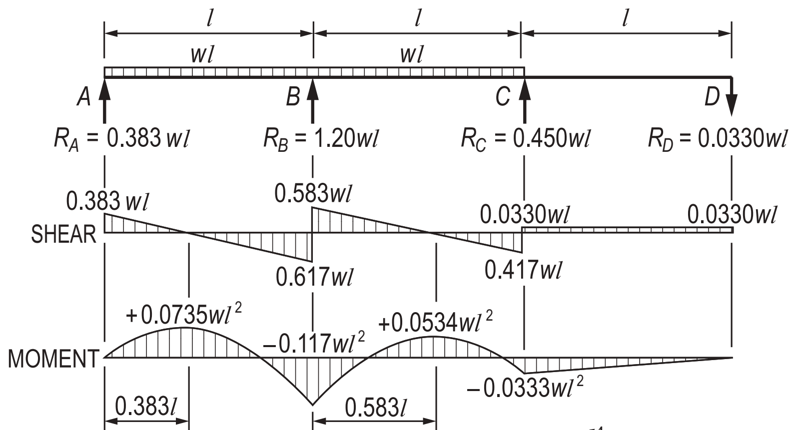
36. Simple Beam—Concentrated Moment at Any Point



Total Equiv. Uniform Load	=	$\frac{8M}{l}$
$R = V$	=	$\frac{M}{l}$
M_x (when $x < a$)	=	Rx
M_x (when $x > a$)	=	$R\,(l - x)$
Δ_x (when $x < a$)	=	$\frac{M}{6EI}\left[\left(6a - \frac{3a^2}{l} - 2l\right)x - \frac{x^3}{l}\right]$
Δ_x (when $x > a$)	=	$\frac{M}{6EI}\left[3\left(a^2 + x^2\right) - \frac{x^3}{l} - x\left(2l + \frac{3a^2}{l}\right)\right]$

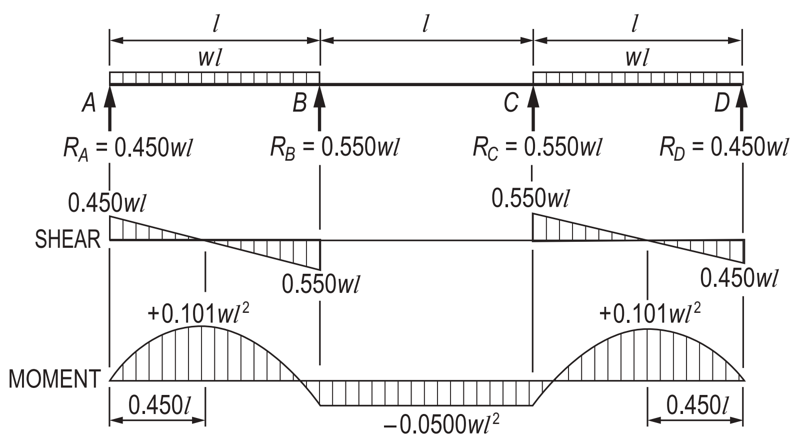
Shears, Moments, and Deflections (cont'd)

37. Continuous Beam—Three Equal Spans—One End Span Unloaded



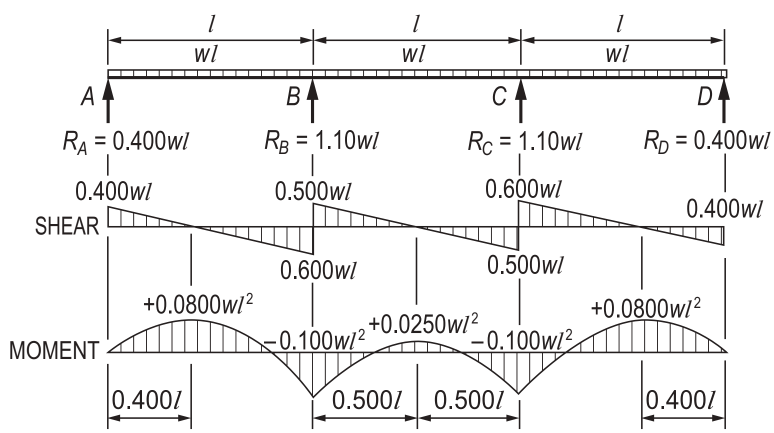
$$\Delta_{max}(0.430l \text{ from } A) = 0.0059\frac{wl^4}{EI}$$

38. Continuous Beam—Three Equal Spans—End Spans Loaded



$$\Delta_{max}(0.479l \text{ from } A \text{ or } D) = 0.0099\frac{wl^4}{EI}$$

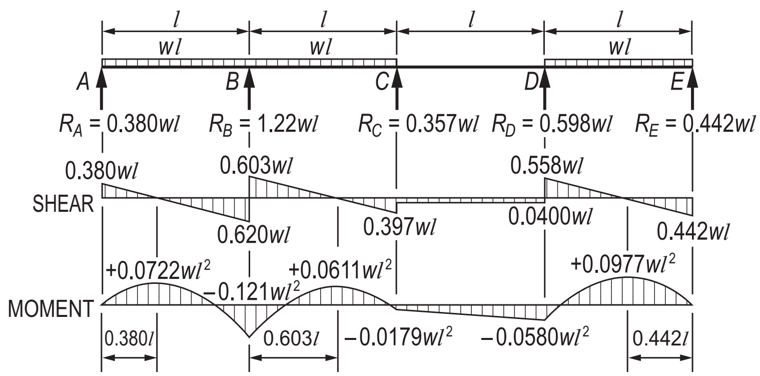
39. Continuous Beam—Three Equal Spans—All Spans Loaded



$$\Delta_{max}(0.446l \text{ from } A \text{ or } D) = 0.0069\frac{wl^4}{EI}$$

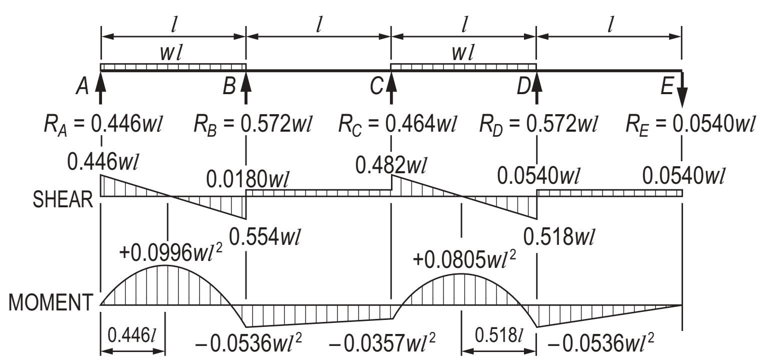
Shears, Moments, and Deflections (cont'd)

40. Continuous Beam—Four Equal Spans—Third Span Unloaded



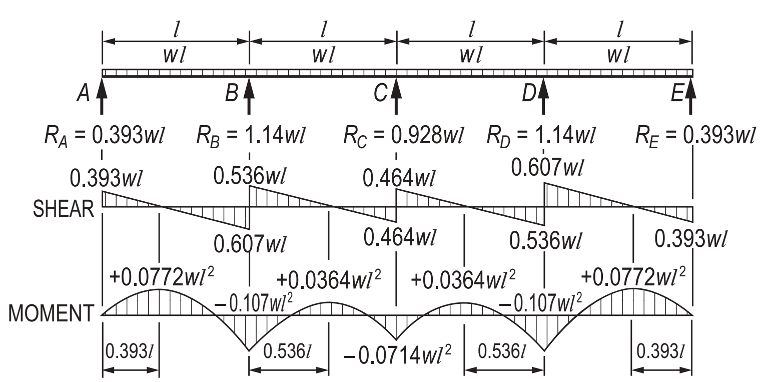
$$\Delta_{max}(0.475l \text{ from } E) = 0.0094\frac{wl^4}{EI}$$

41. Continuous Beam—Four Equal Spans—First and Third Spans Loaded



$$\Delta_{max}(0.477l \text{ from } A) = 0.0097\frac{wl^4}{EI}$$

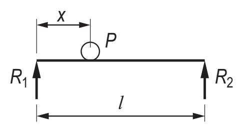
42. Continuous Beam—Four Equal Spans—All Spans Loaded



$$\Delta_{max}(0.440l \text{ from } A \text{ to } E) = 0.0065\frac{wl^4}{EI}$$

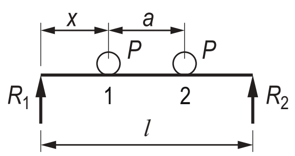
Shears, Moments, and Deflections (cont'd)

43. Simple Beam—One Concentrated Moving Load



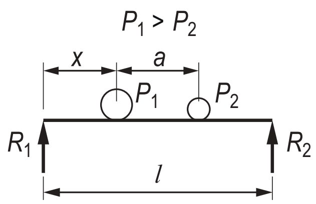
$R_{1\,max} = V_{1\,max}$ (at $x = 0$)	=	P
$M_{max}\left(\text{at point of load, when } x = \dfrac{l}{2}\right)$	=	$\dfrac{Pl}{4}$

44. Simple Beam—Two Equal Concentrated Moving Loads



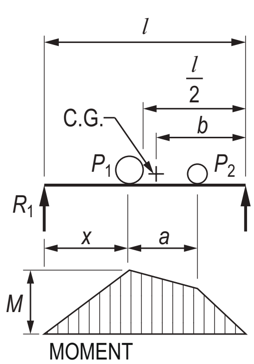
$R_{1\,max} = V_{1\,max}$ (at $x = 0$)	=	$P\left(2 - \dfrac{a}{l}\right)$
M_{max} [when $a < l\left(2-\sqrt{2}\right) = 0.586l$, under load 1 at $x = \dfrac{1}{2}\left(l - \dfrac{a}{2}\right)$]	=	$\dfrac{P}{2l}\left(l - \dfrac{a}{2}\right)^2$
M_{max} [when $a > l\left(2-\sqrt{2}\right) = 0.586\,l$, with one load at center of span (Case 43)]	=	$\dfrac{Pl}{4}$

45. Simple Beam—Two Unequal Concentrated Moving Loads



$R_{1\,max} = V_{1\,max}$ (at $x = 0$)	=	$P_1 + P_2\,\dfrac{l-a}{l}$
M_{max} [under P_1, at $x = \dfrac{1}{2}\left(l - \dfrac{P_2 a}{P_1 + P_2}\right)$]	=	$\left(P_1 + P_2\right)\dfrac{x^2}{l}$
M_{max} [M_{max} may occur with larger load at center of span and other load off span (Case 43)]	=	$\dfrac{P_1 l}{4}$

General Rules for Simple Beams Carrying Moving Concentrated Loads



The maximum shear due to moving concentrated loads occurs at one support when one of the loads is at that support. With several moving loads, the location that will produce maximum shear must be determined by trial.

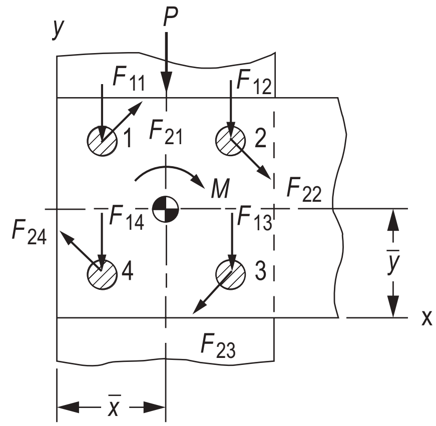
The maximum bending moment produced by moving concentrated loads occurs under one of the loads when that load is as far from one support as the center of gravity of all the moving loads on the beam is from the other support.

In the accompanying diagram, the maximum bending moment occurs under load P_1 when $x = b$. It should also be noted that this condition occurs when the centerline of the span is midway between the center of gravity of the loads and the nearest concentrated load.

Source: Adapted from American Institute of Steel Construction, *Steel Construction Manual*, 15th ed, 2017, Table 3-23. pp. 3-208 through 3-223.

4.2 Steel Design

4.2.1 Fastener Groups in Shear



The location of the centroid of a fastener group with respect to any convenient coordinate frame is

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i}, \quad \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

where

n = total number of fasteners

i = the index number of a particular fastener

A_i = cross-sectional area of the ith fastener

r_i = radius from the centroid of bolt group to the ith fastener

x_i = x-coordinate of the center of the ith fastener

y_i = y-coordinate of the center of the ith fastener

The total shear force on a fastener is the **vector** sum of the force due to direct shear P and the force due to the moment M acting on the group at its centroid.

The magnitude of the direct shear force due to P is

$$|F_{1i}| = \frac{P}{n}$$

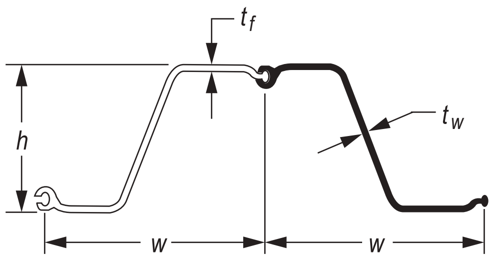
This force acts in the same direction as P.

The magnitude of the shear force due to M is

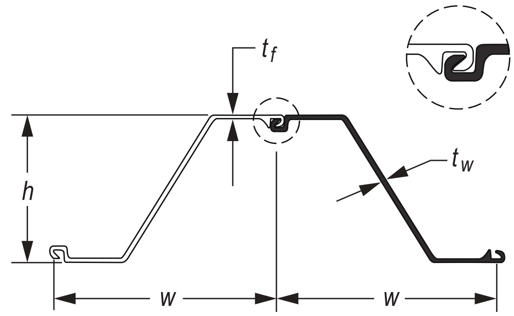
$$|F_{2i}| = \frac{M r_i}{\sum_{i=1}^{n} r_i^2}$$

This force acts perpendicular to a line drawn from the group centroid to the center of a particular fastener. Its sense is such that its moment is in the same direction (CW or CCW) as M.

4.2.2 Steel Sheet Pile Properties



PZ



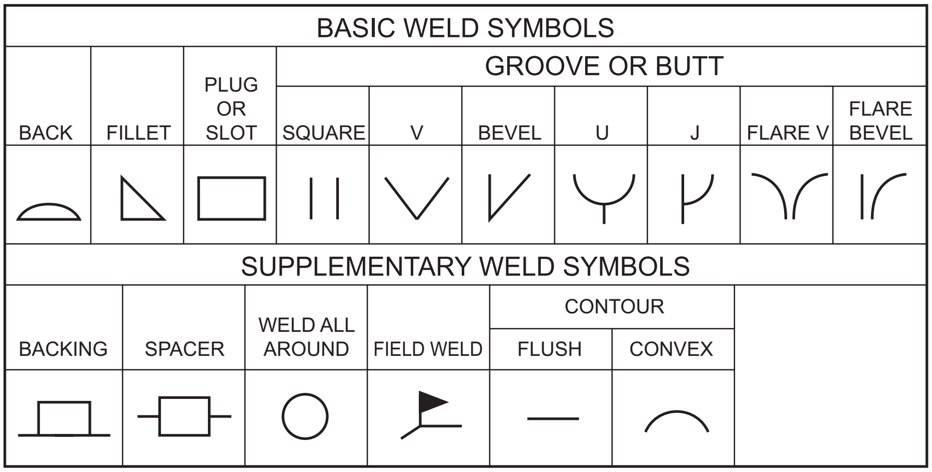
NZ

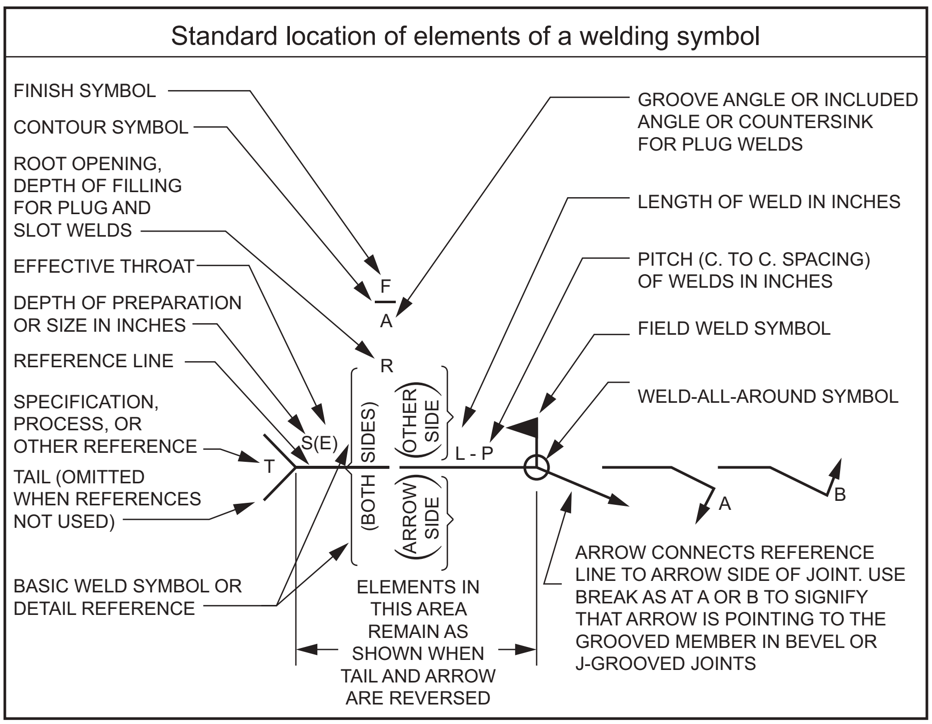
Properties of NZ and PZ Steel Sheet Piles

Section	Width w (in.)	Height h (in.)	Thickness		Cross Sectional Area (in^2/ft)	Weight		Section Modulus		Moment of Inertia (in^4/ft)	Coating Area	
			Flange t_f (in.)	Wall t_w (in.)		Pile (lb/ft)	Wall (lb/ft^2)	Elastic (in^3/ft)	Plastic (in^3/ft)		Both Sides (ft^2/ft of single)	Wall Surface (ft^2/ft^2 of wall)
NZ 14	30.31	13.39	0.375	0.375	6.40	55	21.77	25.65	30.50	171.7	6.10	1.20
NZ 19	27.56	16.14	0.375	0.375	7.07	55	24.05	35.08	41.33	283.1	6.18	1.35
NZ 20	27.56	16.16	0.394	0.394	7.34	57	24.82	36.24	42.80	292.8	6.18	1.35
NZ 21	27.56	16.20	0.433	0.433	7.80	61	26.56	38.69	45.85	313.4	6.18	1.35
NZ 22	27.56	16.25	0.480	0.480	8.57	67	29.20	41.47	49.34	336.9	6.18	1.35
NZ 26	27.56	17.32	0.500	0.500	9.08	71	30.99	48.50	57.01	419.9	6.49	1.41
NZ 28	27.56	17.38	0.560	0.560	9.98	78	33.96	52.62	62.16	457.4	6.49	1.41
NZ 38	27.56	19.69	0.689	0.500	11.00	86	37.45	70.84	81.57	697.3	6.58	1.43
NZ 40	27.56	19.73	0.735	0.551	11.77	92	40.06	74.97	86.75	739.6	6.58	1.43
NZ 42	27.56	19.77	0.769	0.589	12.41	97	42.24	78.17	90.80	772.5	6.58	1.43
PZ 22	22.0	9.0	0.375	0.375	6.47	40.3	22.0	18.1	21.79	84.38	4.48	1.22
PZ 27	18.0	12.0	0.375	0.375	7.94	40.5	27.0	30.2	36.49	184.20	4.48	1.49
PZ 35	22.6	14.9	0.600	0.500	10.29	66.0	35.0	48.5	57.17	361.22	5.37	1.42
PZ 40	19.7	16.1	0.600	0.500	11.77	65.6	40.0	60.7	71.92	490.85	5.37	1.64

Nucor Skyline. "NZ and PZ Hot-Rolled Sheet Piles Datasheet." Accessed June 11, 2021. https://www.nucorskyline.com/documentlibrary/datasheets

4.2.3 Basic Welding Symbols





Notes:

Size, weld symbol, length of weld, and spacing must read in that order, from left to right, along the reference line. Neither orientation of reference nor location of the arrow alters this rule.

The perpendicular leg of ◺, ⩗, ⊬, and |⌈ weld symbols must be at left.

Dimensions of fillet welds must be shown on both the arrow side and the other side.

Symbols apply between abrupt changes in direction of welding unless governed by the "all around" symbol or otherwise dimensioned.

These symbols do not explicitly provide for the case that frequently occurs in structural work, where duplicate material (such as stiffeners) occurs on the far side of a web or gusset plate. The fabricating industry has adopted this convention: when the billing of the detail material discloses the existence of a member on the far side as well as on the near side, the welding shown for the near side shall be duplicated on the far side.

Source: Adapted from American Institute of Steel Construction, *Steel Construction Manual*, 15th ed, 2017, Table 8-2, p. 8-35.

4.3 Concrete Design

4.3.1 Reinforcement Properties

ASTM Standard Reinforcing Bars

Bar Size	Diameter (in.)	Area (in^2)	Weight (lb/ft)
#3	0.375	0.11	0.376
#4	0.500	0.20	0.668
#5	0.625	0.31	1.043
#6	0.750	0.44	1.502
#7	0.875	0.60	2.044
#8	1.000	0.79	2.670
#9	1.128	1.00	3.400
#10	1.270	1.27	4.303
#11	1.410	1.56	5.313
#14	1.693	2.25	7.650
#18	2.257	4.00	13.60

ASTM Standard Prestressing Strands (Seven-Wire Strand, Grade 270)

Nominal Diameter (in.)	Nominal Area (in^2)	Nominal Weight (lb/ft)
3/8	0.085	0.29
7/16	0.115	0.39
1/2 (0.500)	0.153	0.52
0.6	0.217	0.74

ASTM Standard Plain Wire Reinforcement

Size Number	Nominal Diameter (in.)	Nominal Area (in^2)
W 0.5	0.080	0.005
W 1.2	0.124	0.012
W 1.4	0.134	0.014
W 2	0.160	0.020
W 2.5	0.178	0.025
W 2.9	0.192	0.029
W 3.5	0.211	0.035
W 4	0.226	0.040
W 4.5	0.239	0.045
W 5	0.252	0.050
W 5.5	0.265	0.055
W 6	0.276	0.060
W 8	0.319	0.080
W 10	0.357	0.100
W 11	0.374	0.110
W 12	0.391	0.120
W 14	0.422	0.140
W 16	0.451	0.160
W 18	0.479	0.180
W 20	0.505	0.200
W 22	0.529	0.220
W 24	0.553	0.240
W 26	0.575	0.260
W 28	0.597	0.280
W 30	0.618	0.300
W 31	0.628	0.310
W 45	0.757	0.450

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ASTM Standard Deformed Wire Reinforcement

Deformed Wire Size	Nominal Diameter (in.)	Nominal Area (in^2)	Nominal Weight (lb/ft)
D1	0.113	0.010	0.034
D2	0.160	0.020	0.068
D3	0.195	0.030	0.102
D4	0.226	0.040	0.136
D5	0.252	0.050	0.170
D6	0.276	0.060	0.204
D7	0.299	0.070	0.238
D8	0.319	0.080	0.272
D9	0.339	0.090	0.306
D10	0.357	0.100	0.340
D11	0.374	0.110	0.374
D12	0.391	0.120	0.408
D13	0.407	0.130	0.442
D14	0.422	0.140	0.476
D15	0.437	0.150	0.510
D16	0.451	0.160	0.544
D17	0.465	0.170	0.578
D18	0.479	0.180	0.612
D19	0.492	0.190	0.646
D20	0.505	0.200	0.680
D21	0.517	0.210	0.714
D22	0.529	0.220	0.748
D23	0.541	0.230	0.782
D24	0.553	0.240	0.816
D25	0.564	0.250	0.850
D26	0.575	0.260	0.884
D27	0.586	0.270	0.918
D28	0.597	0.280	0.952
D29	0.608	0.290	0.986
D30	0.618	0.300	1.02
D31	0.628	0.310	1.05
D45	0.757	0.450	1.53

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4.3.2 Design Provisions

4.3.2.1 Definitions

a = depth of equivalent rectangular stress block (in.)

A_g = gross area of concrete section (in^2)

A_s = area of longitudinal tension reinforcement (in^2)

A_{st} = total area of longitudinal reinforcement (in^2)

A_v = area of shear reinforcement within a distance s (in.)

b = width of compression face of member (in.)

β_1 = ratio of depth of rectangular stress block a to depth to neutral axis c:

f'_c (psi)	β_1	
$2{,}500 \le f'_c \le 4{,}000$	0.85	(a)
$4{,}000 < f'_c < 8{,}000$	$0.85 - \dfrac{0.05\left(f'_c - 4{,}000\right)}{1{,}000}$	(b)
$f'_c \ge 8{,}000$	0.65	(c)

c = distance from extreme compression fiber to neutral axis (in.)

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement (in.)

d_t = distance from extreme compression fiber to extreme tension steel (in.)

E_c = modulus of elasticity of concrete (psi)

$E_c = 33 w_c^{1.5} \sqrt{f'_c}$

E_s = modulus of elasticity of reinforcement (psi)

ε_t = net tensile strain in extreme layer of longitudinal tension reinforcement

f'_c = compressive strength of concrete (psi)

f_y = yield strength of steel reinforcement (psi)

M_n = nominal flexural strength at section (in.-lb)

ϕM_n = design flexural strength at section (in.-lb)

M_u = factored moment at section (in.-lb)

P_n = nominal axial compressive load strength of member (lb)

ϕP_n = design axial compressive load strength of member (lb)

P_u = factored axial force: to be taken as positive for compression and negative for tension (lb)

ρ_g = ratio of total reinforcement area to cross-sectional area of column

$\rho_g = \frac{A_{st}}{A_g}$

s = center-to-center spacing of longitudinal shear or torsional reinforcement (in.)

V_c = nominal shear strength provided by concrete (lb)

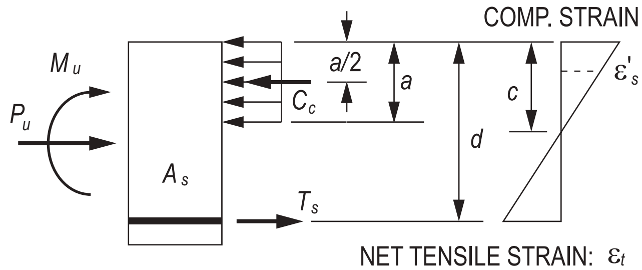
V_n = nominal shear strength at section (lb)

ϕV_n = design shear strength at section (lb)

V_s = nominal shear strength provided by reinforcement (lb)

V_u = factored shear force at section (lb)

4.3.2.2 Beams—Flexure Strength



Single-Reinforced Beam, Service—Internal Forces and Strains

Single-Reinforced Beams:

$$a = \frac{A_s\, f_y}{0.85\, f'_c\, b}$$

$$M_n = 0.85\, f'_c\, ab\left(d - \frac{a}{2}\right) = A_s\, f_y\left(d - \frac{a}{2}\right)$$

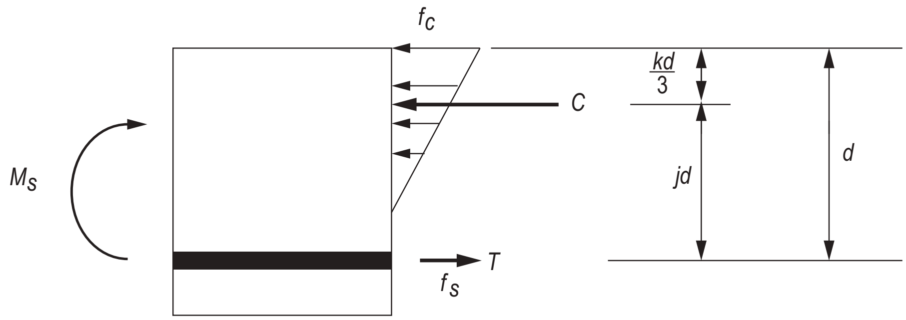
$$\rho = \frac{A_s}{bd}$$

$$m = \frac{f_y}{0.85 f'_c}$$

$$R_n = \frac{M_u}{\phi b d^2}$$

$$\rho = \left(\frac{1}{m}\right)\left[1 - \sqrt{\left(1 - \frac{2mR_n}{f_y}\right)}\right]$$

4.3.2.3 Beams—Flexure Service



Single-Reinforced Beam, Service—Internal Forces and Strains

$$n = \frac{E_s}{E_c}$$

$$\rho = \frac{A_s}{bd}$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$j = 1 - \frac{k}{3}$$

$$f_s = \frac{M_s}{A_s\, jd}$$

$$f_c = \frac{2M_s}{kbjd^2}$$

5 TRANSPORTATION

5.1 Traffic Engineering (Capacity Analysis and Transportation Planning)

5.1.1 Uninterrupted Flow (e.g., Level of Service, Capacity)

5.1.1.1 Traffic Flow, Density, Headway, and Speed Relationships

Traffic flow:

$$q = \frac{n}{t}$$

where

q = traffic flow in vehicles per unit time

n = number of vehicles passing some designated roadway point during time t

t = duration of time interval

Traffic flow is the equivalent hourly rate at which vehicles pass a point on a highway during a period less than 1 hour:

$$q = \frac{n(3{,}600)}{T}$$

where

n = number of vehicles passing a point in the roadway in T sec

q = equivalent hourly flow

Traffic density is defined as

$$k = \frac{n}{l}$$

where

k = traffic density in vehicles per unit distance

n = number of vehicles occupying some length of roadway at some specified time

l = length of roadway

Headway

$$t = \sum_{i=1}^{n} h_i$$

where

t = duration of time interval

h_i = time headway of the ith vehicle (time transpired between arrivals of vehicle i and $i - 1$)

n = number of measured vehicle time headways at some designated roadway point

$$q = \frac{n}{\sum_{i=1}^{n} h_i}$$

$$q = \frac{1}{\bar{h}}$$

where $\bar{h}$ = average time headway, $\frac{\sum h_i}{n}$, in unit time per vehicle

5.1.1.2 Space Mean Speed

$$u_s = \frac{q}{k} = \frac{\text{flow}}{\text{density}}$$

$$u_s = \frac{nL}{\sum_{i=1}^{n} t_i}$$

where

u_s = space mean speed (mph)

n = number of vehicles

L = length of section of highway (miles)

t_i = time it takes the ith vehicle to travel across a section of highway

q = equivalent hourly flow (vph)

flow = density × space mean speed

$$q = k \times u_s$$

5.1.1.3 Lane Occupancy Used in Freeway Surveillance

$$R = \frac{\text{sum of length of vehicles}}{\text{length of roadway section}} = \frac{\sum L_i}{D}$$

where R can be divided by the average length of a vehicle to get an estimate of density k.

Source: Khisty-Lall. *Transportation Engineering: An Introduction*. 3rd ed. New York: Pearson Prentice Hall, 2003, p. 123.

5.1.1.4 Greenshields Maximum Flow Rate Relationship

$$q_{\max} = \frac{k_j u_f}{4}$$

where

$q_{\max}$ = maximum flow rate for Greenshields relationship (vph)

k_j = jam density (veh/mi)

u_f = mean free speed (mph)

Source: Garber, Nicholas J. and Lester A. Hoel. *Traffic & Highway Engineering*. 5th ed. 2014. Cengage Learning, Inc. Reproduced by permission. www.cengage.com/permissions.

5.1.2 Street Segment Interrupted Flow (e.g., Level of Service, Running Time, Travel Speed)

5.1.2.1 Speed-Density Model

$$u_s = u_f\left(1 - \frac{k}{k_j}\right)$$

where

u_f = free-flow speed (mph)

k = density (veh/mi)

k_j = jam density (veh/mi)

5.1.2.2 Flow-Density Model

$$q = u_f\left(k - \frac{k^2}{k_j}\right)$$

where

$$q_{\text{cap}} = u_{\text{cap}} \times k_{\text{cap}}$$

$$k_{\text{cap}} = \frac{k_j}{2}$$

$$u_{\text{cap}} = \frac{u_f}{2}$$

$$q_{\text{cap}} = u_f \frac{k_j}{4}$$

where

q_{cap} = flow at capacity

k_{cap} = density at the capacity flow rate

5.1.2.3 Speed-Flow Model

$$k = k_j\left(1 - \frac{u_s}{u_f}\right)$$

$$q = k_j\left(u_s - \frac{u_s^2}{u_f}\right)$$

5.1.2.4 Time Mean Speed

The TMS $\overline{u_t}$ is computed as the arithmetic average of individual vehicle speeds.

$$\overline{u_t} = \frac{\sum_i^t u_i}{n}$$

where

$\overline{u_t}$ = time mean speed in unit distance per unit time

u_i = spot speed of the *i*th vehicle

n = number of measured vehicle spot speeds

Source: Mannering, Fred L. and Scott S. Washburn. *Principles of Highway Engineering and Traffic Analysis*. 6th ed. Hoboken, NJ: John Wiley and Sons, 2016.

5.1.2.5 Average Speed (Mean Speed)

$$\overline{x} = \frac{\sum n_i S_i}{N}$$

where

$\overline{x}$ = average or mean speed (mph)

n_i = frequency of observations in group i

S_i = middle speed of group i (mph)

N = total number of individual speed observations

Source: Roess, Roger, William McShane, and Elena Prassas. *Traffic Engineering*. 2nd ed. New York: Pearson, 1998, p. 161.

5.1.2.6 Segment Running Time Simplified

The *Highway Capacity Manual*, v. 6, 2016, equation 18-7, p. 18-31, which "is used to compute segment running time on the basis of consideration of through movement control at the boundary intersection, free-flow speed, vehicle proximity, and various midsegment delay sources" may be simplified to the following equation if all of the lost time and boundary control delays are accounted for or given as part of the through movement average travel speed.

Average running time = Segment length / Average travel speed

5.1.3 Traffic Analysis (e.g., Volume Studies, Peak Hour Factor, Speed Studies, Modal Split)

5.1.3.1 Average Annual Daily Traffic Estimation

Average annual daily traffic (AADT)

$$\text{AADT} = V_{24ij} \times DF_i \times MF_j$$

where

AADT = average annual daily traffic (vpd)

V_{24ij} = 24-hour volume for day i in month j (vehicles)

DF_i = daily adjustment factor for day i

MF_j = monthly adjustment factor for month j

$$DF = \frac{V_{\text{avg}}}{V_{\text{day}}}$$

where

V_{avg} = average daily count for all days of the week (vehicles)

V_{day} = average daily count for each day of the week (vehicles)

$$MF_i = \frac{\text{AADT}}{\text{ADT}_i}$$

where

MF_i = monthly adjustment factor for month i

AADT = average annual daily traffic (vehicles/day) (estimated as the average of 12 monthly ADTs)

ADT_i = average daily traffic for month i (vehicles/day)

5.1.3.2 Estimating Annual Vehicle-Miles Traveled

$$VMT_{365} = \text{AADT} \times L \times 365$$

where

VMT_{365} = annual vehicle-miles traveled over the segment

AADT = vehicles per day for the segment

L = length of the segment

The AAWT is computed as the total weekday volume divided by 260 days, or:

$$\text{AAWT} = \frac{\text{total weekday traffic}}{260}$$

Sources: McShane, William, Roger Roess, and Elena Prassas. *Traffic Engineering*. 4th ed. New York: Pearson, 2011 & Mannering, Fred L. and Scott S. Washburn. *Principles of Highway Engineering and Traffic Analysis*. 6th ed. Hoboken, NJ: John Wiley and Sons, 2016.

5.1.3.3 Peak-Hour Factor

$$v = \frac{V}{PHF} \qquad PHF = \frac{V}{V_{15} \times 4}$$

where

v = rate of flow for a peak 15-min period (vph)

PHF = peak-hour factor

V = hourly volume for hour of analysis

V_{15} = maximum 15-min flow rate within peak hour

4 = number of 15-min periods per hour

5.1.4 Accident Analysis (e.g., Conflict Analysis, Accident Rates, Collision Diagrams)

5.1.4.1 Acceleration

Acceleration Assumed Constant

When the acceleration of the vehicle is assumed to be constant:

$$a = \frac{dS}{dt} = \frac{d^2x}{dt^2}$$

where

$dS = a\,dt$

$S = at + S_0$

S_0 = initial speed

$$S = at + S_0 = \frac{dx}{dt}$$

$$dx = \left(at + S_0\right)dt$$

$$x = \frac{1}{2}at^2 + S_0 t + x_0$$

where x_0 = initial position

Source: Khisty-Lall. *Transportation Engineering: An Introduction*. 3rd ed. New York: Pearson Prentice Hall, 2003, p. 100-110.

5.1.4.2 Acceleration Characteristics of Typical Car Versus Typical Truck on Level Terrain

Vehicle Acceleration Rates

Speed Range (mph)	Acceleration Rate (ft/sec^2) for:	
	Typical Car (30 lb/hp)	Typical Truck (200 lb/hp)
0–20	7.5	1.6
20–30	6.5	1.3
30–40	5.9	0.7
40–50	5.2	0.7
50–60	4.6	0.3

Source: McShane, William, Roger Roess, and Elena Prassas. *Traffic Engineering*. 4th ed. New York: Pearson, 2011, Table 2.5, p. 30.

5.1.4.3 Estimating Speed of Vehicle from Skid Marks

The following equation is used to find the initial speed, v_1, of the vehicle based on the known or estimated final speed, v_2:

$$d_b = \frac{v_1^2 - v_2^2}{30(f \pm G)}$$

$$v_1 = \sqrt{d_b(30)(f \pm G) + (v_2)^2}$$

where

d_b = horizontal distance traveled (ft) in reducing speed of vehicle from v_1 to v_2 during braking maneuvers

v_1 = vehicle speed when brakes are applied (initial speed) (mph)

v_2 = vehicle speed at end of travel (final speed) (mph)

$f = \frac{a}{g}$ = coefficient of friction (unitless) between tires and road pavement

a = deceleration of vehicle when brakes are applied (ft/sec^2)

g = acceleration due to gravity (32.2 ft/sec^2)

G = grade of roadway (decimal form; negative if downhill)

Source: McShane, William, Roger Roess, and Elena Prassas. *Traffic Engineering*. 4th ed. New York: Pearson, 2011, p. 30.

5.1.5 Traffic Forecast

The gravity model is based on the gravitational modeling principles covered in physics (the gravitational forces of planets) where the likelihood of a trip going to a destination is a function of the distance from the trip origin and some measure of attractiveness (the equivalent of mass in gravitational theory) of the destination.

$$T'_{ab} = T'_a \frac{A_b f_{ab} K_{ab}}{\sum_{\forall b} A_b f_{ab} K_{ab}}$$

where

T'_{ab} = total number of trips from traffic analysis zone (TAZ) a to TAZ b

T'_a = total number of trips from TAZ a

A_b = total number of trips attracted from TAZ b

f_{ab} = distance/travel cost "friction factor"

K_{ab} = estimated parameter to ensure results balance

Source: Mannering, Fred L. and Scott S. Washburn. *Principles of Highway Engineering and Traffic Analysis*. 6th ed. Hoboken, NJ: John Wiley and Sons, 2016.

5.1.6 Design Traffic

Use expected cumulative Equivalent Single Axle Loads (ESAL) to calculate design traffic by direction and lanes.

$$ESAL_{DL} = D_D \times D_L \times ESAL$$

where

$ESAL_{DL}$ = ESAL in the design lane

ESAL = cumulative two-directional 18-kip Equivalent Single Axle Loads units predicted during the analysis period

D_D = directional distribution factor, expressed as a ratio, that accounts for the distribution of ESAL units by direction

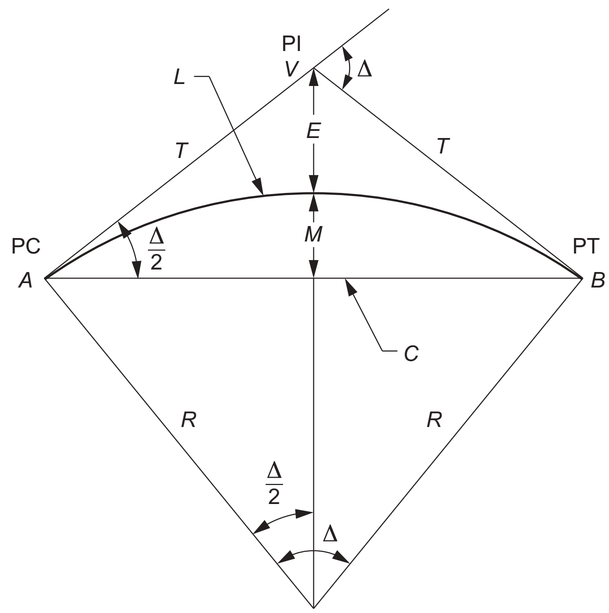
D_L = lane distribution factor, expressed as a ratio, that accounts for distribution of traffic when two or more lanes are available in one direction. If no other information is provided, the following table may be used as a guide:

Number of Lanes in Each Direction	Percent ESAL in Design Lane
1	100
2	80–100
3	60–80
4	50–75

Source: Based on information from AASHTO Guide for Design of Pavement Structures, 1993, published by the American Association of State Highway and Transportation Officials, Washington, D.C.

5.2 Horizontal Design

5.2.1 Basic Curve Elements (e.g., Middle Ordinate, Length, Chord, Radius)



Parts of a Circular Curve

R = radius of circular curve

T = tangent length

Δ = intersection angle/central angle/deflection angle (degrees)

M = middle ordinate

PC = point of curve

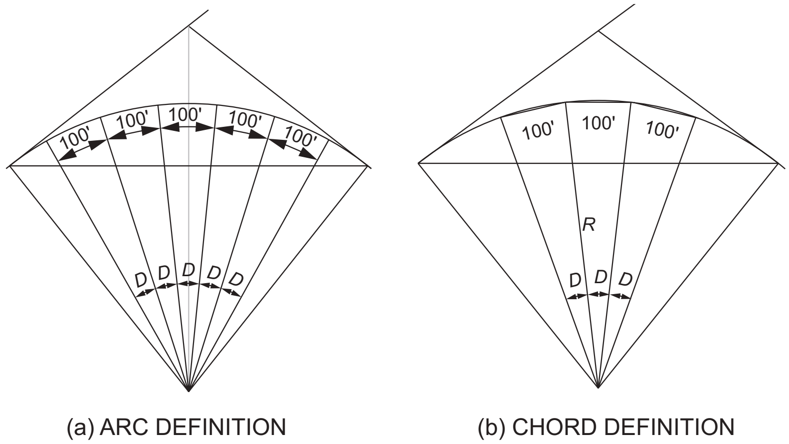
PT = point of tangent

PI = point of intersection

E = external distance

Line $AB = C$ = chord length

L = arc length = curve length measured along the arc from PC to PT

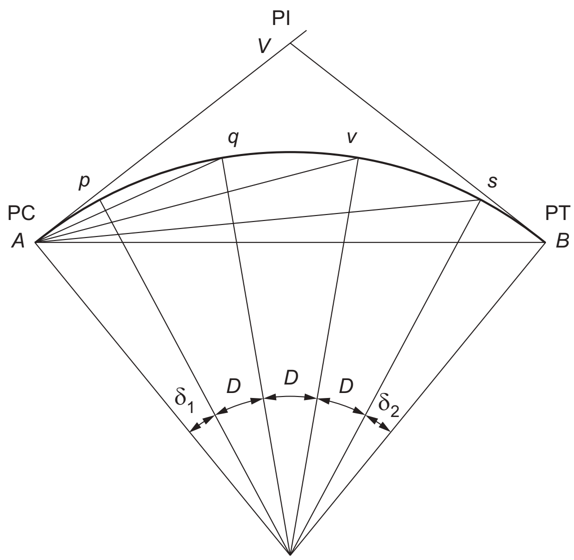


Arc and Chord Definitions for a Circular Curve

where

D_a = degree of curvature by arc definition

D_c = degree of curvature by chord definition



Deflection Angles on a Simple Circular Curve

where

$$R = \frac{5,729.6}{D_a}$$

$$D_a = \frac{100\left(\frac{180}{\pi}\right)}{R} = \frac{18,000}{\pi R}$$

$$R = \frac{50}{\sin \frac{D_c}{2}}$$

$$T = R \tan \frac{\Delta}{2}$$

$$C = 2R \sin \frac{\Delta}{2}$$

$$E = R \sec \frac{\Delta}{2} - R = T\left(\tan \frac{\Delta}{4}\right) = R\left(\frac{1}{\cos \frac{\Delta}{2}} - 1\right)$$

$$M = R - R \cos \frac{\Delta}{2}$$

$$= R\left(1 - \cos \frac{\Delta}{2}\right)$$

$$L = \frac{R \Delta \pi}{180}$$

$$l_1 = \frac{R \delta_1 \pi}{180}$$

$$\frac{\delta_1}{\Delta} = \frac{l_1}{L}$$

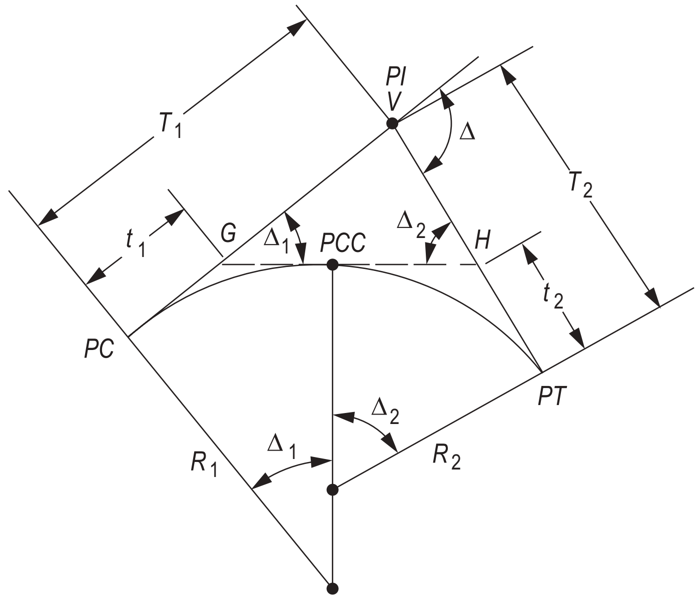
$$L = 100 \frac{\Delta}{D_a}$$

where

l_1 = length of curve from PC to point p subtended by central angle δ_1

$C_1 = 2R \sin \frac{\delta_1}{2}$ = chord length from PC to point p

5.2.2 Layout of Two-Centered Compound Curves



Two-Centered Compound Curve

where

R_1, R_2 = radii of simple curves forming compound curve

Δ_1, Δ_2 = deflection angles of simple curves

Δ = deflection angle of compound curve

$= \Delta_1 + \Delta_2$

t_1, t_2 = tangent lengths of simple curves

T_1, T_2 = tangent lengths of compound curves

PCC = point of compound curve

PI = point of intersection

PC = point of curve

PT = point of tangent

$$\frac{\overline{VG}}{\sin \Delta_2} = \frac{\overline{VH}}{\sin \Delta_1} = \frac{t_1 + t_2}{\sin(180 - \Delta)} = \frac{t_1 + t_2}{\sin \Delta}$$

where

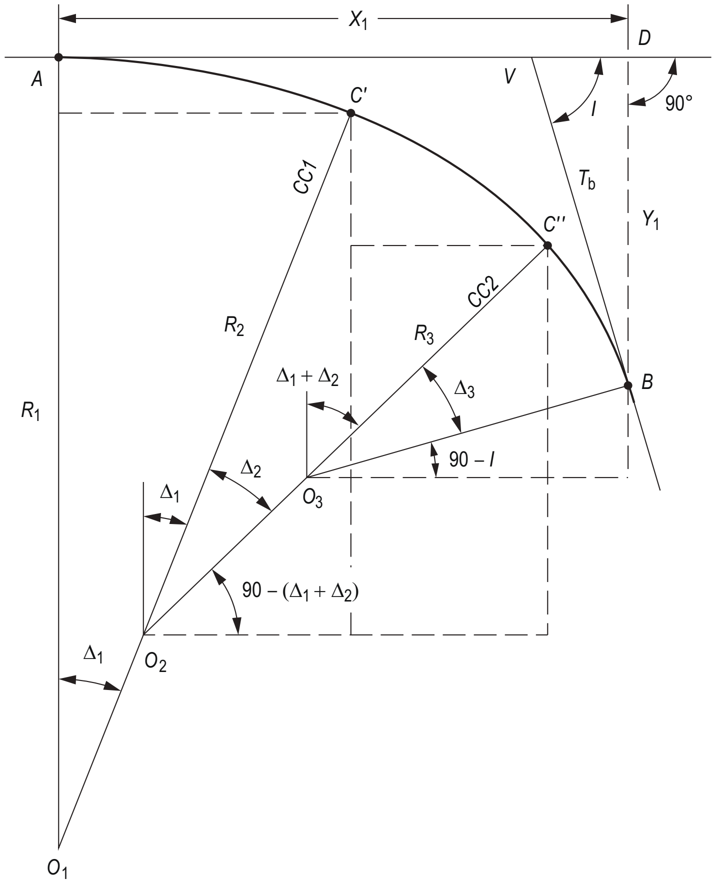
$$T_1 = \overline{VG} + t_1$$

$$t_1 = R_1 \tan \frac{\Delta_1}{2}$$

$$t_2 = R_2 \tan \frac{\Delta_2}{2}$$

$$T_2 = \overline{VH} + t_2$$

5.2.3 Layout of Three-Centered Compound Curves



Three-Centered Compound Curve

Source: Hickerson, Thomas. *Route Location and Design*. 5th ed. 1967, Fig. 47, p. 131.

A three-centered compound curve has centers at O_1, O_2, and O_3 with central angles equal to Δ_1, Δ_2, and Δ_3.

I = total central angle = $\Delta_1 + \Delta_2 + \Delta_3$

Proceeding from flatter to sharper curve, the radii are R_1, R_2, and R_3.

The first PCC is at C', the second at C''.

T_a = AV = long tangent

T_b = VB = short tangent

PCC = point of compound curve

X_1, Y_1 equals the coordinates of point B with reference to A as origin and AV as X axis, where X_1 = AD and Y_1 = DB. Then,

$$Y_1 = T_b \times \sin I$$

$$X_1 = T_a + T_b \times \cos I$$

$$T_b = \frac{Y_1}{\sin I}$$

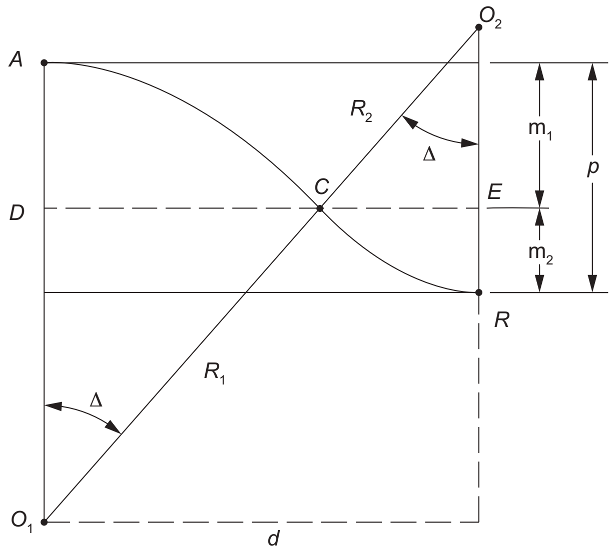
$$T_a = X_1 - T_b \times \cos I$$

$$X_1 = R_1 \sin \Delta_1 + R_2 \sin\left(\Delta_1 + \Delta_2\right) - R_2 \sin \Delta_1 + R_3 \sin I - R_3 \sin\left(\Delta_1 + \Delta_2\right)$$

OR

$$X_1 = \left(R_1 - R_2\right) \sin \Delta_1 + \left(R_2 - R_3\right) \sin\left(\Delta_1 + \Delta_2\right) + R_3 \sin I$$

5.2.4 Layout of Reverse Horizontal Curves Between Parallel Tangents



Reverse Horizontal Curves Between Parallel Tangents
Source: Hickerson, Thomas. *Route Location and Design*. 5th ed. 1967, Fig. 56, p. 142.

where

d = $DE = DC + CE$

p = $m_1 + m_2$ = distance between parallel tracks

Then radii R_1 and R_2 may be equal or unequal.

If equal:

$$R_1 = R_2 = R$$

$$DC = CE = \frac{d}{2} \text{ and } m_1 = m_2 = \frac{p}{2}$$

$$d = R_1 \sin\Delta + R_2 \sin\Delta = (R_1 + R_2)\sin\Delta$$

$$p = R_1(1 - \cos\Delta) + R_2(1 - \cos\Delta) = (R_1 + R_2)(1 - \cos\Delta)$$

If any three are given, the other two may be found.

5.2.5 Method of Designating Directions

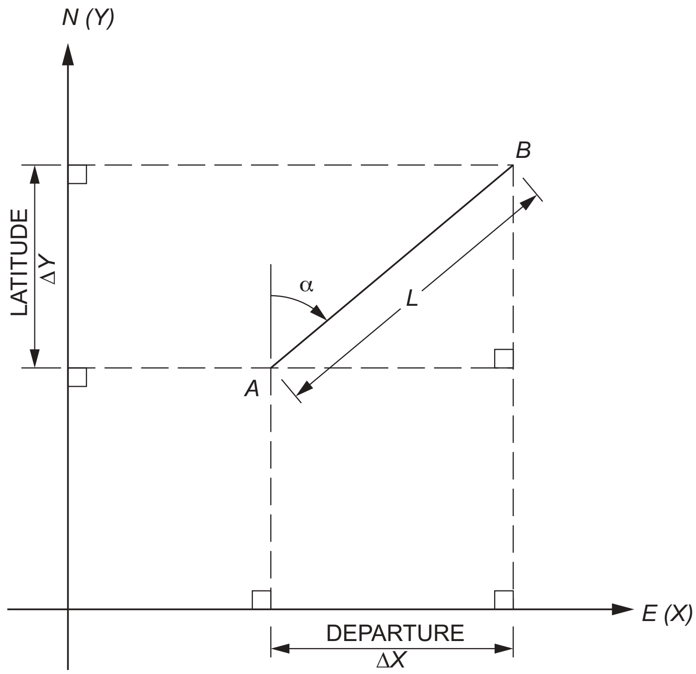
The direction of a line is expressed as the angle between a meridian and the line. The meridian may be a *true meridian*, which is a great circle of the earth passing through the poles, a *magnetic meridian*, the direction of which is defined by a compass needle, or a *grid meridian*, which is established for a plane coordinate system.

The direction of a line may be expressed as its *bearing* or its *azimuth*.

- The bearing of a line is the horizontal acute angle between the meridian and the line
- Because the bearing of a line cannot exceed 90°, the full horizontal circle is divided into four quadrants: northeast, southeast, southwest, and northwest.

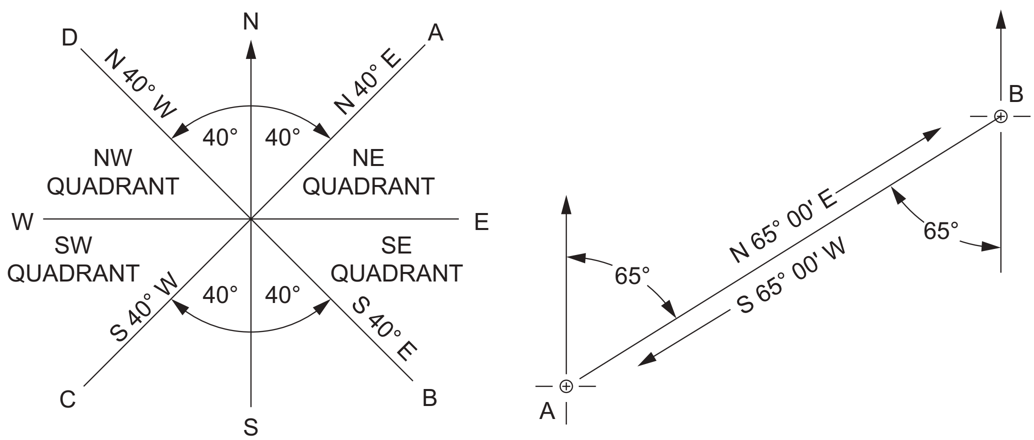
  Northing = latitude = Y = distance × cos (bearing)

  Easting = departure = X = distance × sin (bearing)



Relationship of Latitude and Departure

Ghilani, Charles D. *Elementary Surveying: An Introduction To Geomatics*. 15th ed. 2018. Reprinted by permission of Pearson Education, Inc.



Example Bearings

Example Directions for Lines in the Four Quadrants

Quadrant	Formulas for Computing Bearing Angles from Azimuths
I (NE)	bearing = azimuth
II (SE)	bearing = 180° – azimuth
III (SW)	bearing = azimuth – 180°
IV (NW)	bearing = 360° – azimuth

Source: Ghilani, Charles D. *Elementary Surveying: An Introduction To Geomatics*. 15th ed. 2018. Reprinted by permission of Pearson Education, Inc.

5.3 Vertical Design

5.3.1 Symmetrical Vertical Curve Formula

$$y = ax^2$$

$$A = |g_2 - g_1|$$

$$a = \frac{g_2 - g_1}{2L}$$

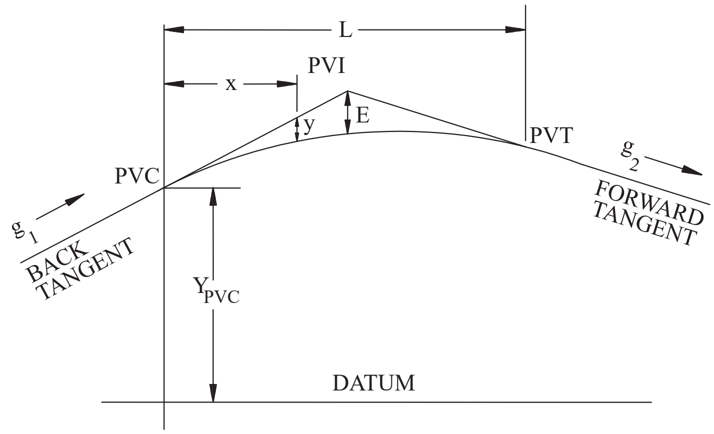
$$E = a\left(\frac{L}{2}\right)^2$$

$$E = \frac{AL}{8}$$

$$r = \frac{g_2 - g_1}{L}$$

$$K = \frac{L}{A}$$

$$x_m = -\frac{g_1}{2a} = \frac{g_1 L}{g_1 - g_2}$$



Vertical Curve Formulas (not to scale)

$$\text{Tangent elevation} = Y_{PVC} + g_1 x = Y_{PVI} + g_2\left(x - \frac{L}{2}\right)$$

$$\text{Curve elevation} = Y_{PVC} + g_1 x + ax^2 = Y_{PVC} + g_1 x + x^2\left(\frac{g_2 - g_1}{2L}\right)$$

where

PVC = point of vertical curvature, or beginning of curve

PVI = point of vertical intersection, or vertex

PVT = point of vertical tangency, or end of curve

A = algebraic difference in grades

a = parabola constant

E = tangent offset at PVI

g_1 = grade of back tangent

g_2 = grade of forward tangent

K = rate of vertical curvature

L = length of curve

r = rate of change of grade

x = horizontal distance from PVC to point on curve

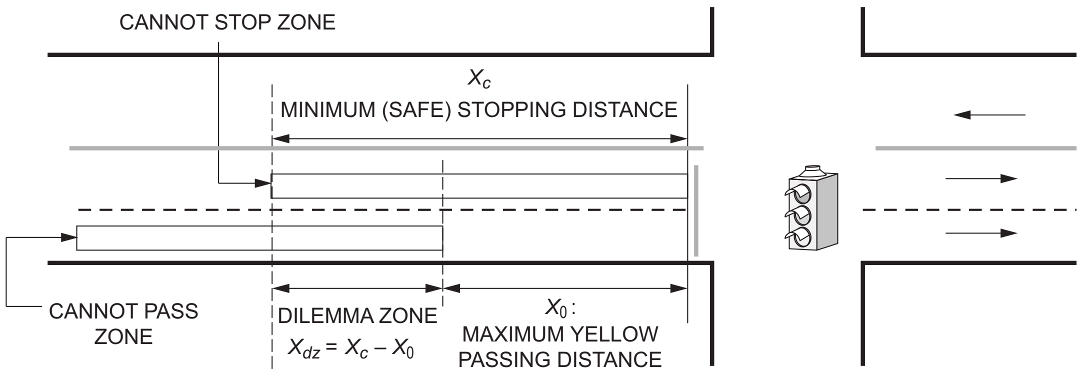
x_m = horizontal distance to min/max elevation on curve

y = tangent offset

5.4 Signal Design

5.4.1 Dilemma Zones

A *dilemma zone* is defined as a zone within which a driver can neither bring his/her vehicle to a stop safely nor go through the signal-controlled intersection before the signal turns red. The formation of a dilemma zone is depicted below:



Dilemma Zone Formation

Sources: Adapted from Mannering, Fred L. and Scott S. Washburn. *Principles of Highway Engineering and Traffic Analysis*. 5th ed. Hoboken, NJ: John Wiley and Sons, 2012, Fig. 7.12, p. 251 & Garber, Nicholas J. and Lester A. Hoel, *Traffic & Highway Engineering*, 5th ed., 2014, Boston, MA: Cengage Learning, Inc., 2015, Fig. 8.16, p. 393.

The physical zone between X_c and X_0, when $X_c > X_0$, is the dilemma zone. In this situation, the word "dilemma" exactly represents such a circumstance, although the driver may not be aware of it.

$$X_c = 1.47V(t_{\text{stop}}) + 1.075\frac{V^2}{a_2}$$

$$X_0 = 1.47VY - W + \frac{1}{2}a_1\left(Y - t_{\text{passing}}\right)^2$$

where

V = vehicle approach speed (mph)

t_{stop} = driver perception time for safe stopping (sec)

t_{passing} = driver perception time for safe passing (sec)

a_2 = maximum vehicle deceleration rate to stop (ft/sec^2)

a_1 = constant vehicle acceleration rate (ft/sec^2)

W = summation of intersection width and vehicle length (ft)

Y = yellow interval (sec)

5.4.2 Offsets

The time difference between a common reference point in the coordinated phases at adjacent signalized intersections is referred to as the *offset.* Assuming a moving platoon, the offset is calculated as:

$$offset = \frac{d_o}{V}$$

where

offset = start of green phase for downstream intersection relative to upstream intersection, for the same traffic movement (sec)

d_o = distance between upstream and downstream for offset calculation (ft)

V = travel speed between upstream and downstream intersection (ft/sec)

Assuming a standing platoon:

$$offset = l_1 + \frac{d_o}{V}$$

With vehicles queued downstream:

$$offset_{\text{adj}} = \frac{d_o}{V} - \left(Qh + l_1\right)$$

where

Q = number of vehicles queued per lane (vehicles)

h = discharge headway of queued vehicles (sec/vehicle)

l_1 = startup lost time (sec)

For good progression in both directions, the cycle length (for both intersections) needs to be twice the travel time from Intersection 1 to Intersection 2:

$$C_{\text{prog}} = \frac{d_o}{V} \times 2$$

where C_{prog} = cycle length necessary for ideal two-way progression (sec)

Sources: Mannering, Fred L. and Scott S. Washburn. *Principles of Highway Engineering and Traffic Analysis*. 5th ed. Hoboken, NJ: John Wiley and Sons, 2012, p. 262 & Garber, Nicholas J. and Lester A. Hoel, *Traffic & Highway Engineering*, 5th ed., 2014, Boston, MA: Cengage Learning, Inc., 2015, p. 250-264.

5.4.3 Interval Timing

ITE *Traffic Engineering Handbook*, 6th edition, included the following recommendations for the timing of yellow and red intervals.

5.4.3.1 Yellow Change Interval

ITE provides the following formula for determining the appropriate yellow time for an approach:

$$Y = t + \frac{v}{2a + 2Gg}$$

where

Y = calculated yellow time

t = reaction time (typically 1 second if no other information is provided)

v = design speed (ft/sec)

a = deceleration rate (typically 10 ft/sec^2 if no other information is provided)

g = acceleration due to gravity

G = grade of approach

5.4.3.2 Red Clearance Interval

ITE provides the following formula for determining the appropriate timing for an optional interval where the approach signal is red and no conflicting traffic is moving.

$$R = \frac{w + L}{v}$$

where

R = calculated red clearance interval time

w = width of intersection measured from near-side stop line to far side point of clearance

L = length of vehicle (typically 20 ft if no other information is provided)

v = design speed (ft/sec)

Source: Adapted from *Traffic Engineering Handbook*, 6th ed., Institute of Transportation Engineers, 2009, pp. 412–413.

5.5 Geotechnical and Pavement

5.5.1 Relative Soil Density

The relative density, usually given as a percentage

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

where

D_r = relative density, usually given as percentage

e = in situ void ratio of the soil

e_{max} = void ratio of the soil in the loosest condition

e_{min} = void ratio of the soil in the densest condition

Using the definition of dry unit weight, relative density in terms of maximum and minimum dry unit weights can be expressed as:

$$D_r = \frac{\left[\frac{1}{\gamma_{d(min)}}\right] - \left[\frac{1}{\gamma_d}\right]}{\left[\frac{1}{\gamma_{d(min)}}\right] - \left[\frac{1}{\gamma_{d(max)}}\right]} = \left[\frac{\gamma_d - \gamma_{d(min)}}{\gamma_{d(max)} - \gamma_{d(min)}}\right]\left[\frac{\gamma_{d(max)}}{\gamma_d}\right]$$

where

$\gamma_{d(min)}$ = dry unit weight in the loosest condition

γ_d = in situ dry unit weight

$\gamma_{d(max)}$ = dry unit weight in the densest condition

Source: Das, Braja M. and Nagaratnam Sivakugan. *Fundamentals of Geotechnical Engineering*. 5th ed. Boston, MA: Cengage Learning, Inc., 2017, p. 70.

5.5.2 Plasticity Index

A one-point method for estimation of liquid limit using the fall cone device is:

$$LL = \frac{w}{0.65 + 0.0175d}$$

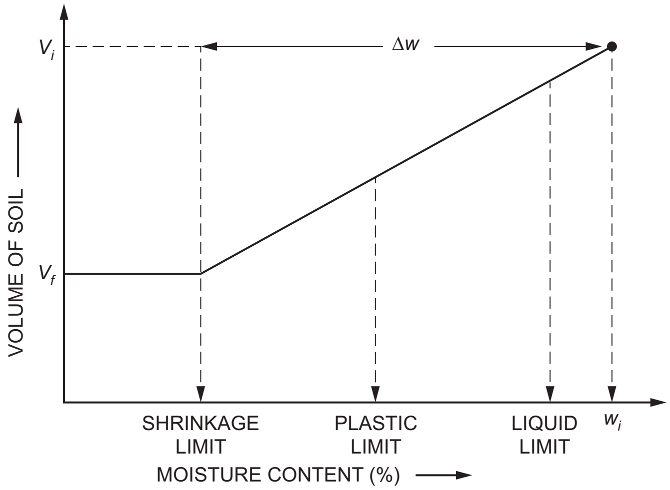
where w = moisture content for 17 mm $\le d \le$ 23 mm

Qualitative Descriptions of Granular Soil Deposits

Relative Density (%)	Description of Soil Deposit
0–15	Very loose
15–35	Loose
35–65	Medium dense
65–85	Dense
85–100	Very dense

Source: Das, Braja M. and Nagaratnam Sivakugan, *Fundamentals of Geotechnical Engineering*, 5th ed., 2017, Cengage Learning, Inc. Reproduced by permission. www.cengage.com/permissions.

5.5.3 Shrinkage of Soil Mass



Shrinkage Limits

Source: Das, Braja M. and Nagaratnam Sivakugan, *Fundamentals of Geotechnical Engineering*, 5th ed., 2017, Cengage Learning, Inc. Reproduced by permission. www.cengage.com/permissions.

The moisture content at which the volume change of the soil mass ceases is defined as the shrinkage limit. The shrinkage limit is determined by the equation:

$$SL = w_i - \Delta w$$

where

SL = shrinkage limit

w_i = initial moisture content when the soil is placed in the shrinkage limit dish (%)

Δw = change in moisture content (%)

However,

$$w_i(\%) = \frac{m_1 - m_2}{m_2} \times 100$$

where

m_1 = mass of the wet soil pat in the dish at the beginning of the test

m_2 = mass of the dry soil pat

$$\Delta w(\%) = \frac{(V_i - V_f)\rho_w}{m_2} \times 100$$

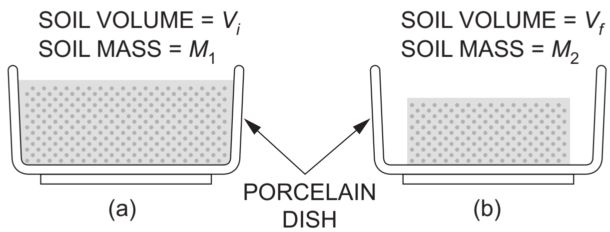
where

V_i = initial volume of the wet soil pat

V_f = volume of the oven-dried soil pat

ρ_w = density of water

$$SL = \left(\frac{m_1 - m_2}{m_2}\right)(100) - \left[\frac{(V_i - V_f)\rho_w}{m_2}\right](100)$$



Shrinkage Limit Test
(a) Soil pat before drying;
(b) Soil pat after drying

Source: Das, Braja M. and Nagaratnam Sivakugan, *Fundamentals of Geotechnical Engineering*, 5th ed., 2017, Cengage Learning, Inc. Reproduced by permission. www.cengage.com/permissions.

5.5.4 Soil Compaction

Compaction – General Principles

Compaction may be measured in terms of dry unit weight.

Water acts as a softening agent during compaction.

The dry unit weight at $w = w_1$ is:

$$\gamma_{d(w=w_1)} = \gamma_{d(w=0)} + \Delta\gamma_d$$

The dry unit weight increases initially as water is added.

After a certain point, ($w = w_2$), the dry unit weight decreases as the moisture content increases.

The moisture content at maximum dry weight is the *optimum moisture content*.

5.5.5 Asphalt Mixture Design

5.5.5.1 Specific Gravity, Bulk Specific Gravity, Bulk Specific Gravity-Saturated Surface Dry

Determined by ASTM C127 and ASTM C128. Both are based on Archimedes' Principle.

5.5.5.2 Specific Gravity of Aggregates

Apparent specific gravity: ratio of the weight of dry aggregate to the weight of water having a volume equal to the solid volume of aggregate (excluding permeable pores) = $\dfrac{A}{A - C}$

Bulk specific gravity: ratio of the weight of dry aggregate to the weight of water having a volume equal to the volume of the aggregate, including permeable and impermeable pores = $\dfrac{A}{B - C}$

Bulk specific gravity-saturated, surface dry (SSD): ratio of the weight of the aggregate, including the weight of water in its permeable voids, to the weight of an equal volume of water = $\dfrac{B}{B - C}$

$$\text{Water adsorption}(\%) = \frac{B - A}{A}(100)$$

where

A = weight of oven-dry sample of aggregate in air

B = weight of saturated, surface-dry sample in air

C = weight of saturated sample in water

5.5.5.3 Specific Gravity of Fine Aggregates

Apparent specific gravity = $\dfrac{A}{B+A-C}$

Bulk specific gravity = $\dfrac{A}{B+D-C}$

Bulk specific gravity (SSD) = $\dfrac{D}{B+D-C}$

Adsorption = $\dfrac{D-A}{A}(100)$

where

A = weight of oven-dry specimen in air

B = weight of pycnometer filled with water

C = weight of pycnometer with specimen and water to calibration mark

D = weight of saturated surface-dry specimen

The percentage of combined aggregate passing a given sieve size (P) is calculated as

$$P = Aa + Bb + Cc + ...$$

where

A, B, C … = percentages of each aggregate that pass a given sieve size

a, b, c … = proportions of each aggregate needed to meet the requirements for material passing the given sieve (given that $a + b + c … = 100$)

The combined specific gravity, G, and absorption are calculated using

$$\text{Combined specific gravity } G = \frac{1}{\dfrac{a}{100G_A} + \dfrac{b}{100G_B} + ...} ...$$

Combined absorption = a Absorption$_A$ + b Absorption$_B$ + …

$$\text{Moisture content} = \frac{(\text{mass wet sand} - \text{mass dry sand})}{\text{mass dry sand}}$$

Saturated surface-dry weight of sand = SSD = Absorption × dry weight + dry weight

Source: Papagiannakis, A. T., and E. A. Masad. *Pavement Design and Materials*. Hobokon, NJ: John Wiley and Sons, 2008, p. 84-86.

5.5.6 Structural Design of Flexible Pavement

5.5.6.1 Asphalt Mixture Volumetrics

Effective specific gravity of aggregate:

$$G_{se} = \frac{100 - P_b}{\dfrac{100}{G_{mm}} - \dfrac{P_b}{G_b}}$$

Maximum specific gravity of the paving mixture:

$$G_{mm} = \frac{100}{\dfrac{P_s}{G_{se}} + \dfrac{P_b}{G_b}}$$

Asphalt absorption:

$$P_{ba} = 100G_b \frac{G_{se} - G_{sb}}{G_{sb}G_{se}}$$

Effective asphalt content:

$$P_{be} = P_b - \frac{P_{ba}}{100} P_s$$

Percent voids in compacted mineral aggregates:

$$VMA = 100 - \frac{G_{mb}P_s}{G_{sb}}$$

Percent air voids in compacted mixture:

$$P_a = \frac{G_{mm} - G_{mb}}{G_{mm}} \times 100$$

where

G_{se} = effective specific gravity of the aggregates

G_{mm} = maximum specific gravity of paving mixture (no air voids)

P_b = asphalt percent by total weight of paving mixture (thus, 100 – P_b is the percent by weight of the base mixture that is not asphalt)

G_b = specific gravity of the asphalt

P_s = percent by weight of aggregates in paving mixture

P_{ba} = amount of asphalt absorbed as a percentage of the total weight of aggregates

G_{sb} = bulk specific gravity of the aggregates

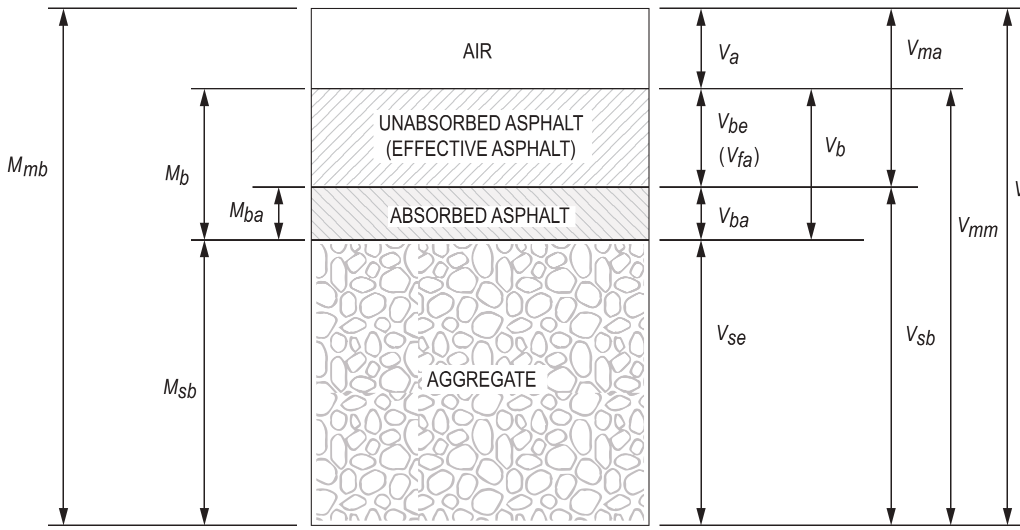
P_{be} = effective asphalt content in paving mixture (percent by weight)

VMA = percent voids in compacted mineral aggregates (percent of bulk volume)

P_a = percent air voids in compacted paving mixture

G_{mb} = bulk specific gravity of the compacted paving mixture

5.5.6.2 Asphalt Concrete Volumetric Terms and Definitions Using Phase Diagram



Phase Diagram

Source: Asphalt Institute. *MS2 Asphalt Mix Design Methods*. 7th ed. Lexington, KY: Asphalt Institute, 2015.

Asphalt Concrete Volumetric Terms

Term	Read as	Defined as
M_{mb}	Bulk mass of mixture	Total mass of component materials (asphalt and aggregate)
M_{sb}	Bulk mass of aggregate	Total mass of combined aggregate in asphalt concrete mixture
M_b	Mass of binder	Total mass of asphalt cement in asphalt concrete mixture
M_{ba}	Mass of absorbed binder	Mass of asphalt cement that is absorbed into the aggregate
V_{sb}	Bulk volume of aggregate	Volume of solid aggregate + total volume of void space in aggregate particle
V_{se}	Effective volume of aggregate	Volume of solid aggregate + total volume of void space in aggregate particle – volume of void space in aggregate particle containing asphalt
V_{sa}	Apparent volume of aggregate	Volume of solid aggregate only (not shown in diagram; $V_{sa} < V_{se} < V_{sb}$)
V_b	Volume of asphalt binder	Total volume of asphalt cement in asphalt concrete mixture
V_{ba}	Volume of absorbed binder	Volume of asphalt absorbed into aggregate
V_{be}	Effective volume of binder	Volume of asphalt binder in mixture that is not absorbed by the aggregate (Note that V_{be} is equal to V_{fa}.)
V_a	Volume of air	Volume of air in asphalt concrete mixture
V_{ma}	Volume of voids in mineral aggregate	Volume of void space between aggregate particles, interparticulate void spaces (does not include void spaces within the individual aggregate particles). This quantity is very similar to the %Voids quantity calculated from the DRUW test.
V_{fa}	Volume of V_{ma} filled with asphalt	Volume of total interparticulate void space that is filled with asphalt cement
V_{mm}	Voidless mix volume	Absolute volume occupied by only aggregate and asphalt cement (does not include volume occupied by air)
V_{mb}	Bulk volume	Total volume occupied by asphalt concrete mixture

Source: Asphalt Institute. *MS2 Asphalt Mix Design Methods*. 7th ed. Lexington, KY: Asphalt Institute, 2015.

Defined Quantities from Volumetric Values

Specific Gravities			
G_{se}	=	Effective specific gravity	$= \dfrac{M_{sb}}{V_{se} \times \rho_{H_2O}}$
G_{sb}	=	Bulk specific gravity (same as G_s(DRY))	$= \dfrac{M_{sb}}{V_{sb} \times \rho_{H_2O}}$
G_{mb}	=	Bulk specific gravity	$= \dfrac{M_{mb}}{V_{mb} \times \rho_{H_2O}}$
G_{mm}	=	Maximum specific gravity	$= \dfrac{M_{mb}}{V_{mm} \times \rho_{H_2O}}$
Volumetric Indices			
VMA	=	Voids in mineral aggregate	$= \dfrac{V_{ma}}{V_{mb}} \times 100$
VFA	=	Voids filled with asphalt	$= \dfrac{V_{fa}}{V_{ma}} \times 100$
VTM (% Air)	=	Air voids	$= \dfrac{V_a}{V_{mb}} \times 100 = \left(1 - \dfrac{G_{mb}}{G_{mm}}\right) \times 100$
Other			
% Absorption	=	Absorbed asphalt content	$= \dfrac{M_{ba}}{M_{sb}} \times 100$

Source: Asphalt Institute. *MS2 Asphalt Mix Design Methods*. 7th ed. Lexington, KY: Asphalt Institute, 2015.

Structural Layer Coefficients (a's)

Pavement Component	Coefficient
Wearing Surface	
Sand-mix asphaltic concrete	0.35
Hot-mix asphaltic concrete	0.44
Base	
Crushed stone	0.14
Dense-graded crushed stone	0.18
Soil cement	0.20
Emulsion/aggregate-bituminous	0.30
Portland-cement/aggregate	0.40
Lime-pozzolan/aggregate	0.40
Hot-mix asphaltic concrete	0.40
Subbase	
Crushed stone	0.11

Source: Mannering, Fred L. and Scott S. Washburn. *Principles of Highway Engineering and Traffic Analysis*. 6th ed. Hoboken, NJ: John Wiley and Sons, 2016, p.128.

5.5.7 Predicting Truck Traffic Volumes

The Average Annual Daily Truck Traffic for vehicles class c ($AADTT_c$) is obtained from:

$$AADTT_c = \frac{1}{7}\sum_{i=1}^{7}\left[\frac{1}{12}\sum_{j=1}^{12}\left(\frac{1}{n}\sum_{k=1}^{n} AADTT_{ijkc}\right)\right]$$

where

$AADTT_{ijkc}$ = average daily traffic volume for vehicle class c, for day k, for day of the week (DOW) i, and for month j

i = DOW, ranging from 1 to 7 for Monday to Sunday, respectively

j = month of the year, ranging from 1 to 12 for January to December, respectively

n = number of times data from a particular DOW is available for computing the average in a given month (i.e., 1, 2, 3, 4, or 5)

5.5.8 Monthly Adjustment Factor

$$MAF_j = \frac{AADTT_c}{VOL_{cj}}$$

where

MAF_j = monthly adjustment factor for month j

$AADTT_c$ = average annual daily truck traffic volume for vehicle class c

VOL_{cj} = average annual daily truck traffic volume for vehicle class c and month j that can be obtained from automatic vehicle classification data

General equation for the accumulation ESAL (Equivalent Single Axle Load) for each category of axle load is:

$$ESAL_i = f_d \times G_{rn} \times AADT_i \times 365 \times N_i \times F_{Ei}$$

where

$ESAL_i$ = equivalent accumulated 18,000-lb (80 kN) single-axle load for the axle category i

f_d = design lane factor

G_{rn} = growth factor for a given growth rate r and design period n

$AADT_i$ = first year annual average daily traffic for axle category i

N_i = number of axles on each vehicle in category i

F_{Ei} = load equivalency factor for axle category i

6 WATER RESOURCES AND ENVIRONMENTAL

6.1 Fluid Mechanics

6.1.1 Constants

6.1.1.1 Ideal Gas Constants

The universal gas constant, designated as $\overline{R}$ in the table below, relates pressure, volume, temperature, and number of moles of an ideal gas. When that universal constant, $\overline{R}$, is divided by the molecular weight of the gas, the result, often designated as R, has units of energy per degree per unit mass [kJ/(kg•K) or ft-lbf/(lbm-ºR)] and becomes characteristic of the particular gas. Some disciplines often use the symbol R to refer to the universal gas constant $\overline{R}$.

6.1.1.2 Fundamental Constants

Quantity		Symbol	Value	Units
electron charge		e	1.6022×10^{-19}	C (coulombs)
Faraday constant		F	96,485	coulombs/(mol)
gas constant	metric	$\overline{R}$	8,314	J/(kmol•K)
gas constant	metric	$\overline{R}$	8.314	kPa•m^3/(kmol•K)
gas constant	USCS	$\overline{R}$	1,545	ft-lbf/(lb mole-ºR)
		$\overline{R}$	0.08206	L•atm/(mole•K)
gravitation-Newtonian constant		G	6.673×10^{-11}	m^3/(kg•s^2)
gravitation-Newtonian constant		G	6.673×10^{-11}	N•m^2/kg^2
gravity acceleration (standard)	metric	g	9.807	m/s^2
gravity acceleration (standard)	USCS	g	32.174	ft/sec^2
molar volume (ideal gas), T = 273.15 K, p = 101.3 kPa		V_m	22,414	L/kmol
speed of light (exact)		c	299,792,458	m/s
Stefan-Boltzmann constant		σ	5.67×10^{-8}	W/(m^2•K^4)

6.1.2 Density, Specific Volume, Specific Weight, and Specific Gravity

The definitions of density, specific weight, and specific gravity follow:

$$SG = \frac{\gamma}{\gamma_w} = \frac{\rho}{\rho_w}$$

where

SG = specific gravity

ρ = density (also called mass density) = $\frac{m}{V}$

m = mass of volume

V = volume of object considered

γ = specific weight (lbf/ft^3) = ρg

ρ_w = mass density of water at standard conditions = 62.4 lbm/ft^3, 1.94 slug/ft^3, or 1,000 kg/m^3

1 slug = 1 lbf-sec^2/ft

γ_w = specific weight of water at standard conditions = 62.4 lbf/ft^3, or 9,807 N/m^3

6.1.3 Stress, Pressure, and Viscosity

Stress is defined as

$$\tau(1) = \underset{\Delta A \to 0}{\text{limit}} \frac{\Delta F}{\Delta A}$$

where

$\tau(1)$ = surface stress vector at Point 1

ΔF = force acting on infinitesimal area ΔA

ΔA = infinitesimal area at Point 1

τ = surface stress = $\frac{F}{A}$

F = force acting on area

A = area

τ_n = $-P$

τ_t = $\mu(dv/dy)$ (one-dimensional; i.e., y)

where

τ_n and τ_t = normal and tangential stress components at Point 1, respectively

P = pressure at Point 1

μ = absolute dynamic viscosity of the fluid [N•s/m^2 or lbf-sec/ft^2]

dv = differential velocity

dy = differential distance, normal to boundary

v = velocity at boundary condition

y = normal distance, measured from boundary

ν = kinematic viscosity (m^2/s or ft^2/sec)

where $\nu = \frac{\mu}{\rho}$

For a thin Newtonian fluid film and a linear velocity profile:

$v(y) = vy/\delta$; $dv/dy = v/\delta$

where

v = velocity of plate on film

δ = thickness of fluid film

For a power law (non-Newtonian) fluid:

$\tau_t = K(dv/dy)^n$

where

K = consistency index

n = power law index

$n < 1 \equiv$ pseudo plastic

$n > 1 \equiv$ dilatant

6.1.4 Characteristics of a Static Liquid

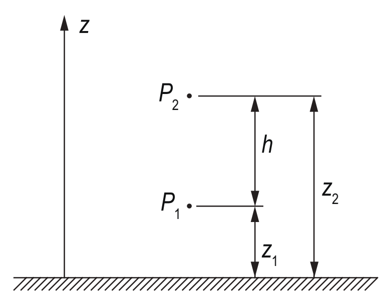
6.1.4.1 The Pressure Field in a Static Liquid

The difference in pressure between two different points is

$$P_2 - P_1 = -\gamma\,(z_2 - z_1) = -\gamma h = -\rho g h$$

Absolute pressure = atmospheric pressure + gauge pressure reading

Absolute pressure = atmospheric pressure – vacuum gauge pressure reading



Source: Bober, W., and Kenyon, R.A., *Fluid Mechanics*, John Wiley and Sons, 1980, Fig. 3-6, p. 49.
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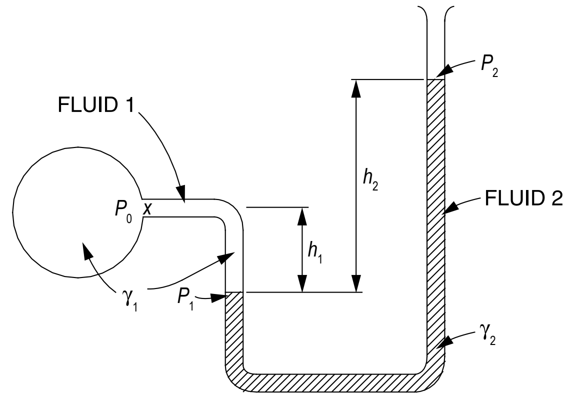
6.1.4.2 Manometers



Source: Bober, W., and Kenyon, R.A., *Fluid Mechanics*, John Wiley and Sons, 1980, Fig. 3-7, p. 49.
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For a simple manometer:

$$P_0 = P_2 + \gamma_2 h_2 - \gamma_1 h_1 = P_2 + g\,(\rho_2\, h_2 - \rho_1\, h_1)$$

If $h_1 = h_2 = h$,

$$P_0 = P_2 + (\gamma_2 - \gamma_1)h = P_2 + (\rho_2 - \rho_1)gh$$

Note that the difference between the two densities is used.

P = pressure

γ = specific weight of fluid

h = height

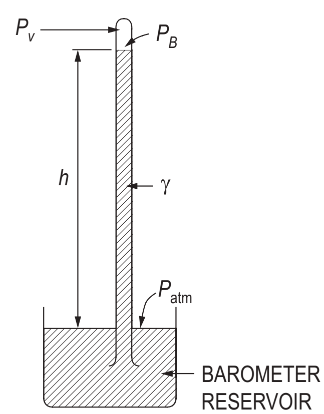
g = acceleration due to gravity

ρ = fluid density

6.1.4.3 Barometers

Another device that works on the same principle as the manometer is the simple barometer.

$$P_{\text{atm}} = P_A = P_v + \gamma h = P_B + \gamma h = P_B + \rho g h$$



P_v = vapor pressure of the barometer fluid

Source: Bober, W., and Kenyon, R.A., *Fluid Mechanics*, John Wiley and Sons, 1980, Fig. 3-8, p. 51.
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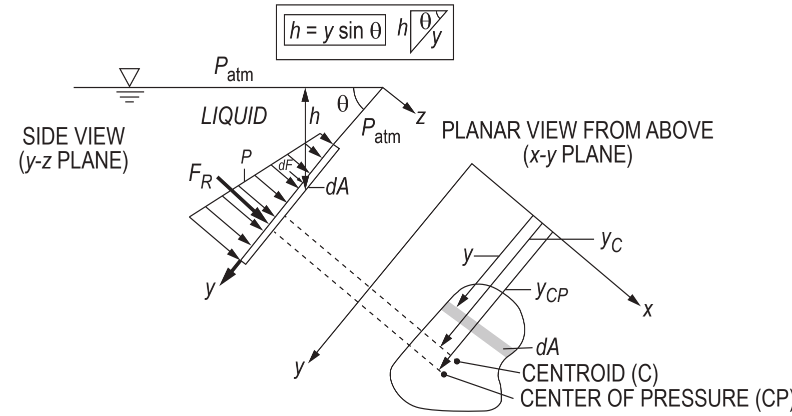
6.1.4.4 Forces on Submerged Surfaces and Center of Pressure



Submerged Plane Surface

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The pressure on a point at a vertical distance h below the surface is

$$P = P_{atm} + \rho g h, \text{ for } h \geq 0$$

where

P = pressure

P_{atm} = atmospheric pressure

P_C = pressure at the centroid of area

P_{CP} = pressure at center of pressure

y_C = slant distance from liquid surface to the centroid of area

y_C = $h_C/\sin\theta$

h_C = vertical distance from liquid surface to centroid of area

y_{CP} = slant distance from liquid surface to center of pressure

h_{CP} = vertical distance from liquid surface to center of pressure

θ = angle between liquid surface and edge of submerged surface

I_{xC} = moment of inertia about the centroidal x-axis

If atmospheric pressure acts above the liquid surface and on the nonwetted side of the submerged surface:

$$y_{CP} = y_C + I_{xC}/y_C A$$

$$y_{CP} = y_C + \rho g \sin\theta \, I_{xC}/P_C A$$

Wetted side: $F_R = (P_{\text{atm}} + \rho g y_C \sin\theta)A$

P_{atm} acting both sides: $F_{R\text{net}} = (\rho g y_C \sin\theta)A$

6.1.4.5 Archimedes Principle and Buoyancy

$$F_B = \gamma V_f$$

where

F_B = buoyancy force (lbf)

γ = specific weight of fluid (lbf/ft^3)

V_f = volume of dissipated fluid (ft^3)

1. The buoyant force exerted on a submerged or floating body is equal to the weight of the fluid displaced by the body.
2. A floating body displaces a weight of fluid equal to its own weight; i.e., a floating body is in equilibrium.

The *center of buoyancy* is located at the centroid of the displaced fluid volume.

In the case of a body lying at the *interface of two immiscible fluids*, the buoyant force equals the sum of the weights of the fluids displaced by the body.

6.1.5 Chemistry

6.1.5.1 Definitions

Avogadro's Number—The number of elementary particles in a mol of a substance

1 mol = 1 gram mole

1 mol = 6.02×10^{23} particles

Molarity of Solutions—The number of gram moles of a substance dissolved in a liter of solution

Molality of Solutions—The number of gram moles of a substance per 1,000 grams of solvent

Normality of Solutions—The product of the molarity of a solution and the number of valence changes taking place in a reaction

Molar Volume of an Ideal Gas [at 0°C (32°F) and 1 atm (14.7 psia)]; 22.4 L/(g mole) [359 ft^3/(lb mole)]:

$$K_{EQ} = \frac{[C]^c[D]^d}{[A]^a[B]^b}$$

where [x] is the thermodynamic activity of x unless otherwise noted

[x] = concentration of x in dilute solution, or

= partial pressure of x, or

= 1 for solids and liquids

Equilibrium Constant of a Chemical Reaction

$$aA + bB \rightleftarrows cC + dD$$

$$K_{eq} = \frac{[C]^c[D]^d}{[A]^a[B]^b}$$

Le Chatelier's Principle for Chemical Equilibrium—When a stress (such as a change in concentration, pressure, or temperature) is applied to a system in equilibrium, the equilibrium shifts in such a way that it tends to relieve the stress.

Heats of Reaction, Solution, Formation, and Combustion—Chemical processes generally involve the absorption or evolution of heat. In an endothermic process, heat is absorbed (enthalpy change is positive). In an exothermic process, heat is evolved (enthalpy change is negative).

Solubility Product of a slightly soluble substance *AB*:

$$A_mB_n \rightarrow mA^{n+} + nB^{m-}$$

Solubility Product Constant = $K_{SP} = [A^+]^m\,[B^-]^n$

Faraday's Equation:

$$m = \left(\frac{Q}{F}\right)\left(\frac{M}{z}\right)$$

where

m = mass (grams) of substance liberated at electrode

Q = total electric charge passed through electrolyte (coulomb or ampere•second)

F = 96,485 coulombs/mol

M = molar mass of the substance (g/mol)

z = valence number

A *catalyst* is a substance that alters the rate of a chemical reaction. The catalyst does not affect the position of equilibrium of a reversible reaction.

The *atomic number* is the number of protons in the atomic nucleus.

Boiling Point Elevation—The presence of a nonvolatile solute in a solvent raises the boiling point of the resulting solution.

Freezing Point Depression—The presence of a solute lowers the freezing point of the resulting solution.

6.1.5.2 Acids, Bases, and pH (Aqueous Solutions)

$$\text{pH} = \log_{10}\left(\frac{1}{[H^+]}\right)$$

where

$[H^+]$ = molar concentration of hydrogen ion (gram moles/liter)

Acids have pH < 7.

Bases have pH > 7.

Periodic Table of Elements

Atomic Number
Symbol
Atomic Weight

I	II											III	IV	V	VI	VII	VIII
1 **H** 1.0079																	2 **He** 4.0026
3 **Li** 6.941	4 **Be** 9.0122											5 **B** 10.811	6 **C** 12.011	7 **N** 14.007	8 **O** 15.999	9 **F** 18.998	10 **Ne** 20.179
11 **Na** 22.990	12 **Mg** 24.305											13 **Al** 26.981	14 **Si** 28.086	15 **P** 30.974	16 **S** 32.066	17 **Cl** 35.453	18 **Ar** 39.948
19 **K** 39.098	20 **Ca** 40.078	21 **Sc** 44.956	22 **Ti** 47.88	23 **V** 50.941	24 **Cr** 51.996	25 **Mn** 54.938	26 **Fe** 55.847	27 **Co** 58.933	28 **Ni** 58.69	29 **Cu** 63.546	30 **Zn** 65.39	31 **Ga** 69.723	32 **Ge** 72.61	33 **As** 74.921	34 **Se** 78.96	35 **Br** 79.904	36 **Kr** 83.80
37 **Rb** 85.468	38 **Sr** 87.62	39 **Y** 88.906	40 **Zr** 91.224	41 **Nb** 92.906	42 **Mo** 95.94	43 **Tc** (98)	44 **Ru** 101.07	45 **Rh** 102.91	46 **Pd** 106.42	47 **Ag** 107.87	48 **Cd** 112.41	49 **In** 114.82	50 **Sn** 118.71	51 **Sb** 121.75	52 **Te** 127.60	53 **I** 126.90	54 **Xe** 131.29
55 **Cs** 132.91	56 **Ba** 137.33	57–71	72 **Hf** 178.49	73 **Ta** 180.95	74 **W** 183.85	75 **Re** 186.21	76 **Os** 190.2	77 **Ir** 192.22	78 **Pt** 195.08	79 **Au** 196.97	80 **Hg** 200.59	81 **Tl** 204.38	82 **Pb** 207.2	83 **Bi** 208.98	84 **Po** (209)	85 **At** (210)	86 **Rn** (222)
87 **Fr** (223)	88 **Ra** 226.02	89–103	104 **Rf** (261)	105 **Db** (262)	106 **Sg** (266)	107 **Bh** (264)	108 **Hs** (269)	109 **Mt** (268)	110 **Ds** (269)	111 **Rg** (272)	112 **Cn** (277)	113 **Uut** unknown	114 **Fl** (289)	115 **Uup** unknown	116 **Lv** (298)	117 **Uus** unknown	118 **Uuo** unknown

Lanthanide Series	57 **La** 138.91	58 **Ce** 140.12	59 **Pr** 140.91	60 **Nd** 144.24	61 **Pm** (145)	62 **Sm** 150.36	63 **Eu** 151.96	64 **Gd** 157.25	65 **Tb** 158.92	66 **Dy** 162.50	67 **Ho** 164.93	68 **Er** 167.26	69 **Tm** 168.93	70 **Yb** 173.04	71 **Lu** 174.97
Actinide Series	89 **Ac** 227.03	90 **Th** 232.04	91 **Pa** 231.04	92 **U** 238.03	93 **Np** 237.05	94 **Pu** (244)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (251)	99 **Es** (252)	100 **Fm** (257)	101 **Md** (258)	102 **No** (259)	103 **Lr** (260)

6.1.5.3 Common Chemicals in Water and Wastewater Processing

Common Chemicals in Water and Wastewater Processing

Name	Formula	Common Application	Molecular Weight	Equivalent Weight
Activated carbon	C	Taste and odor control	12.0	n.a.[a]
Aluminum sulfate	$Al_2(SO_4)_3 \cdot 14.3\ H_2O$	Coagulation	600	100
Ammonia	NH_3	Chloramine disinfection	17.0	n.a.
Ammonium sulfate	$(NH_4)_2SO_4$	Coagulation	132	66.1
Calcium hydroxide	$Ca(OH)_2$	Softening	74.1	37.0
Calcium hypochlorite	$Ca(ClO)_2 \cdot 2H_2O$	Disinfection	179	n.a.
Calcium oxide	CaO	Softening	56.1	28.0
Carbon dioxide	CO_2	Recarbonation	44.0	22.0
Chlorine	Cl_2	Disinfection	71.0	n.a.
Chlorine dioxide	ClO_2	Taste and odor control	67.0	n.a.
Copper sulfate	$CuSO_4$	Algae control	160	79.8
Ferric chloride	$FeCl_3$	Coagulation	162	54.1
Ferric sulfate	$Fe_2(SO_4)_3$	Coagulation	400	66.7
Ferrous sulfate	$FeSO_4 \cdot 7H_2O$	Coagulation	278	139
Fluosilicic acid	H_2SiF_6	Fluoridation	144	n.a.
Magnesium hydroxide	$Mg(OH)_2$	Defluoridation	58.3	29.2
Oxygen	O_2	Aeration	32.0	16.0
Potassium permanganate	$KMnO_4$	Oxidation	158	n.a.
Sodium aluminate	$NaAlO_2$	Coagulation	82.0	n.a.
Sodium bicarbonate	$NaHCO_3$	pH adjustment	84.0	84.0
Sodium carbonate	Na_2CO_3	Softening	106	53.0
Sodium chloride	$NaCl$	Ion exchanger regeneration	58.4	58.4
Sodium fluoride	NaF	Fluoridation	42.0	n.a.
Sodium fluosilicate	Na_2SiF_6	Fluoridation	188	n.a.
Sodium hexameta-phosphate	$(NaPO_3)_n$	Corrosion control	n.a.	n.a.
Sodium hydroxide	$NaOH$	pH adjustment	40.0	40.0
Sodium hypochlorite	$NaClO$	Disinfection	74.4	n.a.
Sodium silicate	Na_2SiO_4	Coagulation aid	184	n.a.
Sodium thiosulfate	$Na_2S_2O_3$	Dechlorination	158	n.a.
Sulfur dioxide	SO_2	Dechlorination	64.1	n.a.
Sulfuric acid	H_2SO_4	pH adjustment	98.1	49.0

[a]Not applicable.

Source: Viessman, W., and M. J. Hammer. *Water Supply and Pollution Control*. 4th ed. New York: Pearson, 1985, Table 11.1, p. 353.

6.1.5.4 Chemical Reaction Equilibria

Definitions

Conversion—moles reacted/moles fed

Extent—For each species in a reaction, the mole balance may be written

$$\text{moles}_{i,\text{out}} = \text{moles}_{i,\text{in}} + v_i\xi$$

where

ξ = extent in moles

v_i = stoichiometric coefficient of the *i*th species, the sign of which is negative for reactants and positive for products

Limiting reactant—Reactant that would be consumed first if the reaction proceeded to completion. Other reactants are *excess reactants*.

Selectivity—Moles of desired product formed/moles of undesired product formed

Yield—Moles of desired product formed/moles that would have been formed if there were no side reactions and the limiting reactant had reacted completely.

6.1.5.5 Data on Selected Elements, Radicals, and Compounds

Data on Selected Elements, Radicals, and Compounds

Name	Symbol or Formula	Atomic or Molecular Weight	Equivalent Weight
Aluminum	Al^{3+}	27.0	9.0
Calcium	Ca^{2+}	40.1	20.0
Carbon	C	12.0	
Chloride	Cl^{-}	35.5	35.5
Hydrogen	H^{+}	1.0	1.0
Magnesium	Mg^{2+}	24.3	12.2
Manganese	Mn^{2+}	54.9	27.5
Nitrogen	N	14.0	
Oxygen	O	16.0	
Phosphorus	P	31.0	
Sodium	Na^{+}	23.0	23.0
Ammonium	NH_4^{+}	18.0	18.0
Bicarbonate	HCO_3^{-}	61.0	61.0
Carbonate	CO_3^{2-}	60.0	30.0
Hydroxyl	OH^{-}	17.0	17.0
Hypochlorite	OCl^{-}	51.5	51.5
Nitrate	NO_3^{-}	62.0	62.0
Orthophosphate	PO_4^{3-}	95.0	31.7
Sulfate	SO_4^{2-}	96.0	48.0
Aluminum hydroxide	$Al(OH)_3$	78.0	26.0
Calcium bicarbonate	$Ca(HCO_3)_2$	162	81.0
Calcium carbonate	$CaCO_3$	100	50.0
Calcium sulfate	$CaSO_4$	136	68.0
Carbon dioxide	CO_2	44.0	22.0
Ferric hydroxide	$Fe(OH)_3$	107	35.6
Hydrochloric acid	HCl	36.5	36.5
Magnesium carbonate	$MgCO_3$	84.3	42.1
Magnesium hydroxide	$Mg(OH)_2$	58.3	29.1
Magnesium sulfate	$MgSO_4$	120	60.1
Sodium sulfate	Na_2SO_4	142	71.0

Source: Adapted from Viessman, W., and M. J. Hammer. *Water Supply and Pollution Control*. 4th ed. New York: Pearson, 1985, Table 11.2, p. 355.

6.1.5.6 Chemical Reaction Equilibrium

For the reaction:

$$aA + bB \rightleftharpoons cC + dD$$

$$\Delta G^{o} = -RT \ln K_a$$

$$K_a = \frac{\left(\hat{a}_C^c\right)\left(\hat{a}_D^d\right)}{\left(\hat{a}_A^a\right)\left(\hat{a}_B^b\right)} = \prod_i \left(\hat{a}_i\right)^{v_i}$$

where

$\hat{a}_i$ = activity of component $i = \dfrac{\hat{f}_i}{f_i^{\circ}}$

$\hat{f}_i$ = fugacity of pure i in its standard state at the equilibrium reaction temperature T

v_i = stoichiometric coefficient of component i

ΔG°= standard Gibbs energy change of reaction

K_a = chemical equilibrium constant

For mixtures of ideal gases:

f_i° = unit pressure, often 1 bar

$\hat{f}_i = y_i P = p_i$

where p_i = partial pressure of component i

Then $K_a = K_p = \dfrac{\left(p_C^c\right)\left(p_D^d\right)}{\left(p_A^a\right)\left(p_B^b\right)} = P^{c+d-a-b} \dfrac{\left(y_C^c\right)\left(y_D^d\right)}{\left(y_A^a\right)\left(y_B^b\right)}$

For solids:

$\hat{a}_i = 1$

For liquids:

$\hat{a}_i = x_i \gamma_i$

The effect of temperature on the equilibrium constant is

$$\frac{d \ln K}{dT} = \frac{\Delta H^{\circ}}{RT^2}$$

where ΔH° = standard enthalpy change of reaction

6.1.5.7 Chemical Reaction Engineering

Nomenclature

A chemical reaction may be expressed by the general equation

$$aA + bB \leftrightarrow cC + dD$$

The rate of reaction of any component is defined as the moles of that component formed per unit time per unit volume.

$$-r_A = -\frac{1}{V}\frac{dN_A}{dt} \qquad \text{(negative because A disappears)}$$

$$-r_A = \frac{-dC_A}{dt} \qquad \text{if } V \text{ is constant}$$

The rate of reaction is frequently expressed by

$$-r_A = kf_r\,(C_A, C_B, ...)$$

where

k = reaction rate constant

C_I = concentration of component I

In the conversion of A, the fractional conversion X_A is defined as the moles of A reacted per mole of A fed.

$$X_A = (C_{A0} - C_A)/C_{A0} \qquad \text{if } V \text{ is constant}$$

The Arrhenius equation gives the dependence of k on temperature

$$k = Ae^{-E_a/\bar{R}T}$$

where

A = pre-exponential or frequency factor

E_a = activation energy (J/mol, cal/mol)

T = temperature (K)

$\bar{R}$ = gas law constant = 8.314 J/(mol•K)

For values of rate constant (k_i) at two temperatures (T_i),

$$E_a = \frac{RT_1T_2}{(T_1 - T_2)}\ln\left(\frac{k_1}{k_2}\right)$$

Reaction Order

If

$$-r_A = kC_A^xC_B^y$$

the reaction is x order with respect to reactant A and y order with respect to reactant B. The overall order is

$$n = x + y$$

6.1.5.8 Batch Reactor, Constant Volume

For a well-mixed, constant-volume batch reactor

$$-r_A = -dC_A/dt$$

$$t = C_{A0}\int_0^{X_A} dX_A/(-r_A)$$

Zero-Order Irreversible Reaction

$$-r_A = kC_A^0 = k(1)$$

$$-dC_A/dt = k \qquad \text{or}$$

$$C_A = C_{A0} - kt$$

$$dX_A/dt = k/C_{A0} \qquad \text{or}$$

$$C_{A0}X_A = kt$$

First-Order Irreversible Reaction

$$-r_A = kC_A$$

$$-dC_A/dt = kC_A \qquad \text{or}$$

$$\ln(C_A/C_{A0}) = -kt$$

$$dX_A/dt = k(1-X_A) \qquad \text{or}$$

$$\ln(1-X_A) = -kt$$

Second-Order Irreversible Reaction

$$-r_A = kC_A^2$$

$$-dC_A/dt = kC_A^2 \qquad \text{or}$$

$$1/C_A - 1/C_{A0} = kt$$

$$dX_A/dt = kC_{A0}(1-X_A)^2 \qquad \text{or}$$

$$X_A/\left[C_{A0}(1-X_A)\right] = kt$$

First-Order Reversible Reactions

$$A \underset{k_2}{\overset{k_1}{\rightleftarrows}} R$$

$$-r_A = -\frac{dC_A}{dt} = k_1 C_A - k_2 C_R$$

$$K_c = k_1/k_2 = \hat{C}_R/\hat{C}_A$$

$$M = C_{R_0}/C_{A_0}$$

$$\frac{dX_A}{dt} = \frac{k_1(M+1)}{M+\hat{X}_A}\left(\hat{X}_A - X_A\right)$$

$$-\ln\left(1-\frac{X_A}{\hat{X}_A}\right) = -\ln\frac{C_A-\hat{C}_A}{C_{A_0}-\hat{C}_A}$$

$$= \frac{(M+1)}{\left(M+\hat{X}_A\right)}k_1 t$$

$\hat{X}_A$ is the equilibrium conversion.

6.1.5.9 Batch Reactor, Variable Volume

If the volume of the reacting mass varies with the conversion (such as a variable-volume batch reactor) according to

$$V = V_{X_A = 0}\left(1 + \varepsilon_A X_A\right)$$

(i.e., at constant pressure)

where

$$\varepsilon_A = \frac{V_{X_A = 1} - V_{X_A = 0}}{V_{X_A = 0}} = \frac{\Delta V}{V_{X_A = 0}}$$

then at any time,

$$C_A = C_{A0}\left[\frac{1 - X_A}{1 + \varepsilon_A X_A}\right]$$

and

$$t = C_{A0}\int_0^{X_A} dX_A/\left[\left(1 + \varepsilon_A X_A\right)\left(-r_A\right)\right]$$

For a first-order irreversible reaction:

$$kt = -\ln\left(1 - X_A\right) = -\ln\left(1 - \frac{\Delta V}{\varepsilon_A V_{XA = 0}}\right)$$

6.1.5.10 Flow Reactors, Steady State

Space-time τ is defined as the reactor volume divided by the inlet volumetric feed rate. Space-velocity SV is the reciprocal of space-time, $SV = 1/\tau$.

Plug-Flow Reactor (PFR)

$$\tau = \frac{C_{A0} V_{PFR}}{F_{A0}} = C_{A0}\int_0^{X_A} \frac{dX_A}{\left(-r_A\right)}$$

where F_{A0} = moles of A fed per unit time

Continuous-Stirred Tank Reactor (CSTR)

For a constant-volume, well-mixed CSTR:

$$\frac{\tau}{C_{A0}} = \frac{V_{CSTR}}{F_{A0}} = \frac{X_A}{-r_A}$$

where $-r_A$ is evaluated at exit stream conditions.

Continuous-Stirred Tank Reactors in Series

With a first-order reaction $A \rightarrow R$, there is no change in volume:

$$\tau_{N\text{-reactors}} = N\tau_{\text{individual}}$$

$$= \frac{N}{k}\left[\left(\frac{C_{A0}}{C_{AN}}\right)^{1/N} - 1\right]$$

where

N = number of CSTRs (equal volume) in series

C_{AN} = concentration of A leaving the Nth CSTR

6.1.6 Population Projection

6.1.6.1 Linear Projection = Algebraic Projection

$$P_t = P_0 + k\Delta t$$

where

P_t = population at time t

P_0 = population at time zero

k = growth rate

Δt = elapsed time relative to time zero (years)

6.1.6.2 Log Growth = Exponential Growth = Geometric Growth

$$P_t = P_0 e^{k\Delta t}$$

$$\ln P_t = \ln P_0 + k\Delta t$$

where

P_t = population at time t

P_0 = population at time zero

k = growth rate

Δt = elapsed time relative to time zero (years)

6.1.6.3 Percent Growth

$$P_t = P_0 (1 + k)^n$$

where

P_t = population at time t

P_0 = population at time zero

k = growth rate

n = number of periods

6.1.6.4 Ratio and Correlation Growth

$$\frac{P_2}{P_{2R}} = \frac{P_1}{P_{1R}} = k$$

where

P_2 = projected population

P_{2R} = projected population of larger region

P_1 = population at last census

P_{1R} = population of larger region at last census

k = growth ratio constant

6.1.6.5 Decreasing-Rate-of-Increase Growth

$$P_t = P_0 + (S - P_0)(1 - e^{-k\,(t - t_0)})$$

where

P_t = population at time t

P_0 = population at time zero

k = growth rate constant

S = saturation population

t = future time

t_0 = initial time

6.2 Hydraulics

6.2.1 Principles of One-Dimensional Fluid Flow

6.2.1.1 Continuity Equation

So long as the flow Q is continuous, the *continuity equation*, as applied to one-dimensional flows, states that the flow passing two points (1 and 2) in a stream is equal at each point, $A_1 v_1 = A_2 v_2$.

$$Q = Av$$

$$\dot{m} = \rho Q = \rho A v$$

where

Q = volumetric flow rate

$\dot{m}$ = mass flow rate

A = cross-sectional area of flow

v = average flow velocity

ρ = fluid density

For steady, one-dimensional flow, $\dot{m}$ is a constant. If, in addition, the density is constant, then Q is constant.

6.2.1.2 Energy Equation

The energy equation for steady incompressible flow with no shaft device is

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f \quad \text{or}$$

$$\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f$$

where h_f = the head loss, considered a friction effect, and all remaining terms are defined above.

If the cross-sectional area and the elevation of the pipe are the same at both sections (1 and 2), then $z_1 = z_2$ and $v_1 = v_2$.

The pressure drop $P_1 - P_2$ is given by the following:

$$P_1 - P_2 = \gamma\, h_f = \rho g h_f$$

6.2.1.3 Field Equation

The field equation is derived when the energy equation is applied to one-dimensional flows. Assuming no friction losses and that no pump or turbine exists between sections 1 and 2 in the system,

$$\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 \quad \text{or}$$

$$\frac{P_2}{\rho} + \frac{v_2^2}{2} + z_2 g = \frac{P_1}{\rho} + \frac{v_1^2}{2} + z_1 g$$

where

P_1, P_2 = pressure at sections 1 and 2

v_1, v_2 = average velocity of the fluid at the sections

z_1, z_2 = vertical distance from a datum to the sections (the potential energy)

γ = specific weight of the fluid = ρg

g = acceleration due to gravity

ρ = fluid density

6.2.1.4 Hydraulic Gradient (Grade Line)

Hydraulic grade line is the line connecting the sum of pressure and elevation heads at different points in conveyance systems. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

6.2.1.5 Energy Line (Bernoulli Equation)

The Bernoulli equation states that the sum of the pressure, velocity, and elevation heads is constant. The energy line is this sum or the "total head line" above a horizontal datum. The difference between the hydraulic grade line and the energy line is the $v^2/2g$ term.

$$H = z_a + \frac{P_a}{\gamma} + \frac{{v_a}^2}{2g}$$

where

H = total head (ft)

$\frac{P_a}{\gamma}$ = pressure head

P_a = pressure (lbf/ft^2)

γ = specific weight of fluid (lbf/ft^3)

$\frac{{v_a}^2}{2g}$ = velocity head

v_a = velocity (ft/sec)

g = acceleration due to gravity (32.2 ft/sec^2)

z_a = elevation (ft)

6.2.1.6 Properties of Water (USCS Units)

Physical Properties of Water

Temperature (°F)	Specific Weight, γ (lbf/ft^3)	Mass Density, ρ (lbf-sec^2/ft^4) (slug/ft^3)	Dynamic/Absolute Viscosity, μ ($\times 10^{-5}$ lbf-sec/ft^2)	Kinematic Viscosity, ν ($\times 10^{-5}$ ft^2/sec)	Vapor Pressure, P_v (psi)	Surface Tension, σ (lbf/ft)
32	62.42	1.940	3.746	1.931	0.09	0.00518
40	62.43	1.940	3.229	1.664	0.12	0.00514
50	62.41	1.940	2.735	1.410	0.18	0.00509
60	62.37	1.938	2.359	1.217	0.26	0.00504
70	62.30	1.936	2.050	1.059	0.36	0.00498
80	62.22	1.934	1.799	0.930	0.51	0.00492
90	62.11	1.931	1.595	0.826	0.70	0.00486
100	62.00	1.927	1.424	0.739	0.95	0.00480
110	61.86	1.923	1.284	0.667	1.24	
120	61.71	1.918	1.168	0.609	1.69	
130	61.55	1.913	1.069	0.558	2.22	
140	61.38	1.908	0.981	0.514	2.89	
150	61.20	1.902	0.905	0.476	3.72	
160	61.00	1.896	0.838	0.442	4.74	
170	60.80	1.890	0.780	0.413	5.99	
180	60.58	1.883	0.726	0.385	7.51	
190	60.36	1.876	0.678	0.362	9.34	
200	60.12	1.868	0.637	0.341	11.52	
212	59.83	1.860	0.593	0.319	14.70	

Source: Vennard, John K., and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., John Wiley and Sons, 1982, p. 663.
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6.2.1.7 Properties of Water (SI-Metric Units)

Properties of Water (SI Metric Units)

Temperature (°C)	Specific Weight γ (kN/m^3)	Density ρ (kg/m^3)	Dynamic/Absolute Viscosity, μ (Pa•s)	Kinematic Viscosity ν (m^2/s)	Vapor Pressure P_v (kPa)
0	9.805	999.8	0.001781	0.000001785	0.61
5	9.807	1000.0	0.001518	0.000001518	0.87
10	9.804	999.7	0.001307	0.000001306	1.23
15	9.798	999.1	0.001139	0.000001139	1.70
20	9.789	998.2	0.001002	0.000001003	2.34
25	9.777	997.0	0.000890	0.000000893	3.17
30	9.764	995.7	0.000798	0.000000800	4.24
40	9.730	992.2	0.000653	0.000000658	7.38
50	9.689	988.0	0.000547	0.000000553	12.33
60	9.642	983.2	0.000466	0.000000474	19.92
70	9.589	977.8	0.000404	0.000000413	31.16
80	9.530	971.8	0.000354	0.000000364	47.34
90	9.466	965.3	0.000315	0.000000326	70.10
100	9.399	958.4	0.000282	0.000000294	101.33

Source: Vennard, John K., and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., John Wiley and Sons, 1982, p. 664. Reproduced with permission of the Licensor through PLSclear.

6.2.2 Fluid Flow Characterization

6.2.2.1 Reynolds Number—Open Channel (Newtonian Fluid)

$$Re = \frac{vR}{\nu}$$

where

Re = Reynolds number

v = mean velocity of flow (ft/sec)

R = hydraulic radius (ft)

ν = kinematic viscosity (ft^2/sec)

The critical Reynolds number $(Re)_c$ is defined to be the minimum Reynolds number at which a flow will turn turbulent.

Flow through an open channel is generally characterized as:

Laminar flow $Re < 500$

Transitional flow $500 < Re < 2{,}000$

Fully turbulent flow $Re \geq 2{,}000$

where f = friction factor

A chart that gives f versus Re for various values of ε/D, known as a Moody, Darcy, or Stanton friction factor diagram, is available in this section.

6.2.2.2 Reynolds Number—Circular Pipes

$$Re = \frac{vD\rho}{\mu} = \frac{vD}{\nu}$$

$$Re' = \frac{v^{(2-n)} D^n \rho}{K\left(\frac{3n+1}{4n}\right)^n 8^{(n-1)}}$$

where

v = fluid velocity

ρ = mass density

D = diameter of the pipe, dimension of the fluid streamline, or characteristic length

μ = dynamic viscosity

ν = kinematic viscosity

Re = Reynolds number (Newtonian fluid)

Re' = Reynolds number (Power law fluid)

K = consistency index

n = power law index

Flow through a closed nonpressurized pipe is generally characterized as

Laminar flow	$Re < 2{,}100$
Transitional flow	$2{,}100 < Re < 10{,}000$
Fully turbulent flow	$Re \geq 10{,}000$

Flow through a closed pressurized pipe is generally characterized as

Laminar flow	$Re < 2{,}000$
Transitional flow	$2{,}000 < Re < 4{,}000$
Fully turbulent flow	$Re \geq 4{,}000$

The velocity distribution for *laminar flow* in circular tubes or between planes is

$$v(r) = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where

r = distance (m) from the centerline

R = radius (m) of the tube, or half the distance between the parallel planes

v = local velocity (m/s) at r

v_{max} = velocity (m/s) at the centerline of the duct

$v_{max} = 1.18\overline{v}$, for fully turbulent flow

$v_{max} = 2\bar{v}$, for circular tubes in laminar flow

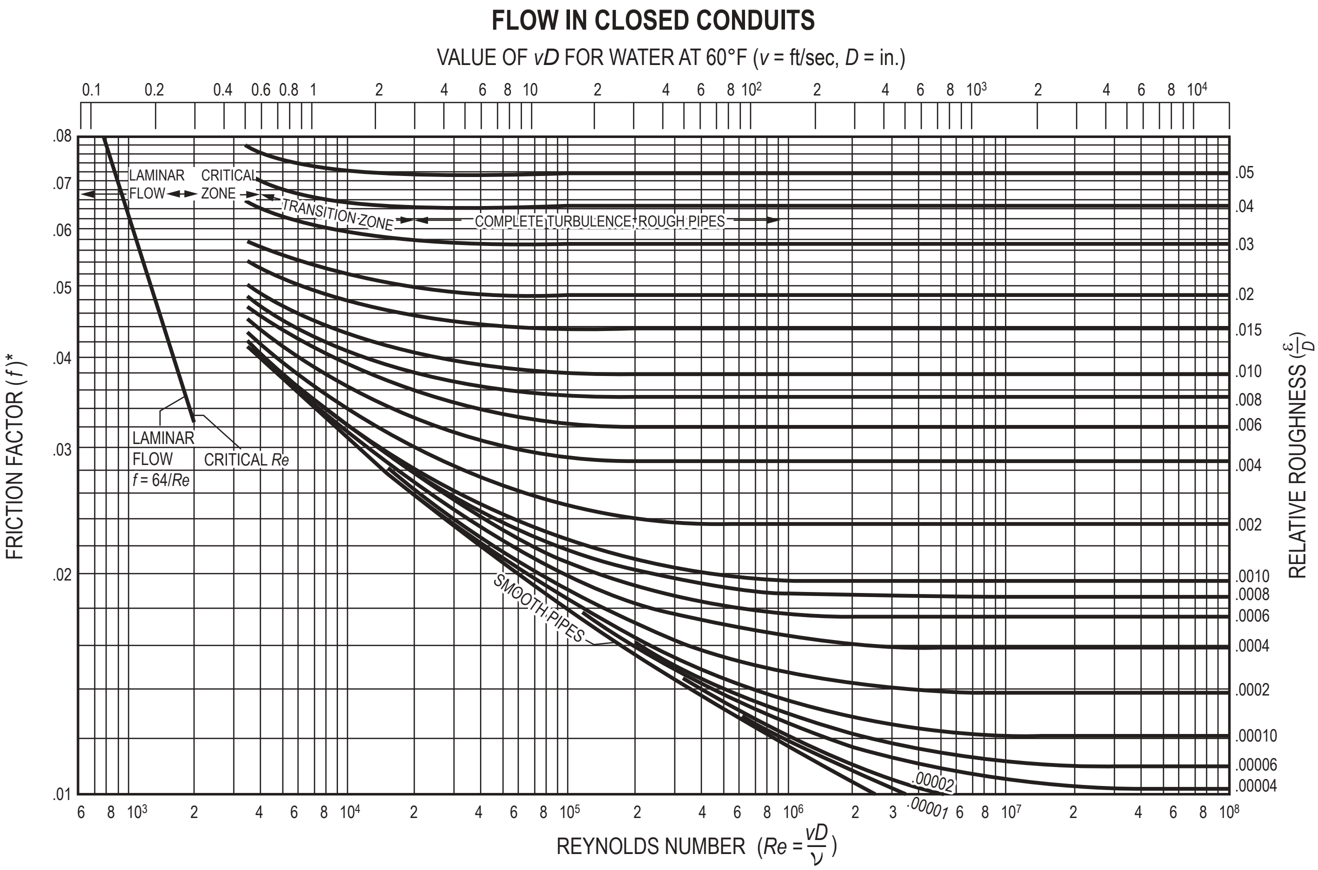
$v_{max} = 1.5\bar{v}$, for parallel planes in laminar flow

where $\bar{v}$ = average velocity in the tube (ft/sec)

The shear stress distribution is

$$\frac{\tau}{\tau_w} = \frac{r}{R}$$

where τ and τ_w = shear stresses at radii r and R, respectively



* The Fanning Friction is this factor divided by 4.

	ε (ft)	ε (mm)
GLASS, DRAWN BRASS, COPPER, LEAD	SMOOTH	SMOOTH
COMMERCIAL STEEL, WROUGHT IRON	0.0001–0.0003	0.03–0.09
ASPHALTED CAST IRON	0.0002–0.0006	0.06–0.18
GALVANIZED IRON	0.0002–0.0008	0.06–0.24
CAST IRON	0.0006–0.003	0.18–0.91
CONCRETE	0.001–0.01	0.30–3.0
RIVETED STEEL	0.003–0.03	0.91–9.1
CORRUGATED METAL PIPE	0.1–0.2	30–61
LARGE TUNNEL, CONCRETE OR STEEL LINED	0.002–0.004	0.61–1.2
BLASTED ROCK TUNNEL	1.0–2.0	300–610

Moody, Darcy, or Stanton Friction Factor Diagram

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6.2.3 Consequences of Fluid Flow (Circular Conduits)

6.2.3.1 Head Loss Due to Flow (Darcy-Weisbach Equation)

The *Darcy-Weisbach equation* is

$$h_f = f\frac{L}{D}\frac{v^2}{2g}$$

where

f = $f(Re, \varepsilon/D)$, the Moody, Darcy, or Stanton friction factor

D = diameter of the pipe

L = length over which the pressure drop occurs

ε = roughness factor for the pipe, and other symbols as defined previously

6.2.3.2 Pressure Drop for Laminar Flow

The equation for Q in terms of the pressure drop ΔP_f is the *Hagen-Poiseuille equation*. This relation is valid only for flow in the laminar region.

$$Q = \frac{\pi R^4 \Delta P_f}{8\mu L} = \frac{\pi D^4 \Delta P_f}{128\mu L}$$

6.2.3.3 Mach Number

The local *speed of sound* in an ideal gas is given by:

$$c = \sqrt{kRT}$$

where

c = local speed of sound

k = ratio of specific heats = $\frac{c_p}{c_v}$

R = specific gas constant = $\overline{R}$/(molecular weight)

T = absolute temperature

Example: Speed of sound in dry air at 1 atm 20°C is 343.2 m/s.

This shows that the acoustic velocity in an ideal gas depends only on its temperature. The *Mach number* (Ma) is the ratio of the fluid velocity to the speed of sound.

$$\text{Ma} = \frac{V}{c}$$

where

V = mean fluid velocity

6.2.4 Flow in Conduits (Circular or Noncircular)

6.2.4.1 Hydraulic Radius

Analysis of flow in conduits uses the hydraulic radius R_H:

$$R_H = \frac{\text{cross-sectional area}}{\text{wetted perimeter}}$$

6.2.4.2 Drag Force

The drag force F_D on objects immersed in a large body of flowing fluid or objects moving through a stagnant fluid is

$$F_D = \frac{C_D \rho v^2 A}{2}$$

where

C_D = drag coefficient

v = velocity (ft/sec) of the flowing fluid or moving object

A = projected area (ft^2) of blunt objects such as spheres, ellipsoids, disks, and plates, cylinders, ellipses, and air foils with axes perpendicular to the flow

ρ = fluid density ($lbf\text{-}sec^2/ft^4$)

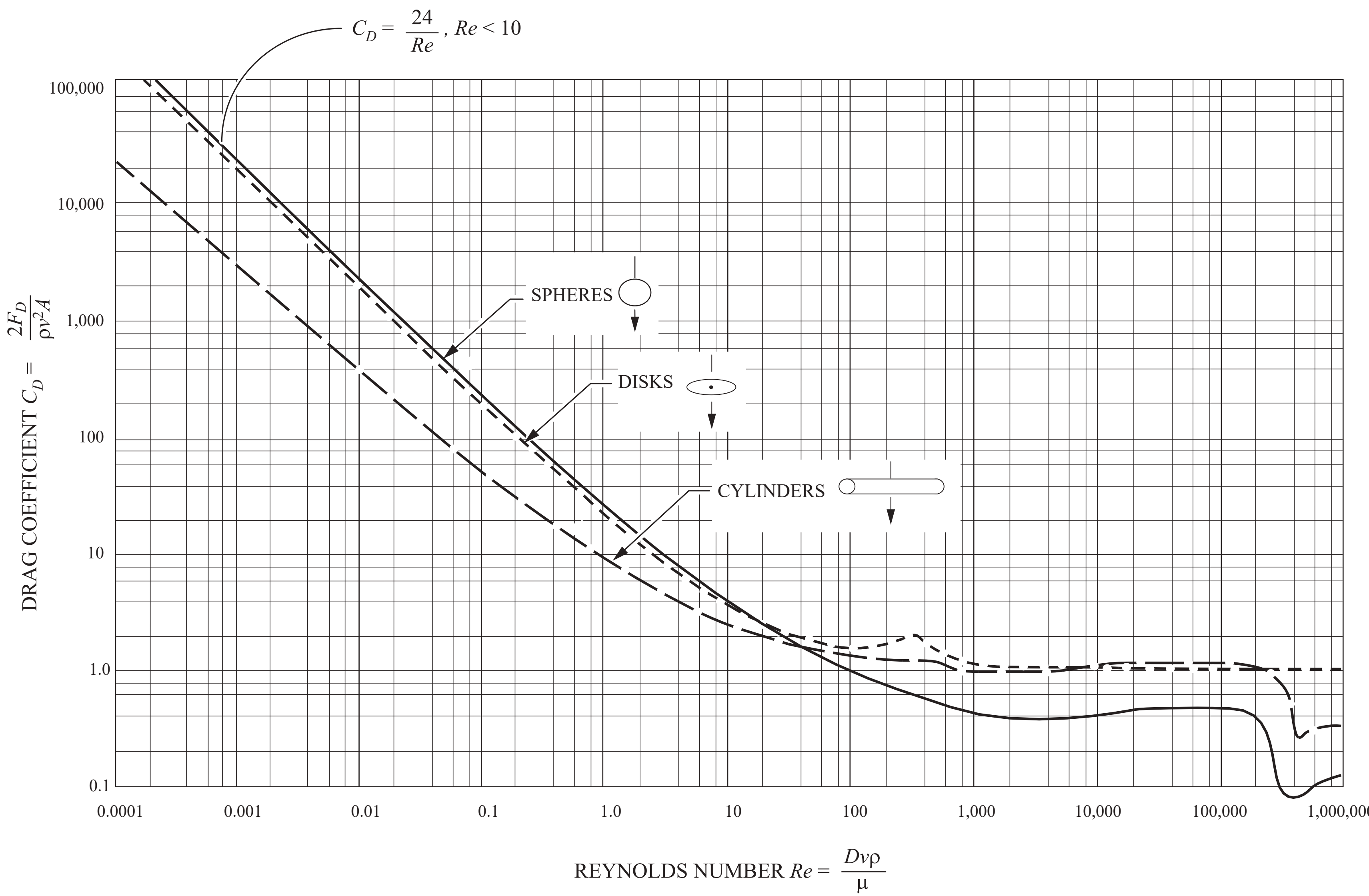
For flat plates placed parallel with the flow:

$C_D = 1.33/Re^{0.5}$ $(10^4 < Re < 5 \times 10^5)$

$C_D = 0.031/Re^{1/7}$ $(10^6 < Re < 10^9)$

The characteristic length in the Reynolds Number (*Re*) is the length of the plate parallel with the flow. For blunt objects, the characteristic length is the largest linear dimension (diameter of cylinder, sphere, disk, etc.) that is perpendicular to the flow.

Drag Coefficient for Spheres, Disks, and Cylinders



Note: Intermediate divisions are 2, 4, 6, and 8

6.2.4.3 Impulse-Momentum Principle

The resultant force in a given direction acting on the fluid equals the rate of change of momentum of the fluid:

$$\Sigma \boldsymbol{F} = \Sigma Q_2 \rho_2 v_2 - \Sigma Q_1 \rho_1 v_1$$

where

$\Sigma \boldsymbol{F}$ = resultant of all external forces acting on the control volume

$\Sigma Q_1 \rho_1 v_1$ = rate of momentum of the fluid flow entering the control volume in the same direction of the force

$\Sigma Q_2 \rho_2 v_2$ = rate of momentum of the fluid flow leaving the control volume in the same direction of the force

6.2.4.4 Similitude

In order to use a model to simulate the conditions of the prototype, the model must be *geometrically, kinematically,* and *dynamically similar* to the prototype system.

To obtain dynamic similarity between two flow pictures, all independent force ratios that can be written must be the same in both the model and the prototype. Thus, dynamic similarity between two flow pictures (when all possible forces are acting) is expressed in the five simultaneous equations below.

$$\left[\frac{F_I}{F_P}\right]_p = \left[\frac{F_I}{F_P}\right]_m = \left[\frac{\rho v^2}{P}\right]_p = \left[\frac{\rho v^2}{P}\right]_m$$

$$\left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m = \left[\frac{vl\rho}{\mu}\right]_p = \left[\frac{vl\rho}{\mu}\right]_m = \left[Re\right]_p = \left[Re\right]_m$$

$$\left[\frac{F_I}{F_G}\right]_p = \left[\frac{F_I}{F_G}\right]_m = \left[\frac{v^2}{lg}\right]_p = \left[\frac{v^2}{lg}\right]_m = \left[Fr^2\right]_p = \left[Fr^2\right]_m$$

$$\left[\frac{F_I}{F_E}\right]_p = \left[\frac{F_I}{F_E}\right]_m = \left[\frac{\rho v^2}{E_v}\right]_p = \left[\frac{\rho v^2}{E_v}\right]_m = \left[Ca\right]_p = \left[Ca\right]_m$$

$$\left[\frac{F_I}{F_T}\right]_p = \left[\frac{F_I}{F_T}\right]_m = \left[\frac{\rho l v^2}{\sigma}\right]_p = \left[\frac{\rho l v^2}{\sigma}\right]_m = \left[We\right]_p = \left[We\right]_m$$

where the subscripts p and m stand for *prototype* and *model* respectively, and

F_I = inertia force

F_P = pressure force

F_V = viscous force

F_G = gravity force

F_E = elastic force

F_T = surface tension force

Re = Reynolds number

We = Weber number

Ca = Cauchy number

Fr = Froude number

l = characteristic length

v = velocity

ρ = density

σ = surface tension

E_v = bulk modulus

μ = dynamic viscosity

P = pressure

g = acceleration due to gravity

6.2.5 Hydraulic Flow Measurement

6.2.5.1 Pitot Tube

From the stagnation pressure equation for an *incompressible fluid*,

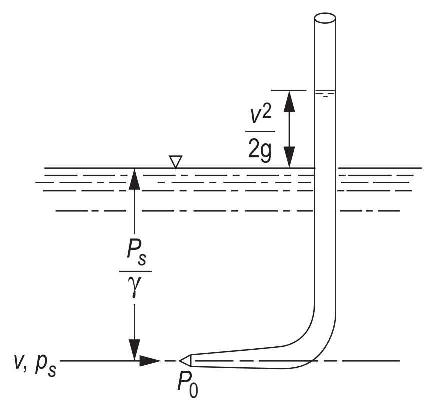
$$v = \sqrt{(2/\rho)(P_0 - P_s)} = \sqrt{2g(P_0 - P_s)/\gamma}$$

where

v = velocity of the fluid

P_0 = stagnation pressure

P_s = static pressure of the fluid at the elevation where the measurement is taken



Source: Vennard, John K., and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., John Wiley and Sons, 1982, Fig. 11.11, p. 512. Reproduced with permission of the Licensor through PLSclear.

For a *compressible fluid*, use the above incompressible fluid equation if the Mach number ≤ 0.3.

6.2.5.2 Venturi Meters

For *incompressible fluids*:

$$Q = \frac{C_v A_2}{\sqrt{1-\left(A_2/A_1\right)^2}} \sqrt{2g\left(\frac{P_1}{\gamma}+z_1-\frac{P_2}{\gamma}-z_2\right)}$$

where

Q = volumetric flow rate (ft^3/sec)

C_v = coefficient of velocity

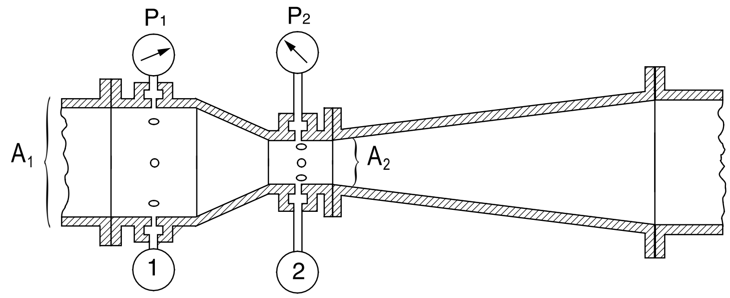
A = cross-sectional area of flow (ft^2)

P = pressure (lbf/ft^2)

γ = ρg (lbf/ft^3)

z_1 = elevation of venturi entrance (ft)

z_2 = elevation of venturi throat (ft)



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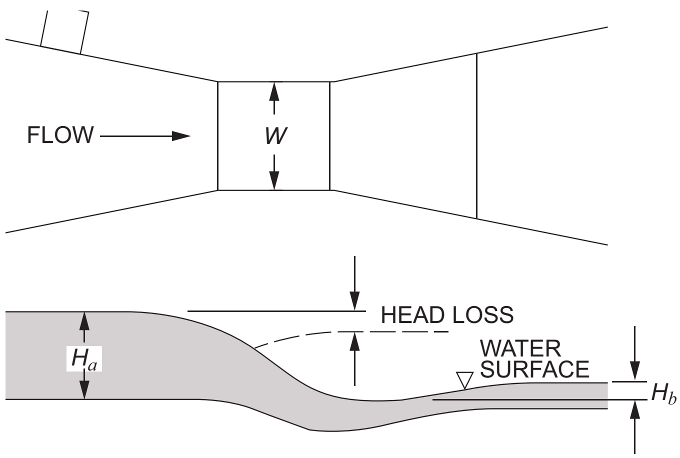
6.2.5.3 Parshall Flume



$$\left.\begin{cases} Q = 2.06\, H_a^{1.58} & \text{for } W = 0.5 \text{ ft} \\ Q = 3.07\, H_a^{1.53} & \text{for } W = 0.75 \text{ ft} \end{cases}\right\} \quad H_b/H_a \le 0.6$$

$$Q = 4WH_a^{1.522W^{0.026}} \quad \text{for } W \text{ from 1 through 8 ft} \quad H_b/H_a \le 0.7$$

$$Q = (3.688W + 2.5)H_a^{1.6} \quad \text{for } W \text{ from 8 through 50 ft} \quad H_b/H_a \le 0.8$$

where

Q = discharge flow (ft^3/sec)

W = width of the flume throat (ft)

H_a = upstream head (ft)

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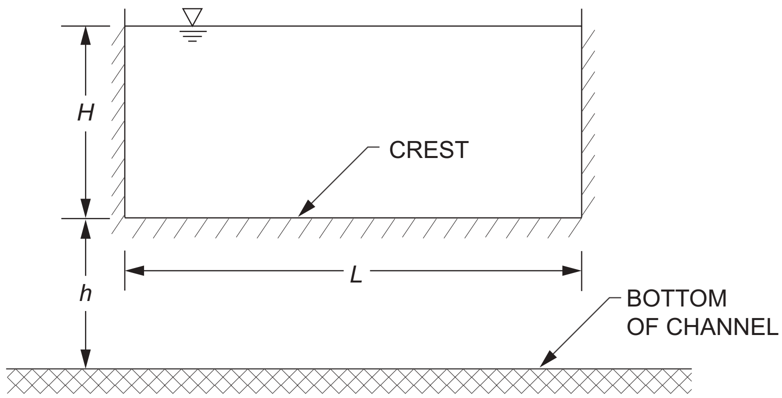
6.2.5.4 Rectangular Weirs



Sharp-Crested Weirs

$$Q = CLH^{3/2}$$

where

Q = discharge flow over weir (length^3/time)

L = weir length (length)

H = head/depth of discharge over weir (length)

C = coefficient of discharge

Rehbock Coefficient of Discharge Equation

$$C = 3.27 + 0.4\frac{H}{h} \qquad \text{(USCS units)}$$

where

$\frac{H}{h} < 10$

h = height of bottom of weir with respect to the floor of the approach channel (ft)

Sharp-Crested Weirs with End Contractions (Francis Equation)

$$Q = C\,[L - n(0.1H)]\,H^{3/2}$$

where

Q = discharge flow over weir (ft^3/sec, typical)

L = weir length (ft)

H = head/depth of discharge over weir (ft)

C = coefficient of discharge $\cong$ 3.33 for rectangular weirs where h becomes negligible (USCS units)

n = number of end contractions

Typical Rectangular Weir Proportions

Broad-crested weir: $0.08 < H/L < 0.33$

Short-crested weir: $0.33 < H/L < 1.5$

Sharp-crested weir: 1.5 < H/L

H = head/depth of discharge over weir (ft)

L = weir length in the direction perpendicular to flow (ft)

Horizontal Broad Crested Weirs

$$Q = \frac{2}{3} C_v L_e \sqrt{2g}\, H_e^{3/2}$$

where

Q = discharge (ft^3/sec)

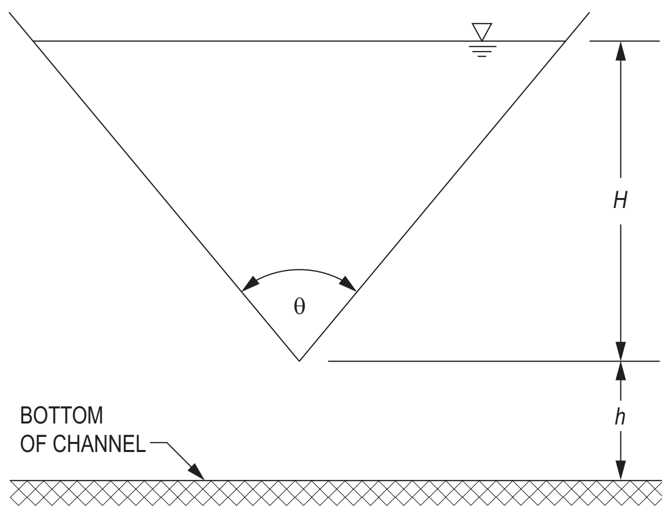
L_e = effective weir length (ft)

H_e = energy head upstream of weir (ft)

C_v = velocity coefficient of discharge

$C_v = 0.602 + 0.075\frac{H}{h}$ (Rehbock Eq.)

6.2.5.5 Triangular (V-Notch) Weirs



General V-Notch

$$Q = \frac{8}{15} C_d \sqrt{2g}\, \tan\frac{\theta}{2} H^{5/2}$$

where

Q = discharge (ft^3/sec)

θ = v-notch angle (degrees)

H = head/depth of discharge over weir (ft)

C_d = coefficient of discharge

g = acceleration due to gravity (ft/sec^2)

h = height of notch vertex with respect to the floor of the approach channel (ft)

90° V-Notch (Cone Equation)

$Q = C\,H^{5/2}$ (USCS units)

where

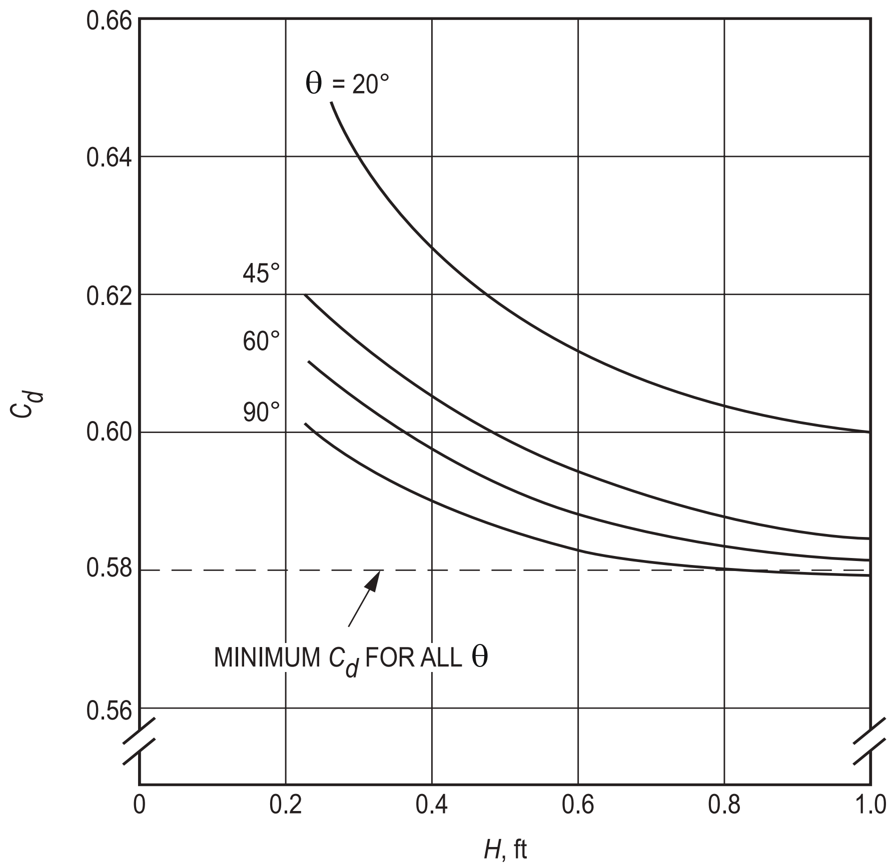
Weir is small and fully contracted.

All of weir must be located at least $2H$ from the approach boundaries.

C = coefficient of discharge, $2.48 \le C \le 2.57$

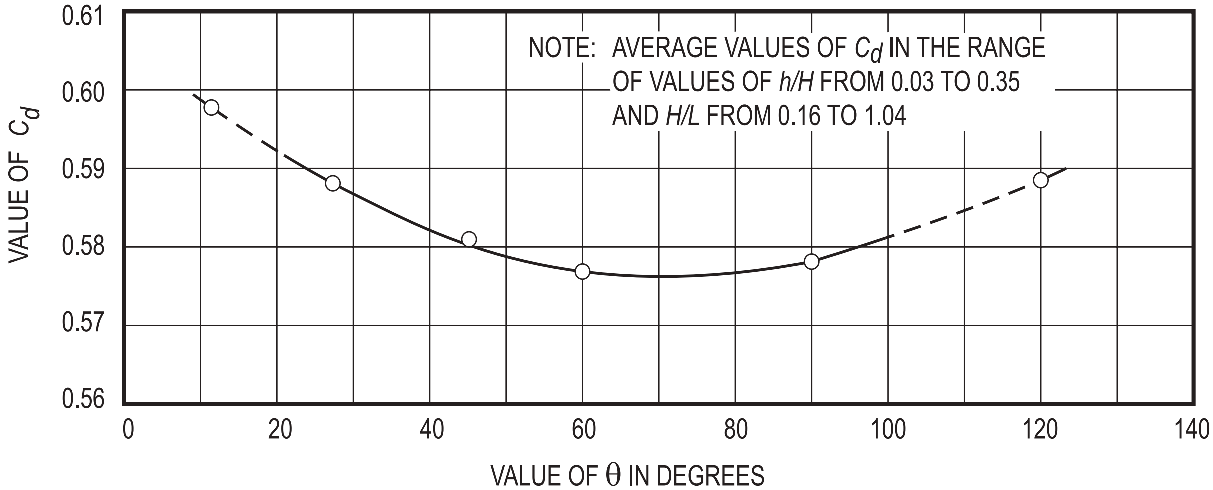
$0.20 \le H \le 1.24$ ft

6.2.5.6 Common Coefficients of Discharge for Triangular Weirs



Coefficient of Discharge vs. Head Over Discharge Chart

Source: U.S. Bureau of Reclamation, 1984.



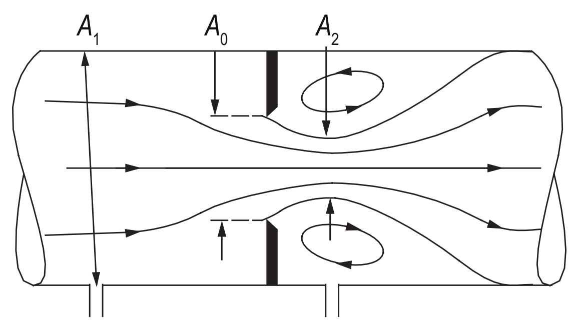
Coefficient of Discharge as a Function of Q Chart

Source: U.S. Geological Survey. Discharge Characteristics of Triangular-notch Thin-plate Weirs by John Shen. Geological Survey Water Supply Paper: 1617-B. Washington, DC: U.S. Department of the Interior, 1981, Figure 12, p. B29. https://pubs.usgs.gov/wsp/1617b/report.pdf.

6.2.6 Orifices

6.2.6.1 Coefficient of Contraction

The cross-sectional area at the vena contracta A_2 is characterized by a *coefficient of contraction* C_c and given by $C_c A_0$.



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6.2.6.2 Coefficient of Discharge

$$Q = C\ A_0\sqrt{2g\left(\frac{P_1}{\gamma}+z_1-\frac{P_2}{\gamma}-z_2\right)}$$

where

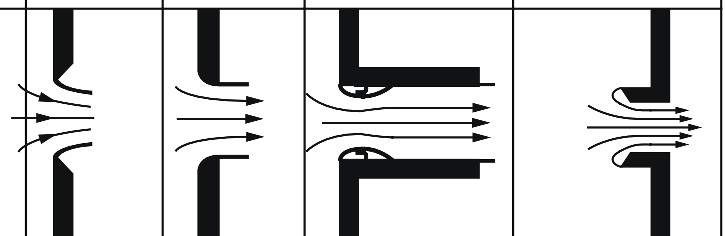
$$C = \frac{C_d}{\sqrt{1-C_c^2\left(A_0/A_1\right)^2}}$$

where

C_d = coefficient of discharge or meter coefficient = $C_c \times C_v$

C_c = coefficient of contraction

C_v = flow coefficient

	SHARP EDGED	ROUNDED	SHORT TUBE	BORDA
				
C_d	0.61	0.98	0.80	0.51
C_c	0.62	1.00	1.00	0.52
C_v	0.98	0.98	0.80	0.98

Orifices and Their Nominal Coefficients

Source: Vennard, John K., and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., John Wiley and Sons, 1982, Fig. 11.29, p. 535.
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For incompressible flow through a horizontal orifice installation

$$Q = C_d A_0 \sqrt{\frac{2}{\rho}\left(P_1 - P_2\right)}$$

where

Q = flow rate (ft^3/sec)

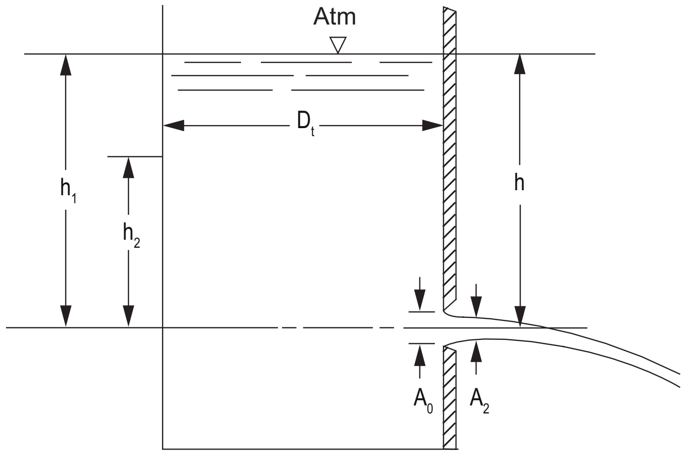
C_d = coefficient of discharge

A_0 = area of orifice (ft^2)

P = pressure (lbf/ft^2)

ρ = density of fluid ($lbf\text{-}sec^2/ft^4$)

6.2.6.3 Orifice Discharging Freely into Atmosphere



Source: Vennard, John K., and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., John Wiley and Sons, 1982, Fig. 11.28, p. 535. Reproduced with permission of the Licensor through PLSclear.

$$Q = C_d A_0 \sqrt{2gh}$$

in which h is measured from the liquid surface to the centroid of the orifice opening.

where

Q = flow (ft^3/sec)

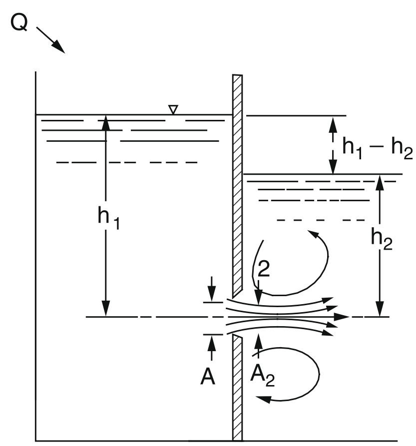
C_d = coefficient of discharge

A_0 = cross-sectional area of flow (ft^2)

g = acceleration due to gravity (ft/sec^2)

h = height of fluid above orifice (ft)

6.2.6.4 Submerged Orifices (Steady Flow)



Source: Vennard, John K., and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., John Wiley and Sons, 1982, Fig. 11.27, p. 534. Reproduced with permission of the Licensor through PLSclear.

$$Q = A_2 v_2 = C_c C_v A \sqrt{2g(h_1 - h_2)} = C_d A \sqrt{2g(h_1 - h_2)}$$

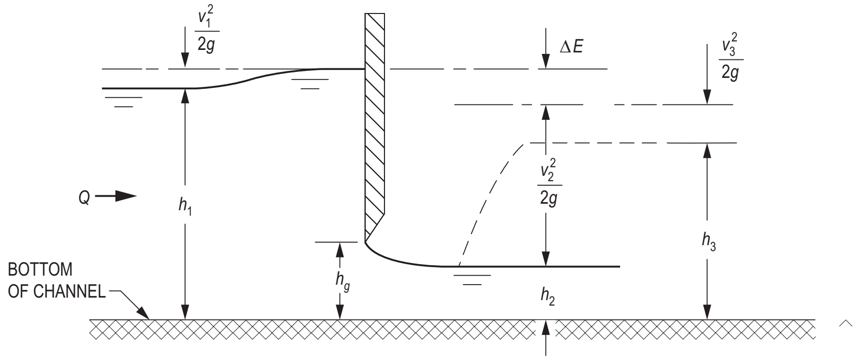
in which the product of C_c and C_v is defined as the coefficient of discharge of the orifice

where

v_2 = velocity of the fluid exiting the orifice (ft/sec)

g = acceleration due to gravity (ft/sec^2)

6.2.6.5 Sluice Gates



$$Q = C_d L h_g \sqrt{2g(h_1 - h_2)}$$

where

Q = flow (ft^3/sec)

C_d = coefficient of discharge

L = length or width of the sluice gate (ft)

h_g = height of gate opening (ft)

g = acceleration due to gravity (ft/sec^2)

h_1 = head upstream of gate (ft)

h_2 = head downstream of gate (ft)

6.2.7 Spillways

6.2.7.1 Waterways Experiment Station (WES) Standard Spillway

$$Q = CLH_e^{3/2}$$

where

Q = discharge (ft^3/sec)

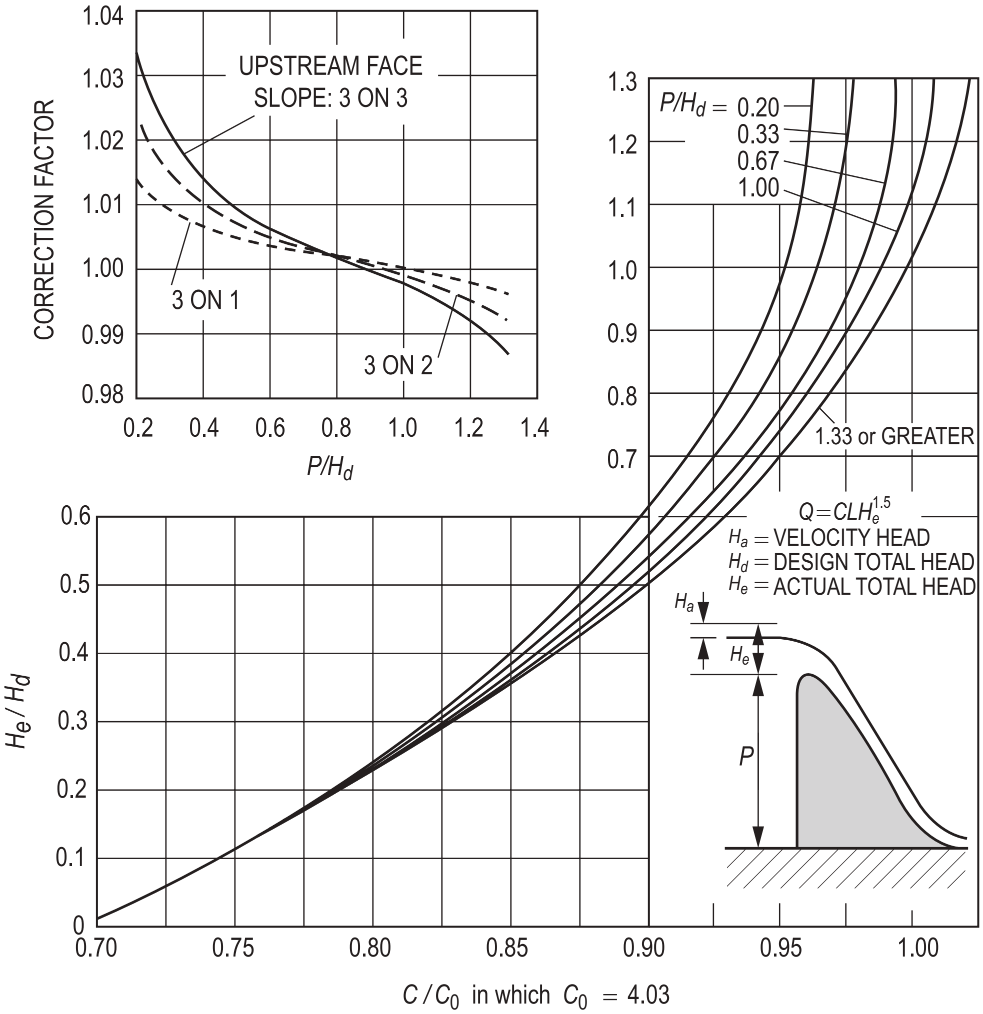
L = net effective spillway crest length (ft)

H_e = actual head over spillway (ft)

H_d = design head over spillway (ft)

P = height of spillway above floor (ft)

C = coefficient of discharge



Discharge Coefficient for the WES Standard Spillway Shape (Chow 1959)

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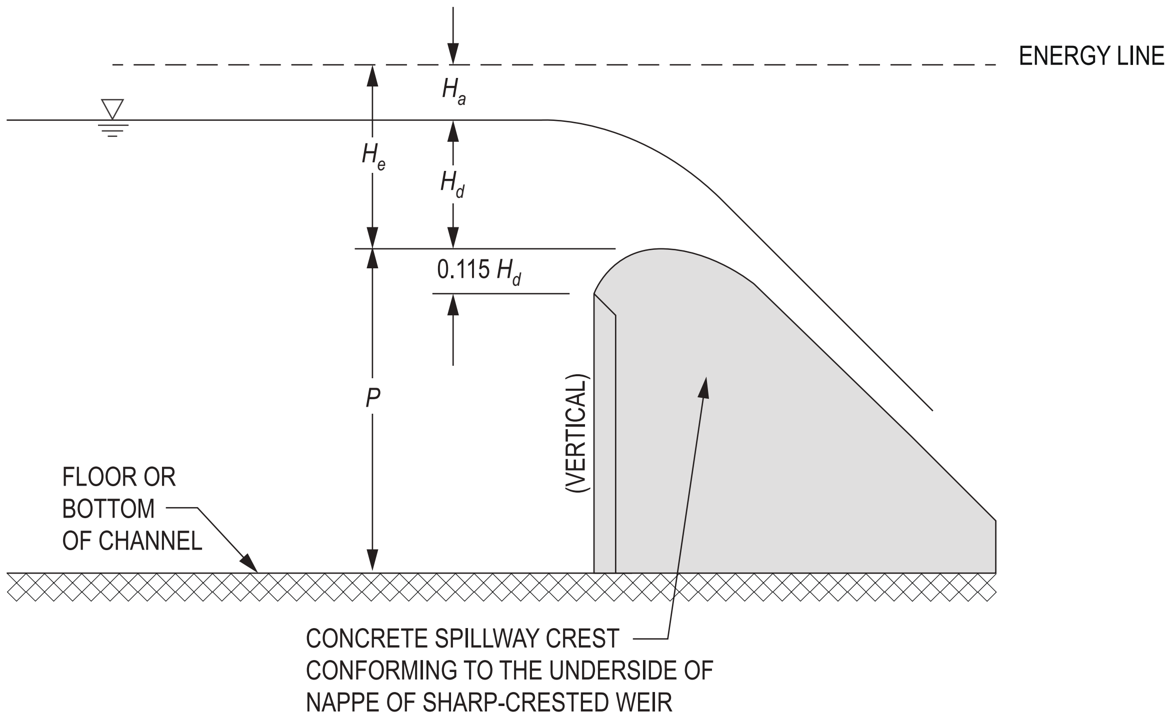
6.2.7.2 Ogee Spillway (Overflow Spillway)



$$Q = C\sqrt{2g}\,H_e^{3/2}$$

where

H_e = total energy head over crest = $H_d + H_a$

H_d = free flowing or design head

H_a = approach velocity head

C = dimensionless coefficient of discharge

$$Q = C_d L H_d^{\ 3/2}$$

where

H_d / P approaches zero

Q = discharge (ft^3/sec)

L = net effective spillway crest length (ft)

H_d = head over spillway crest (ft)

P = height of spillway above floor (ft)

C_d = coefficient of discharge

6.3 Closed Conduit Flow and Pumps

6.3.1 Hazen-Williams Equation

$$V = k_1 C R_H^{0.63} S^{0.54}$$

where

$$Q = k_1 C A R_H^{0.63} S^{0.54}$$

where

C = roughness coefficient

k_1 = 0.849 for SI units

k_1 = 1.318 for USCS units

R_H = hydraulic radius (ft or m)

S = slope of energy grade line (ft/ft or m/m) = $\frac{h_f}{L}$

V = velocity (ft/sec or m/s)

Q = discharge (ft^3/sec or m^3/s)

6.3.1.1 Circular Pipe Flow

$$Q = 0.432\ C D^{2.63} S^{0.54} \qquad \text{(USCS units)}$$

where

D = pipe diameter (ft)

$$Q = 0.432 C D^{2.63} \left(\frac{h_f}{L}\right)^{0.54}$$

6.3.1.2 Circular Pipe Head Loss (as feet)

$$h_f = \frac{4.73\ L}{C^{1.852}\ D^{4.87}}\ Q^{1.852}$$

where

h_f = head loss (ft)

L = pipe length of head loss (ft)

D = pipe diameter (ft)

Q = flow (cfs)

C = Hazen-Williams coefficient

6.3.1.3 Circular Pipe Head Loss (as pressure)

$$P = \frac{4.52\, Q^{1.85}}{C^{1.85}\, D^{4.87}}$$

where

P = pressure loss per unit length of pipe (psi per foot of pipe)

Q = flow (gpm)

D = pipe diameter (inches)

C = Hazen-Williams coefficient

6.3.1.4 Values of Hazen-Williams Coefficient C

Values of Hazen-Williams Coefficient *C*

Pipe Material	C
Ductile iron	140
Concrete (regardless of age)	130
Cast iron:	
New	130
5 yr old	120
20 yr old	100
Welded steel, new	120
Wood stave (regardless of age)	120
Vitrified clay	110
Riveted steel, new	110
Brick sewers	100
Asbestos-cement	140
Plastic	150
Galvanized iron (concrete-lined)	120

6.3.2 Darcy-Weisbach Equation (Head Loss)

$$h_f = f_a \frac{L_a}{D_a}\frac{v_a^2}{2g} = f_b \frac{L_b}{D_b}\frac{v_b^2}{2g}$$

$$\left(\pi D^2/4\right)v = \left(\pi D_A^2/4\right)v_A + \left(\pi D_B^2/4\right)v_B$$

where

h_f = head loss (ft)

f = $f\,(Re,\ \varepsilon/D)$, friction factor from Moody, Darcy, or Stanton diagram

D = diameter of the pipe (ft)

L = length of which the pressure drop occurs (ft)

v = velocity (ft/sec)

g = acceleration due to gravity (ft/sec^2)

6.3.3 Minor Losses in Pipe Fittings, Contractions, and Expansions

Head losses also occur as the fluid flows through pipe fittings (i.e., elbows, valves, couplings, etc.) and sudden pipe contractions and expansions.

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f,\,\text{fitting}}$$

$$\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f,\,\text{fitting}}$$

where

$$h_{f,\,\text{fitting}} = C\frac{v^2}{2g}$$

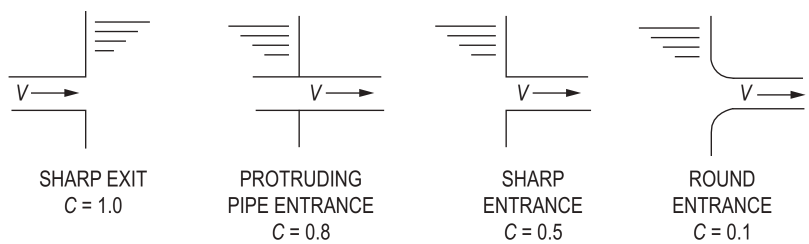
$$\frac{v^2}{2g} = 1 \text{ velocity head}$$

Specific fittings have characteristic values of C, which will be provided in the problem statement. A generally accepted *nominal value* for head loss in *well-streamlined gradual contractions* is

$$h_{f,\,\text{fitting}} = 0.04\frac{v^2}{2g}$$

6.3.3.1 Entrance and Exit Head Losses

The *head loss* at either an *entrance* or *exit* of a pipe from or to a reservoir is also given by the $h_{f,\,\text{fitting}}$ equation. Values for C for various cases are shown as follows:



Source: Bober, W., and Kenyon, R.A., *Fluid Mechanics*, John Wiley and Sons, 1980, Fig. 6-10, p. 297.
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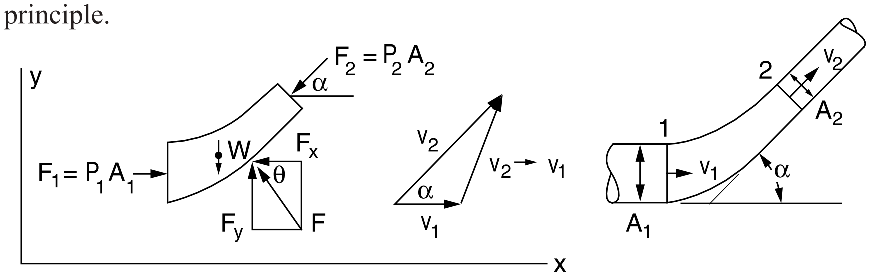
6.3.3.2 Minor Head Loss Coefficients

Component	Loss Coefficient
Globe valve, fully open	10.0
Swing check valve, fully open	2.5
Gate valve, fully open	0.2
Short-radius elbow	0.9
Medium-radius elbow	0.8
Long-radius elbow	0.6
45° elbow	0.4
Closed return bend	2.2
Standard tee – flow through run	0.6
Standard tee – flow through branch	1.8
Exit	1.0

Source: U.S. Environmental Protection Agency. Office of Research and Development. EPANET 2 Users Manual by Lewis Rossman. EPA/600/R-00/057. Washington, DC: U.S. EPA, 2000, Table 3.3, p. 32. http://nepis.epa.gov/Adobe/PDF/P1007WWU.pdf.

6.3.4 Pipe Bends, Enlargements, and Contractions

The force exerted by a flowing fluid on a bend, enlargement, or contraction in a pipeline may be computed using the impulse-momentum principle.



Source: Vennard, John K., and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., John Wiley and Sons, 1982, Fig. 6.3, p. 213. Reproduced with permission of the Licensor through PLSclear.

$$P_1A_1 - P_2A_2\cos\alpha - \boldsymbol{F_x} = Q\rho\,(v_2\cos\alpha - v_1)$$

$$\boldsymbol{F_y} - \mathrm{W} - P_2A_2\sin\alpha = Q\rho\,(v_2\sin\alpha - 0)$$

where

$\boldsymbol{F}$ = force exerted by the bend on the fluid (force exerted by fluid on the bend is equal in magnitude and opposite in sign), $\boldsymbol{F_x}$ and $\boldsymbol{F_y}$ are the x-component and y-component of the force $\boldsymbol{F} = \sqrt{\boldsymbol{F}_x^2 + \boldsymbol{F}_y^2}$ and $\theta = \tan^{-1}\left(\frac{\boldsymbol{F_y}}{\boldsymbol{F_x}}\right)$

P = internal pressure in the pipe line

A = cross-sectional area of the pipe line

W = weight of the fluid

v = velocity of the fluid flow

α = angle the pipe bend makes with the horizontal

ρ = density of the fluid

Q = fluid volumetric flow rate

6.3.5 Fire Hydrant Flow

6.3.5.1 Calculating Rated Capacity at 20 psi from Fire Hydrant

$$Q_R = Q_F \times (H_R/H_F)^{0.54}$$

where

Q_R = rated capacity (gpm) at 20 psi

Q_F = total test flow

H_R = P_S – 20 psi

H_F = $P_S - P_R$

P_S = static pressure

P_R = residual pressure

Source: Reproduced with permission of NFPA from NFPA 291, *Recommended Practice for Fire Flow Testing and Marking of Hydrants*, 2019 edition. Copyright© 2018, National Fire Protection Association. For a full copy of NFPA 291, please go to www.nfpa.org.

6.3.5.2 Fire Hydrant Discharging to Atmosphere

$$Q = 29.8\, D^2 C_d\, P^{1/2}$$

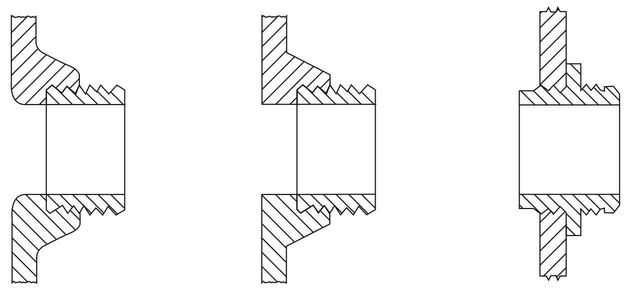
where

Q = discharge (gpm)

D = outlet diameter (in.)

P = pressure detected by pitot gauge (psi)

C_d = hydrant coefficient based on hydrant outlet geometry



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6.3.6 Flow Through a Packed Bed

6.3.6.1 Characteristic

A porous, fixed bed of solid particles can be characterized by:

L = length/depth of the packed bed (ft)

D_p = effective particle diameter (ft)

ϕ_s = sphericity of particles

ε = interparticle void fraction of the bed

6.3.6.2 Ergun Equation

The Ergun equation can be used to estimate pressure loss through a packed bed under laminar and turbulent flow conditions.

$$\frac{\Delta P}{L} = \frac{150 v_0 \mu (1-\varepsilon)^2}{k \phi_s^2 D_p^2 \varepsilon^3} + \frac{1.75 \rho v_0^2 (1-\varepsilon)}{k \phi_s D_p \varepsilon^3}$$

where

ΔP = pressure loss across packed bed (lbf/in^2, psi)

v_0 = superficial (flow through empty vessel) linear velocity (ft/hr)

ρ = fluid density (lbf-sec^2/ft^4)

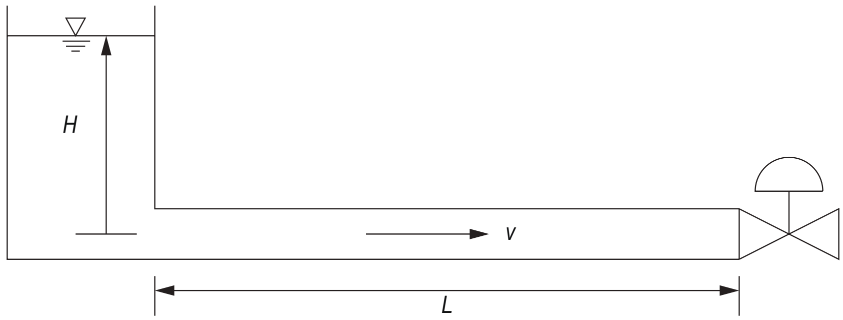
μ = fluid viscosity (lbm/hr-ft)

(centipoise × 2.42 = lbm/hr-ft)

(centistokes × 0.3876 × density, lb/ft^3 = lbm/hr-ft)

k = conversion factor (144 in^2/ft^2)

6.3.7 Water Hammer



6.3.7.1 Critical Time

$$t_c = \frac{2L}{a}$$

where

t_c = critical time of valve closure (sec)

L = upstream pipe length (ft)

a = wave/sound speed (ft/sec)

6.3.7.2 Rapid Valve Closure—Joukowsky Equation

When $t_{act} \le t_c$,

$$\Delta P = -\rho a \Delta v$$

where

ΔP = change in pressure (lbf/ft^2)

t_{act} = actual time of valve closure (sec)

t_c = critical time of valve closure (sec)

ρ = fluid density (lbf-sec^2/ft^4)

a = wave/sound speed (ft/sec)

Δv = change in velocity (ft/sec)

6.3.7.3 Slow Valve Closure

When $t_{act} > t_c$,

$$\Delta P = \rho a \Delta v \frac{t_c}{t_{act}}$$

where

ΔP = change in pressure (lbf/ft^2)

t_{act} = actual time of valve closure (sec)

t_c = critical time of valve closure (sec)

ρ = fluid density (lbf-sec^2/ft^4)

a = wave/sound speed (ft/sec)

Δv = change in velocity (ft/sec)

6.3.8 Pump Application and Analysis, Including Wet Wells, Lift Stations, and Cavitation

6.3.8.1 Total Dynamic Pumping Head

$$TDH = H_L + H_F + H_V$$

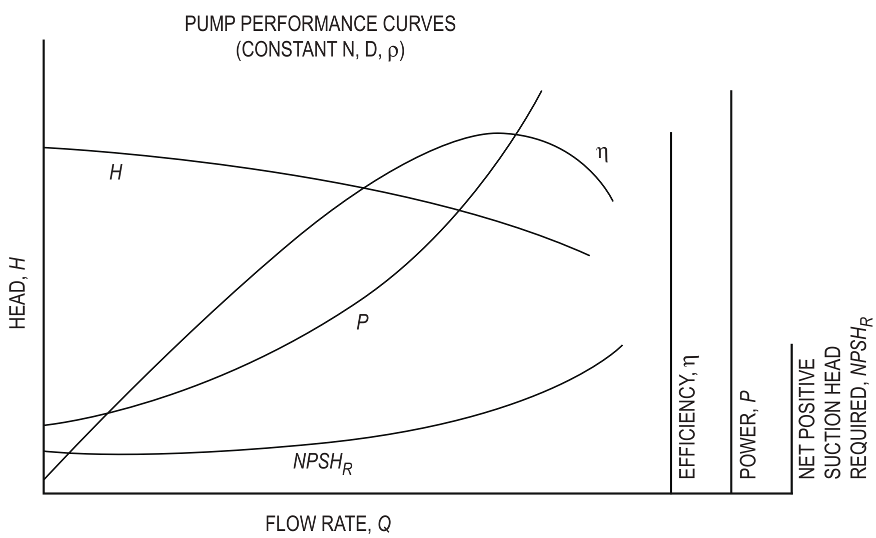
where

H_L = total static head (ft)

H_F = total friction head loss (ft)

H_V = velocity head (ft) = $\dfrac{v^2}{2g}$

6.3.8.2 Centrifugal Pump Characteristics



6.3.8.3 Net Positive Suction Head Available ($NPSH_A$)

$$NPSH_A = H_{pa} + H_s - \sum h_L - H_{vp} = \frac{P_{\text{inlet}}}{\rho g} + \frac{v_{\text{inlet}}^2}{2g} - \frac{P_{\text{vapor}}}{\rho g}$$

where

H_{pa} = atmospheric pressure head on the surface of the liquid in the sump (ft)

H_s = static suction head, height of the surface of the liquid above the centerline of the pump impeller (ft)

Σh_L = total friction head losses in the suction line (ft)

H_{vp} = vapor pressure head of the liquid at operating temperature (ft)

v = fluid velocity at pump inlet (ft/sec)

P_{vapor} = fluid vapor pressure at pump inlet (lbm/ft^3)

ρ = fluid density (lbf-sec^2/ft^4)

g = acceleration due to gravity (ft/sec^2)

SF = safety factor (ft)

6.3.8.4 Fluid Power Equations

Fluid power $\dot{W}_{\text{fluid}} = \rho g H Q$

Pump (brake) power $\dot{W} = \dfrac{\rho g H Q}{\eta_{\text{pump}}}$

Purchased power $\dot{W}_{\text{purchased}} = \dfrac{\dot{W}}{\eta_{\text{motor}}}$

where

η_{pump} = pump efficiency (0 to 1)

η_{motor} = motor efficiency (0 to 1)

H = head increase provided by pump

Pump Power Equation

$$\dot{W} = Q\gamma h/\eta_t = Q\rho g h/\eta_t$$

where

Q = volumetric flow (m^3/s or ft^3/sec)

h = head (m or ft) the fluid has to be lifted

η_t = total efficiency = $\eta_{\text{pump}} \times \eta_{\text{motor}}$

$\dot{W}$ = power (kg•m^2/s^3 or ft-lbf/sec)

Pump Brake Horsepower

$$BHP = \frac{HP}{\eta}$$

BHP = pump break horsepower (hp)

HP = hydraulic horsepower (hp)

η = pump efficiency

$$BHP = \frac{Q\gamma H}{\left(\frac{550 \text{ ft-lbf/sec}}{\text{hp}}\right)\eta}$$

where

BHP = pump break horsepower (hp)

Q = volumetric flow rate (ft^3/sec)

γ = specific weight of fluid (lbf/ft^3)

H = total head added by pump (ft)

η = pump efficiency

$$BHP = \frac{Q\ H\ SG}{3{,}956\ \eta}$$

where

BHP = pump break horsepower (hp)

Q = volumetric flow rate (gpm)

H = total head added by pump (ft)

SG = specific gravity of fluid

η = pump efficiency

	Specific Gravity
Fresh water	1.0
Salt water	1.024
Lube oil	0.85
Diesel oil	0.85

$$BHP = \frac{QP}{1{,}714\ \eta}$$

where

BHP = pump break horsepower (hp)

Q = volumetric flow rate (gpm)

P = pressure rise (lbf/in^2)

η = pump efficiency

6.3.8.5 Scaling and Affinity Laws (Performance of Components)

$$\left(\frac{Q}{ND^3}\right)_2 = \left(\frac{Q}{ND^3}\right)_1$$

$$\left(\frac{\dot{m}}{\rho ND^3}\right)_2 = \left(\frac{\dot{m}}{\rho ND^3}\right)_1$$

$$\left(\frac{H}{N^2D^2}\right)_2 = \left(\frac{H}{N^2D^2}\right)_1$$

$$\left(\frac{P}{\rho N^2D^2}\right)_2 = \left(\frac{P}{\rho N^2D^2}\right)_1$$

$$\left(\frac{\dot{W}}{\rho N^3D^5}\right)_2 = \left(\frac{\dot{W}}{\rho N^3D^5}\right)_1$$

where

Q = flow rate (ft^3/sec)

$\dot{m}$ = mass flow rate (mass/sec)

H = total head added by pump (ft)

P = pressure rise (lbf/ft^2)

$\dot{W}$ = power (ft-lbf/sec)

ρ = fluid density (lbf-sec^2/ft^4)

N = rotational speed (rev/time)

D = impeller diameter (length)

Subscripts 1 and 2 refer to different but similar machines or to different operating conditions of the same machine.

6.3.8.6 Pump Cavitation

Suction Head Available

$$H_s = H_{pa} - \text{NPSH}_\text{r} - H_{vp} - \sum h_L$$
$$= \frac{P_a - P_\text{vapor}}{\gamma} - \text{NPSH}_\text{r} - \sum h_L$$

where

H_s = static suction head of liquid (ft)

H_{pa} = atmospheric pressure head on surface of liquid (ft)

NPSH_r = net positive suction head required by pump, maximum (ft)

H_{vp} = vapor pressure head of the liquid at the operating temperature (ft)

$\sum h_L$ = total friction losses in the suction line (ft)

P_a = atmospheric pressure head on surface of liquid (ft of liquid, typ.)

P_vapor = vapor pressure head of the liquid at the operating temperature (ft of liquid, typ.)

γ = specific weight of fluid (lbf/ft^3)

Cavitation Parameter

Cavitation can be expected to occur below this critical value of the cavitation parameter in all pumps:

$$\sigma = \frac{\text{NPSH}}{h_p}$$

where

σ = cavitation parameter

h_p = head added by the pump (ft)

6.3.8.7 Multiple Pumps in Series

$$H = H_A + H_B + \ldots$$

$$Q = Q_A = Q_B = \ldots$$

$$\eta = \frac{H_A + H_B + \ldots}{H_A/\eta_A + H_B/\eta_B + \ldots}$$

$$\dot{W} = \frac{\gamma Q\left(H_A + H_B + \ldots\right)}{\eta}$$

where

A, B, … refer to different pumps

$\dot{W}$ = power (ft-lbf/sec)

Q = flow rate (ft^3/sec)

γ = specific weight of fluid (lbf/ft^3)

H = total head added by pump (ft)

η = efficiency of pump network

6.3.8.8 Multiple Pumps in Parallel

$$H = H_A = H_B = \ldots$$

$$Q = Q_A + Q_B + \ldots$$

$$\eta = \frac{Q_A + Q_B + \ldots}{Q_A/\eta_A + Q_B/\eta_B + \ldots}$$

$$\dot{W} = \frac{\gamma H\left(Q_A + Q_B + \ldots\right)}{\eta}$$

where

A, B, … refer to different pumps

$\dot{W}$ = power (ft-lbf/sec)

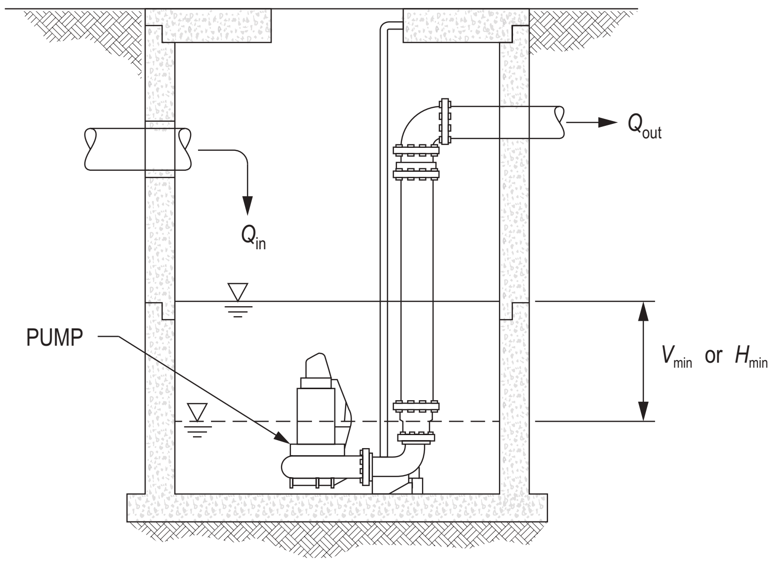
Q = flow rate (ft^3/sec)

γ = specific weight of fluid (lbf/ft^3)

H = total head added by pump (ft)

η = efficiency of pump network

6.3.9 Lift Station Pumping and Wet Wells



Source: Jensen Precast. "Pump Station Wet Wells: Minimum Storage Volume." Published September 11, 2015. https://www.jensenprecast.com/news/pump-station-wet-wells-minimum-storage-volume/.

6.3.9.1 Single Pump System Cycle Time

$$T_{\min} = \frac{V_{\min}}{Q_{\text{in}}} + \frac{V_{\min}}{Q_{\text{out}} - Q_{\text{in}}}$$

where

$T_{\min}$ = recommended minimum cycle time between two consecutive pump starts, including filling time (min)

Q_{in} = influent flow to wet well (gal/min)

Q_{out} = pumping rate of a single pump in operation (gal/min)

$V_{\min}$ = minimum volume required to operate the pump (gal)

$H_{\min}$ = minimum submergence required to operate the pump (ft)

where

$$Q_{\text{in}} = \frac{Q_{\text{out}}}{2}$$

6.3.9.2 Ideal Minimum Wet Well Volume

$$V_{\min} = \frac{T_{\min} \cdot Q_{\text{out}}}{4}$$

6.3.9.3 Minimum Submergence of Pump Inlet

$$S_{\min} = D + \frac{0.574Q}{D^{1.5}}$$

where

$S_{\min}$ = minimum necessary submergence of inlet to avoid vortexing (in.)

D = pump inlet diameter (in.)

Q = rate of flow (gal/min)

$$S_{\min} = D(1 + 2.3Fr)$$

where

Fr = Froude number

where

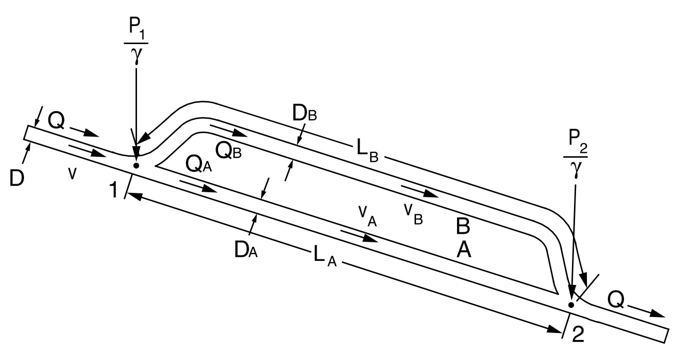
D = pump inlet diameter (ft) with symmetrical inlet approach

$$Fr = \frac{v}{(gD)^{0.5}}$$

v = velocity at pump inlet (ft/sec)

g = acceleration due to gravity (ft/sec^2)

6.3.10 Pipe Network Analysis



Source: Vennard, John K., and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., John Wiley and Sons, 1982 Fig. 9.25, p. 407. Reproduced with permission of the Licensor through PLSclear.

6.3.10.1 Network Flow Continuity

Principle: The total flow entering into a network of pipes equals the total flow out of the network.

$$\sum Q_{\text{in}} = \sum Q_{\text{out}}$$

6.3.10.2 Network Head Loss Continuity

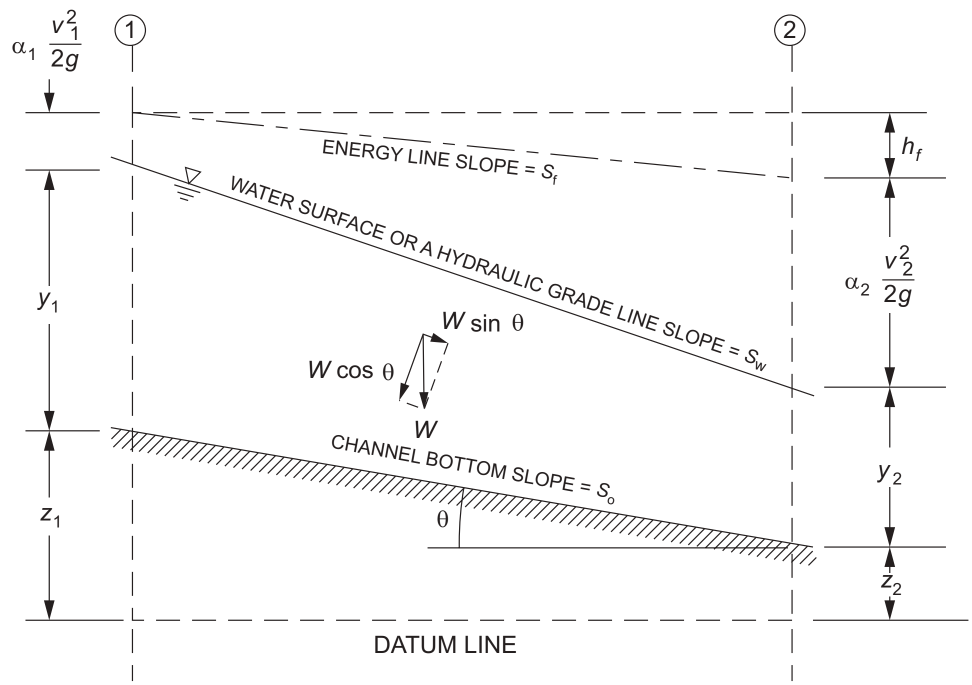
Principle: Connected, parallel pipes have equal head losses.

$$h_L = f_A \frac{L_A}{D_A}\frac{v_A^2}{2g} = f_B \frac{L_B}{D_B}\frac{v_B^2}{2g}$$

$$\frac{\pi D^2}{4} v = \frac{\pi D_A^2}{4} v_A + \frac{\pi D_B^2}{4} v_B$$

6.4 Open-Channel Flow

6.4.1 Conservation of Energy



$$z_1 + y_1 + \frac{\alpha_1 v_1^2}{2g} = z_2 + y_2 + \frac{\alpha_2 v_2^2}{2g} + h_f$$

where

z_1, z_2 = elevations of the channel bottom (ft)

y_1, y_2 = depths of flow (ft)

v_1, v_2 = velocities (ft/sec)

α_1, α_2 = kinetic energy coefficients

h_f = frictional loss

Energy coefficients: Used for the average velocity over the depth to compute the total kinetic energy

Integrating the cubed incremental velocities is not equal to the cube of the integrated incremental velocities.

6.4.2 Total Head and Specific Energy

$$H = \alpha \frac{v^2}{2g} + y + z = \frac{\alpha Q^2}{2gA^2} + y + z$$

$$E = \alpha \frac{v^2}{2g} + y$$

where

H = total head or energy head at any location along an open-channel flow

E = specific energy; the energy head where the channel bottom is the datum ($z = 0$)

Q = discharge (ft^3/sec)

v = velocity (ft/sec)

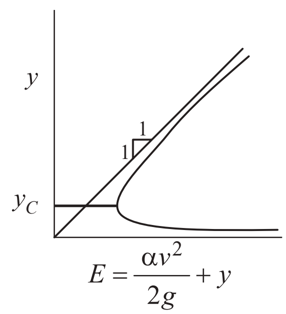
y = depth of flow (ft)

A = cross-sectional area of flow (ft^2)

α = kinetic energy correction factor, usually 1.0

z = elevation of channel bottom above datum (ft)

6.4.2.1 Specific Energy Diagram



Alternate depths: depths with the same specific energy

y_1 = flow depth at supercritical flow

y_2 = flow depth at subcritical flow

Upstream and downstream conditions for rectangular channel hydraulic jump.

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8\,Fr_1^2} - 1\right)$$

6.4.3 Normal and Critical Flow

Normal flow and depth: Occurs with uniform flow in a prismatic open channel

Critical flow and depth: Occurs when velocity of the water is the same as the speed at which disturbances of free surface will move through the shallow water

Uniform flow: a flow condition where depth and velocity do not change along a channel

6.4.3.1 Froude Number

Froude Number = ratio of inertial forces to gravity forces:

$$Fr = \frac{v}{\sqrt{gy}} = \sqrt{\frac{Q^2 T}{gA^3}}$$

where

y = depth of flow = $\frac{A}{T}$

T = width of the water surface at top (ft)

Supercritical flow: $Fr > 1$

Subcritical flow: $Fr < 1$

Critical flow: $Fr = 1$

6.4.3.2 Critical Depth

For Rectangular Channels

Critical State of Flow:

$$\frac{v_c^2}{2g} = \frac{y_c}{2}$$

where

v_c = flow velocity at critical depth (ft/sec)

y_c = critical depth (ft)

g = acceleration due to gravity (ft/sec^2)

Critical Flow:

$$Q_c = \left(\frac{A^3 g}{b}\right)^{1/2}$$

where

Q_c = critical flow in a channel at minimum specific energy (ft^3/sec)

g = acceleration due to gravity (ft/sec^2)

A = area of channel flow (ft^2)

b = channel width (ft)

Critical Depth:

$$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3}$$

where

y_c = critical depth of flow (ft)

Q = flow (ft^3/sec)

g = acceleration due to gravity (ft/sec^2)

y = actual depth of flow (ft)

y_n = normal depth of flow (ft)

b = channel width (ft)

$$y_c = \frac{2}{3}E$$

$$\frac{v^2}{2g} = \frac{E}{3}$$

where

E = specific energy (ft-lbf/lbm)

v = velocity (ft/sec)

6.4.4 Momentum Depth Relationship

6.4.4.1 Momentum Equation

$$M = \frac{Q^2}{gA} + Ay$$

where

y = vertical distance from liquid surface to centroid of area (ft)

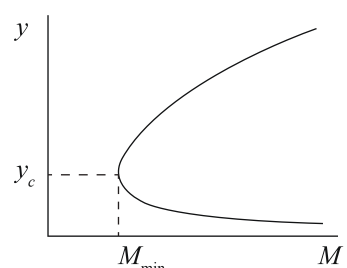
M = momentum (ft^3)

Q = channel flow (ft^3/sec)

A = area of channel (ft^2)

Sequent (conjugate) depths: depths with the same momentum

6.4.4.2 Momentum Depth Diagram



6.4.5 Steady Uniform Flow

6.4.5.1 Manning's Equation

$$Q = \frac{1.486}{n} A R_H^{2/3} S^{1/2}$$

$$v = \frac{1.486}{n} R_H^{2/3} S^{1/2}$$

where

Q = discharge or flow rate (ft^3/sec)

v = flow velocity (ft/sec)

n = Manning's roughness coefficient

A = cross-sectional area of flow (ft^2)

R_H = hydraulic radius (ft) = $\frac{A}{P}$

P = wetted perimeter (ft)

S = slope of the energy grade line (ft/ft)

= $\frac{h_L}{L}$

h_L = head loss (ft)

L = pipe length (ft)

Conveyance

$$K = \frac{1.486}{n} A R_H^{2/3}$$

$$K = \frac{Q}{\sqrt{S}}$$

where K = conveyance-flow-carrying capacity of cross section directly proportional to discharge (cfs)

Approximate Values of Manning's Roughness Coefficient

Material		Manning *n*
Closed Conduit or Built-up Channel	Metal:	
	Brass	0.01
	Copper	0.011
	Steel – welded	0.012
	Steel – riveted	0.016
	Cast iron – coated	0.013
	Wrought iron – galvanized	0.016
	Corrugated metal (storm drain)	0.024
	Nonmetal:	
	Glass	0.01
	Cement	0.011
	Cement mortar	0.013
	Concrete culvert	0.013
	Concrete-lined channel/pipe	0.015
	Wood	0.012
	Clay	0.013
	Brickwork	0.013
	Brickwork with cement mortar	0.015
	Masonry/rubble masonry	0.025
	Sanitary sewer coated with slime	0.013
	Asphalt	0.013
	Plastic	0.013
	PVC	0.009–0.011
	Polyethylene	0.009–0.015
Excavated or Dredged Channel	Straight and clean	0.022
	Winding and sluggish	0.025
	Dredged	0.028
	Rock cut/stony	0.035
	Earth bottom, rubble sides	0.03
	Unmaintained/uncut brush	0.08
Natural Streams	On plain, clean, straight, no pools	0.03
	On plain, clean, winding, some pools	0.04
	On plain, sluggish, weedy, deep pools	0.07
	On mountain, few boulders	0.04
	On mountain, large boulders	0.05

6.4.5.2 Chezy Equation

$$v = C\sqrt{R_H S}$$

$$Q = CA\sqrt{R_H S}$$

where

v = velocity in channel

Q = flow in channel

C = Chezy's resistance coefficient

R_H = hydraulic radius

S = slope of hydraulic surface

$$C = \frac{k}{n} R_H^{1/6}$$

where

C = Chezy's resistance coefficient ($\text{ft}^{0.5}$/sec)

R_H = slope of hydraulic surface (ft/ft)

k = unit conversion factor

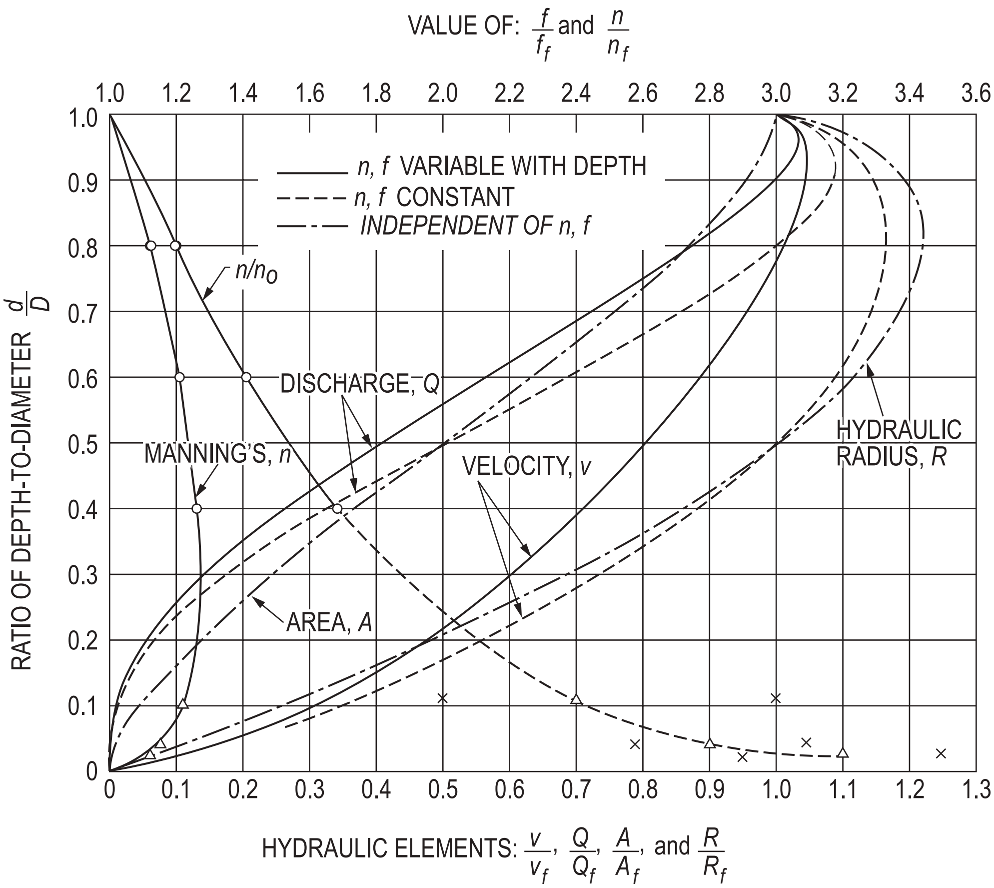
= 1 (SI units)

= 1.486 (USCS units)

n = Manning's roughness coefficient ($\text{ft}^{-1/3}$/sec)

6.4.5.3 Flow in Circular Pipe

Hydraulic Elements Graph for Circular Pipe (Partial Flow Diagram)



Source: American Society of Civil Engineers. *Design and Construction of Sanitary and Storm Sewers*. Reston, VA: ASCE, 1970, pg. 87, fig. 24.

Minimum Diameter of Circular Pipe Under Full Flow Conditions

$$D = \left[\frac{C_0 Q n}{\sqrt{S}}\right]^{3/8}$$

where

Depth y is constant.

D = diameter of pipe (ft)

C_0 = 2.16 for USCS units

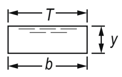
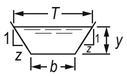
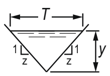
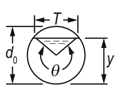
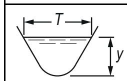
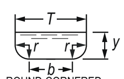
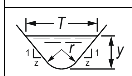
Q = flow (ft^3/sec)

n = Manning's roughness coefficient

S = slope (ft/ft)

6.4.5.4 Flow in Channels

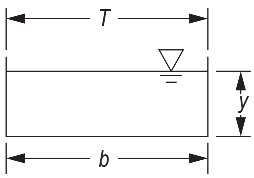
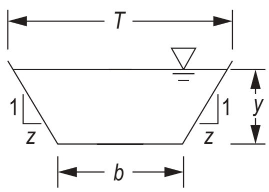
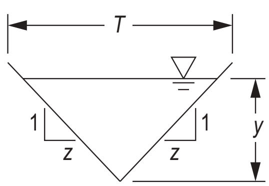
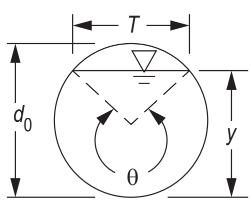
Geometric Elements of Channel Sections

Section	Area A	Wetted Perimeter P	Hydraulic Radius R	Top Width T	Hydraulic Depth D	Section Factor Z
 RECTANGLE	by	$b + 2y$	$\frac{by}{b+2y}$	b	y	$by^{1.5}$
 TRAPEZOID	$(b + zy)y$	$b+2y\sqrt{1+z^2}$	$\frac{(b+zy)y}{b+2y\sqrt{1+z^2}}$	$b + 2zy$	$\frac{(b+zy)y}{b+2zy}$	$\frac{[(b+zy)y]^{1.5}}{\sqrt{b+2zy}}$
 TRIANGLE	zy^2	$2y\sqrt{1+z^2}$	$\frac{zy}{2\sqrt{1+z^2}}$	$2zy$	$1/2y$	$\frac{\sqrt{2}}{2}zy^{2.5}$
 CIRCLE	$1/8(\theta - \sin\theta)d_0^2$	$1/2\theta d_0$	$1/4\left(1-\frac{\sin\theta}{\theta}\right)d_0$	$(\sin 1/2\,\theta)d_0$ or $2\sqrt{y(d_0-y)}$	$1/8\left(\frac{\theta-\sin\theta}{\sin 1/2\,\theta}\right)d_0$	$\frac{\sqrt{2}}{32}\frac{(\theta-\sin)^{1.5}}{(\sin 1/2\,\theta)^{0.5}}d_0^{2.5}$
 PARABOLA	$2/3Ty$	$T+\frac{8}{3}\frac{y^2}{T}$ *	$\frac{2T^2y}{3T^2+8y^2}$ *	$\frac{3}{2}\frac{A}{y}$	$2/3y$	$2/9\sqrt{6}\,Ty^{1.5}$
 ROUND-CORNERED RECTANGLE (y > r)	$\left(\frac{\pi}{2}-2\right)r^2+(b+2r)y$	$(\pi-2)r+b+2y$	$\frac{(\pi/2-2)r^2+(b+2r)y}{(\pi-2)r+b+2y}$	$b + 2r$	$\frac{(\pi/2-2)r^2}{b+2r}+y$	$\frac{[(\pi/2-2)r^2+(b+2r)y]^{1.5}}{\sqrt{b+2r}}$
 ROUND-BOTTOMED TRIANGLE	$\frac{T^2}{4z}-\frac{r^2}{z}(1-z\cot^{-1}z)$	$\frac{T}{z}\sqrt{1+z^2}-\frac{2r}{z}(1-z\cot^{-1}z)$	$\frac{A}{P}$	$2\left[z(y-r)+r\sqrt{1+z^2}\right]$	$\frac{A}{T}$	$A\sqrt{\frac{A}{T}}$

*Satisfactory approximation for the interval $0 < x \le 1$, where $x = 4y/T$. When $x > 1$, use the exact expression $P = (T/2)\left[\sqrt{1+x^2}+1/x\ln\left(x+\sqrt{1+x^2}\right)\right]$.

Source: Chow, Ven Te. *Open-Channel Hydraulics*. Estate of Ven Te Chow, 1959, Table 2-1, p. 21.

Channel Section Critical Depths

Cross Section	Uniform Flow Section Factor $AR^{2/3} = \frac{Qn}{K_n S^{0.5}}$	Critical Flow Section Factor $Z_c = \frac{Q}{\sqrt{g}} = A_c\sqrt{D_c}$	Critical Depth y_c
 RECTANGLE	$\left[\frac{b^5 y^5}{(b+2y)^2}\right]^{1/3}$	$by^{1.5}$	$\left(\frac{Z_c}{b}\right)^{2/3}$
 TRAPEZOID	$\left[\frac{(b+zy)^5 y^5}{\left(b+2y\sqrt{1+z^2}\right)^2}\right]^{1/3}$	$\frac{[(b+zy)y]^{1.5}}{\sqrt{b+2zy}}$	$0.81\left[\frac{Z_c^4}{z^{1.5}b^{2.5}}\right]^{0.135} - \frac{b}{30z}$ for $0.1 < \frac{Q}{b^{2.5}} < 0.4$
 TRIANGLE	$\left[\frac{z^5 y^8}{4(1+z^2)}\right]^{1/3}$	$\frac{\sqrt{2}}{2} zy^{2.5}$	$\left(\frac{\sqrt{2}\,Z_c}{z}\right)^{0.4}$
 CIRCLE	$\frac{1}{16}\left[\frac{(\theta - \sin\theta)^5 d_0^3}{2\theta^2}\right]^{1/3}$	$\frac{\sqrt{2}}{32}\left[\frac{(\theta - \sin\theta)^{1.5}}{\left(\sin\frac{1}{2}\theta\right)^{0.5}}\right] d_0^{2.5}$	$\frac{1.01}{d_0^{0.26}} Z_c^{0.5}$ for $0.02 \le \frac{y_c}{d_0} \le 0.85$

θ is in radians

Source: Republished with permission of McGraw-Hill. From "Hydraulics for Excess Water Management" by Ben C. Yen in *Water Resources Handbook*, edited by Larry W. Mays, 1996. Permission conveyed through Copyright Clearance Center, Inc.

Best Hydraulic Efficient Sections Without Freeboard

Cross Section	Area A	Wetted Perimeter P	Hydraulic Radius R	Top Width T	Hydraulic Depth D	Section Factor Z
Trapezoid, half of a hexagon	$\sqrt{3}\,y^2$	$2\sqrt{3}\,y$	$\frac{1}{2}y$	$\frac{4}{3}\sqrt{3}\,y$	$\frac{3}{4}y$	$1.5y^{2.5}$
Rectangle, half of a square	$2y^2$	$4y$	$\frac{1}{2}y$	$2y$	y	$2y^{2.5}$
Triangle, half of a square	y^2	$2\sqrt{2}\,y$	$\frac{1}{4}\sqrt{2}\,y$	$2y$	$\frac{1}{2}y$	$\frac{\sqrt{2}}{2}y^{2.5}$
Semicircle	$\frac{\pi}{2}y^2$	πy	$\frac{1}{2}y$	$2y$	$\frac{\pi}{4}y$	$\frac{\pi}{4}y^{2.5}$
Parabola, $B = 2\sqrt{2}\,y$	$\frac{4}{3}\sqrt{2}\,y^2$	$\frac{8}{3}\sqrt{2}\,y$	$\frac{1}{2}y$	$2\sqrt{2}\,y$	$\frac{2}{3}y$	$\frac{8\sqrt{3}}{9}y^{2.5}$
Hydrostatic catenary	$1.39586\,y^2$	$2.9836\,y$	$0.46784\,y$	$1.917532\,y$	$0.72795\,y$	$1.19093\,y^{2.5}$

Source: Republished with permission of McGraw-Hill. From "Hydraulics for Excess Water Management" by Ben C. Yen in *Water Resources Handbook*, edited by Larry W. Mays, 1996. Permission conveyed through Copyright Clearance Center, Inc.

6.4.6 Hydraulic Classification of Slopes

Name	Type	Condition
Mild	M	$y_n > y_c$
Steep	S	$y_n < y_c$
Critical	C	$y_n = y_c$
Horizontal	H	$y_n = \infty$
Adverse	A	$S_0 < 0$

6.4.6.1 Flow Profiles on Slopes

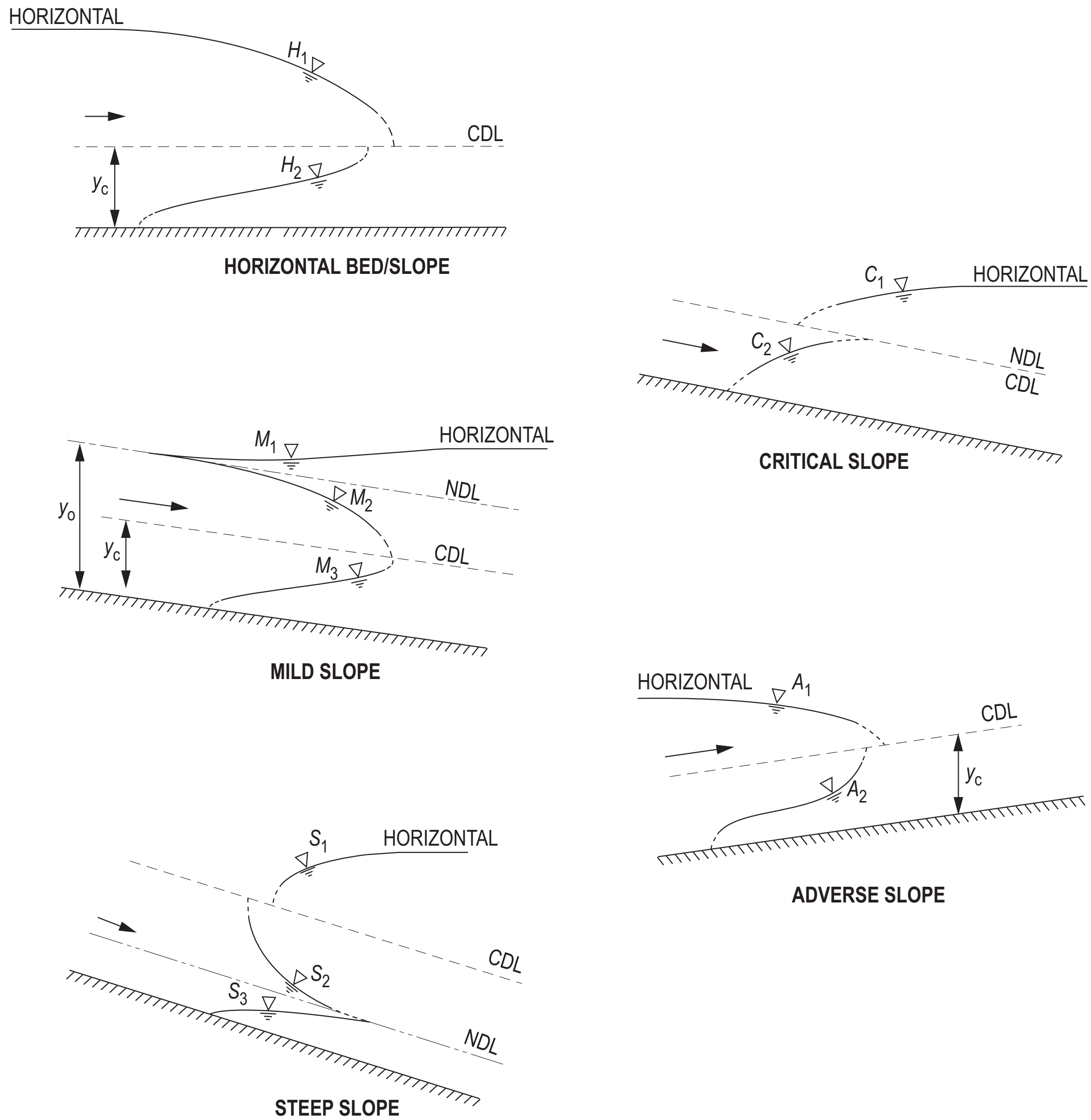
Type 1: Actual depth is greater than y_c and y_n; flow is subcritical.

Type 2: Actual depth is between y_c and y_n; flow can be either subcritical or supercritical.

Type 3: Actual depth is less than both y_c and y_n; flow is supercritical.

6.4.7 Gradually Varied Flow

6.4.7.1 Gradually Varied Flow Profile Diagrams



where

CDL = critical depth line

NDL = normal depth line

6.4.8 Rapidly Varied Flow and Hydraulic Jump

6.4.8.1 Depths and Flows

Assumes channel has rough rectangular shape and

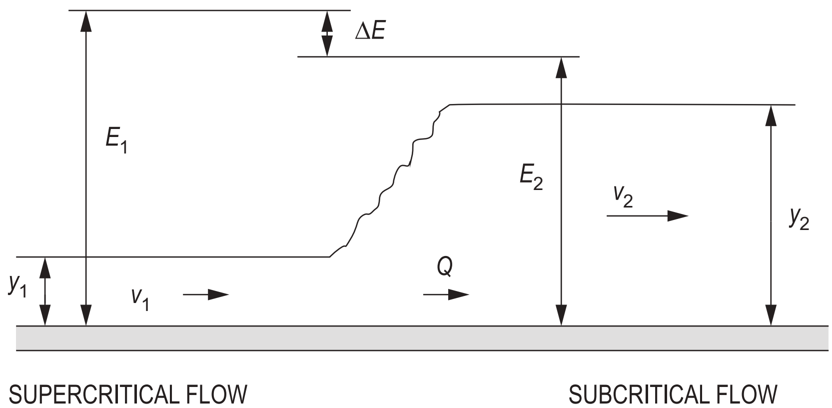
$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8\,Fr_1^2} - 1\right)$$

where

y_1 = flow depth at upstream supercritical flow location (ft)

y_2 = flow depth at downstream subcritical flow location (ft)

Fr_1 = Froude number at upstream supercritical flow location



$$y_1 = -\frac{1}{2}y_2 + \sqrt{\frac{2v_2^2 y_2}{g} + \frac{y_2^2}{4}}$$

$$y_2 = -\frac{1}{2}y_1 + \sqrt{\frac{2v_1^2 y_1}{g} + \frac{y_1^2}{4}}$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left(\sqrt{1 + 8(Fr_2)^2} - 1\right)$$

$$v_1^2 = \left(\frac{gy_2}{2y_1}\right)(y_1 + y_2)$$

$$\Delta E = \left(y_1 + \frac{v_1^2}{2g}\right) - \left(y_2 + \frac{v_2^2}{2g}\right)$$

where

y = depth at point (ft)

g = acceleration due to gravity = 32.2 ft/sec^2 or 9.81 m/s^2

Fr = Froude number

v = velocity (ft/sec)

ΔE = change in energy (ft-lbf/lbm)

Hydraulic Jump Height

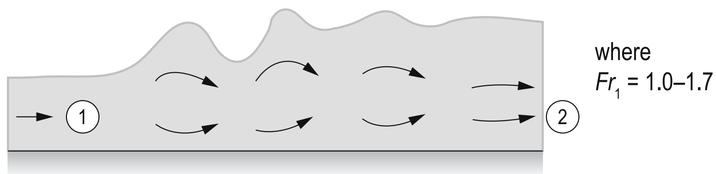
$$h_j = \Delta h = y_2 - y_1$$

where

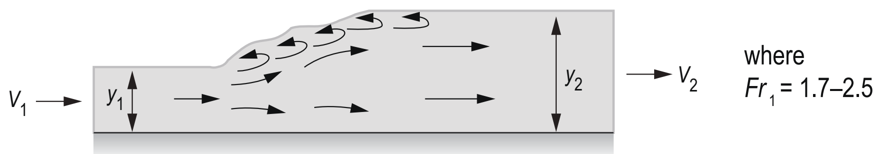
h_j = difference in depths after and before the hydraulic jump

6.4.8.2 Classification of Hydraulic Jumps

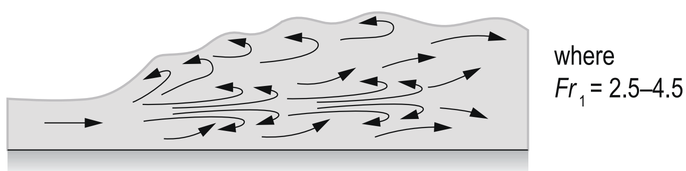
Undular Jumps - Low energy dissipation rate - Smooth downstream water surface



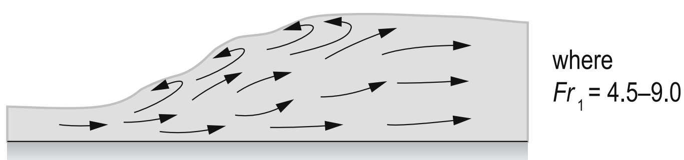
Weak Jumps - Low energy dissipation rate - Smooth downstream water surface



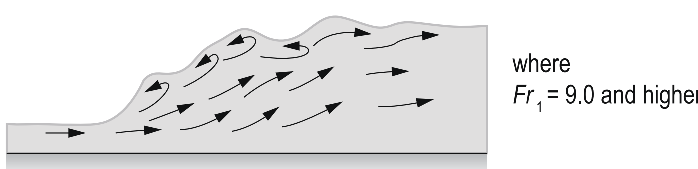
Oscillating Jumps - Irregular fluctuations of flow - Causes turbulence downstream



Steady Jumps - Jump forms steadily at same location and is well balanced - Turbulence is confined within the jump



Strong Jumps - Large change in depth of the water surface - High energy dissipation rate



Source: Chow, Ven Te. *Open-Channel Hydraulics*. Estate of Ven Te Chow, 1959, Table 15.2, p. 395.

6.4.8.3 Pre-Hydraulic Jump (Upstream)

Energy level:

$$E_1 = y_1 + \frac{v_1^2}{2g}$$

where

y = depth at point (ft)

g = acceleration due to gravity (32.2 ft/sec^2)

Fr = Froude number

v = velocity (ft/sec)

E = specific energy (ft-lbf/lbm)

6.4.8.4 Energy Loss in Hydraulic Jumps

Relative loss of hydraulic jump:

$$\frac{E_2}{E_1} = 1 - \frac{\Delta E}{E_1} = \frac{\left(1 + 8\,Fr_1^2\right)^{3/2} - 4Fr_1^2 + 1}{8\,Fr_1^2\left(2 + Fr_1^2\right)}$$

6.4.8.5 Hydraulic Jump Efficiency (Relative Jump Height)

$$\frac{h_j}{E_1} = \frac{y_2}{E_1} - \frac{y_1}{E_1} = \frac{\sqrt{1 + 8Fr_1^2} - 3}{Fr_1^2 + 2}$$

6.4.8.6 Energy Loss in Horizontal Hydraulic Jump

$$\Delta E = \frac{\left(y_2 - y_1\right)^3}{4y_1 y_2}$$

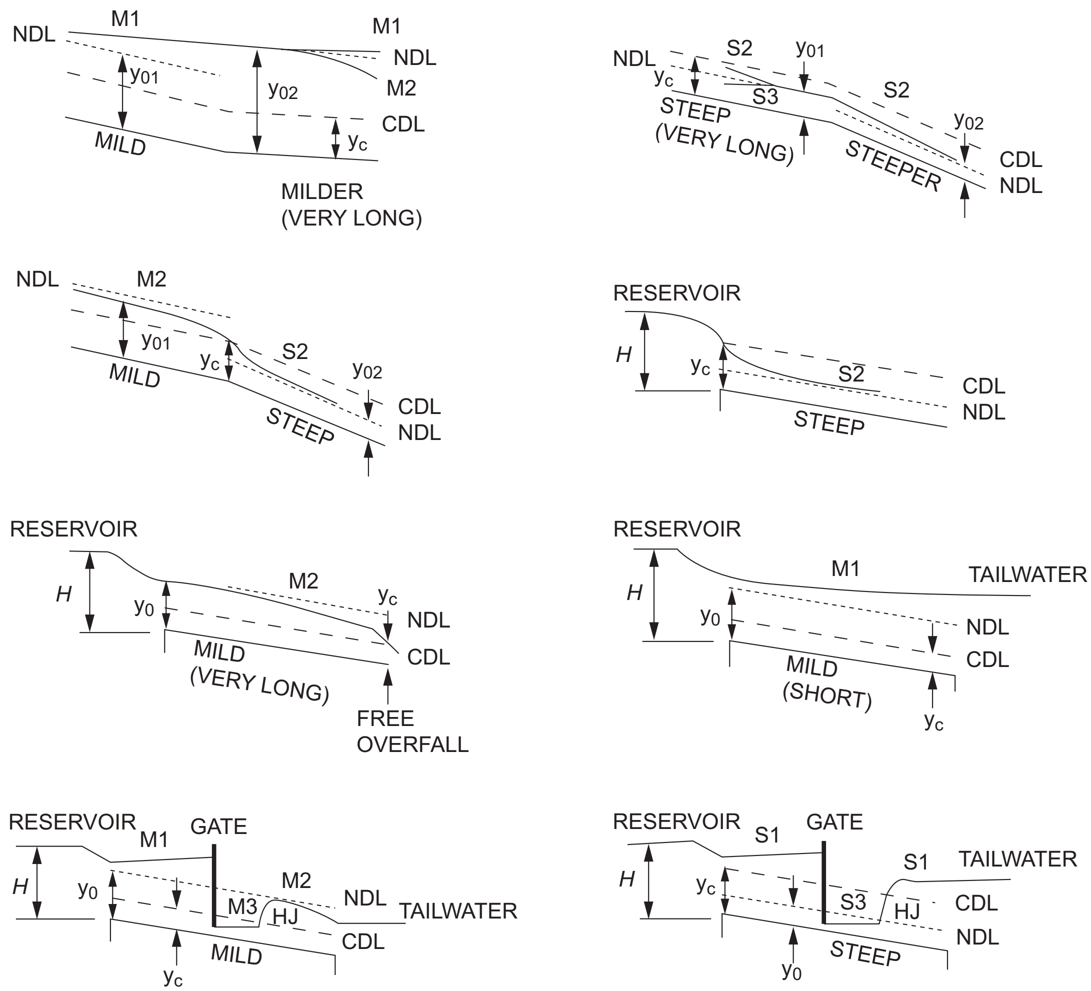
where

y_1 = depth upstream of the jump (ft)

y_2 = depth downstream of the jump (ft)

ΔE = energy loss (ft)

6.4.9 Composite Slopes Channel Profiles



where

H = depth of flow

y_n = normal depth

y_c = critical depth

y_0 = depth at that point

CDL = critical depth line

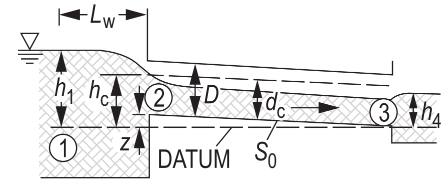
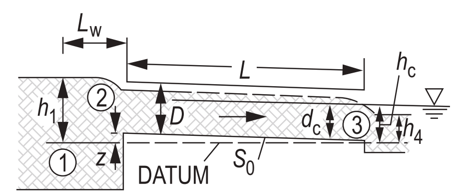
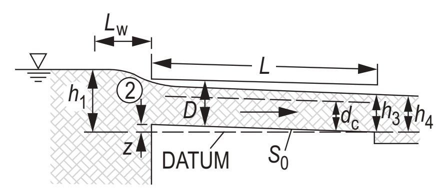
NDL = normal depth line

M and S designations represent points on mild or steep slope, respectively

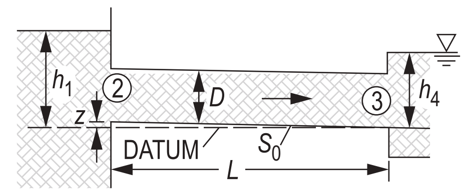
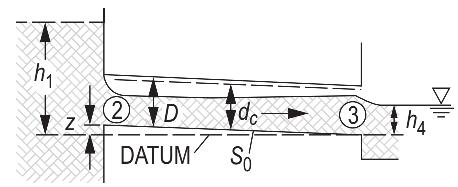
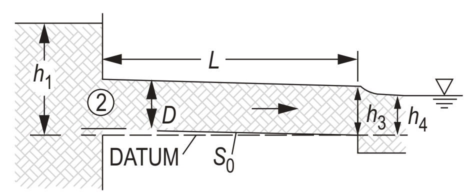
6.4.10 Stormwater Collection and Drainage

6.4.10.1 Classification of Culvert Flow (USGS Method)

Definitions for variables in the equations are listed on p. 360.

USGS Classification	Typical Slopes	Flow Type	Submerged Inlet ($HW > D$)	Submerged Outlet ($TW > D$)	Control Section	Diagram
Type 1 Critical Depth at Inlet	Steep	Partially Filled	No	No	Inlet	$Q = CA_c\sqrt{2g\left(h_1 - z + a_1\frac{V_1^2}{2g} - d_c - h_{f_{1,2}}\right)}$ 
Type 2 Critical Depth at Outlet	Mild	Partially Filled	No	No	Outlet	$Q = CA_c\sqrt{2g\left(h_1 + \alpha_1\frac{V_1^2}{2g} - d_c - h_{f_{1,2}} - h_{f_{2,3}}\right)}$ 
Type 3 Tranquil Flow Throughout	Mild	Partially Filled	No	No		$Q = CA_3\sqrt{2g\left(h_1 + \alpha_1\frac{V_1^2}{2g} - h_3 - h_{f_{1,2}} - h_{f_{2,3}}\right)}$ 

Source: *Measurement of Peak Discharge at Culverts by Indirect Methods*, G.L. Bodhaine (1968)
Chapter A3: "Techniques of Water-Resources Investigations of the United States Geological Survey," figure 2 p. 2, U.S. Department of the Interior.

USGS Classification	Typical Slopes	Flow Type	Submerged Inlet ($HW > D$)	Submerged Outlet ($TW > D$)	Control Section	Diagram
Type 4 Submerged Outlet	Any	Full	Yes	Yes	Outlet	$Q = CA_0 \sqrt{\dfrac{2g(h_1 - h_4)}{1 + \dfrac{29C^2 n^2 L}{R_0^{4/3}}}}$ 
Type 5 Rapid Flow at Inlet	Any	Partial	Yes	No		$Q = CA_0 \sqrt{2g(h_1 - z)}$ 
Type 6 Full Flow Free Outfall	Any	Full	Yes	No	Inlet	$Q = CA_0 \sqrt{2g(h_1 - h_3 - h_{f_{2,3}})}$ 

Source: *Measurement of Peak Discharge at Culverts by Indirect Methods*, G.L. Bodhaine (1968)
Chapter A3: "Techniques of Water-Resources Investigations of the United States Geological Survey," figure 2 p. 2, U.S. Department of the Interior.

where

A_c = area of section of flow at critical depth (ft^2)

A_3 = area of section at flow at exit end of culvert (ft^2)

A_0 = area of culvert barrel (ft^2)

R_o = hydraulic radius of a culvert barrel (ft)

C = coefficient of discharge

d_c = maximum depth in critical-flow section (ft)

g = gravitational constant (acceleration) (ft/sec^2)

h = static or piezometric head above an arbitrary datum (ft)

h_f = head loss due to friction (ft)

L = length of culvert barrel, bridge abutment, or broad-crested weir in direction of flow (ft)

n = Manning's roughness coefficient ($ft^{1/6}$)

Q = total discharge (ft^2/sec)

V = mean velocity of flow in a section (ft/sec)

z = elevation of a point above a datum (ft)

α = velocity-head coefficient = 29 (USCS units) = 19.6 (SI units)

6.4.10.2 Inlet and Outlet Control (Federal Highway Administration)

Culvert Head Loss, Total

$$H = H_e + H_f + H_0 + H_b + H_j + H_g$$

where

H = H_L = total head loss (ft)

H_e = entrance loss (ft)

H_f = friction loss (ft)

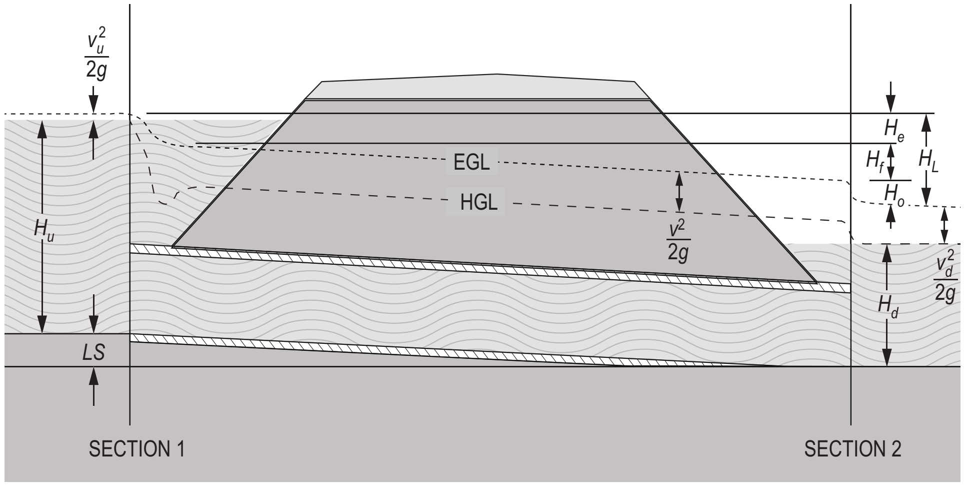
H_o = exit loss (ft)

H_b = bend losses (ft)

H_j = junction losses (ft)

H_g = grate losses (ft)

Headwater and Tailwater Levels



Full Flow Energy and Hydraulic Grade Lines

Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Fig. 3.8, p. 3.11. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.

$$H_u + LS + \frac{v_u^2}{2g} = H_d + \frac{v_d^2}{2g} + H_L$$

where

H_u = headwater depth above entrance invert in outlet control (ft)

LS = elevation drop through the culvert (ft) = length × slope

v_u = upstream/approach velocity (ft/sec)

H_d = tailwater depth above the outlet invert (ft)

v_d = downstream velocity (ft/sec)

H_L = sum of all head losses (ft)

g = acceleration due to gravity (ft/sec^2)

Entrance Head Loss

$$H_e = k_e\left(\frac{v^2}{2g}\right)$$

where

H_e = entrance head loss (ft)

k_e = entrance loss coefficient

v = entrance velocity (ft/sec)

g = acceleration due to gravity (ft/sec^2)

Friction Head Loss

$$H_f = \left(1 + k_e + \frac{K_u n^2 L}{R_H^{1.33}}\right)\frac{v^2}{2g}$$

where

H_f = friction head loss (ft)

K_u = 29 for USCS units

n = Manning's roughness coefficient

L = length of culvert section (ft)

A = cross-sectional area of flow (ft^2)

R_H = hydraulic radius (ft) = $\frac{A}{P}$

P = wetted perimeter (ft)

v = flow velocity (ft/sec)

g = acceleration due to gravity (ft/sec^2)

Composite Culvert Roughness

$$n_c = \left[\frac{\sum_{i=1}^{G}\left(p_i n_i^{1.5}\right)}{P}\right]^{0.57}$$

where

n_c = composite Manning's roughness coefficient

G = number of different roughness materials in the perimeter

p_1 = wetted perimeter influenced by material 1 (ft)

p_2 = wetter perimeter influenced by material 2 (ft), etc…

n_1 = Manning's roughness coefficient for material 1

n_2 = Manning's roughness coefficient for material 2, etc…

P = total wetted perimeter (ft)

Grate (Bar Rack) Head Loss

$$H_g = 1.5\left(\frac{V_g^2 - V_u^2}{2g}\right)$$

$$H_g = K_g \frac{W}{X}\left(\frac{v_u^2}{2g}\right)\sin\theta_g$$

where

H_g = head loss through bar rack (ft)

K_g = dimensionless bar shape factor, values below

2.42 = sharp-edged rectangular bars

1.83 = rectangular bars with semicircular upstream face

1.79 = circular bars

1.67 = rectangular bars with semicircular upstream and downstream faces

W = maximum cross-sectional width of the bars facing the flow (ft)

X = minimum clear spacing between bars (ft)

θ_g = angle of the grate with respect to the horizontal (degrees)

g = acceleration due to gravity (ft/sec^2)

Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, p. 2.17. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.

or

Refer to Wastewater Collection and Treatment.

Inlet Control Design

Unsubmerged Weir Discharge:

$$\frac{HW_i}{D} = \frac{H_c}{D} + K\left[\frac{K_u Q}{AD^{0.5}}\right]^M + K_S S \quad \text{up to } Q/AD^{0.5} = 3.5$$

where

HW_i = headwater depth above inlet control section invert (ft or m)

D = interior height of culvert barrel (ft or m)

H_c = specific head at critical depth (ft or m) = $d_c + \frac{v_c^2}{2g}$

Q = discharge (ft^3/sec or m^3/s)

A = full cross-sectional area of culvert barrel (ft^2 or m^2)

S = culvert barrel slope (ft/ft or m/m)

K, M, c, Y = constants from the following tables

K_u = unit conversion 1.0 (1.811 SI)

K_S = slope correction, –0.5 (mitered inlets + 0.7)

Submerged Orifice Discharge:

$$\frac{HW_i}{D} = c\left[\frac{K_u Q}{AD^{0.5}}\right]^2 + Y + K_S S \quad \text{when } Q/AD^{0.5} = 4.0$$

Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, p. A.1 and A.2. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.

Constants for Inlet Control Equations for Charts

Chart No.	Shape and Material	Nomograph Scale	Inlet Configuration	Equation Form	Unsubmerged *K*	Unsubmerged *M*	Submerged *c*	Submerged *Y*	References
1	Circular Concrete	1	Square edge w/headwall	1	0.0098	2.0	0.0398	0.67	1, 2
1	Circular Concrete	2	Groove end w/headwall	1	0.0018	2.0	0.0292	0.74	1, 2
1	Circular Concrete	3	Groove end projecting	1	0.0045	2.0	0.0317	0.69	1, 2
2	Circular CM	1	Headwall	1	0.0078	2.0	0.0379	0.69	1, 2
2	Circular CM	2	Mitered to slope	1	0.0210	1.33	0.0463	0.75	1, 2
2	Circular CM	3	Projecting	1	0.0340	1.50	0.0553	0.54	1, 2
3	Circular	A	Beveled ring, 45° bevels	1	0.0018	2.50	0.0300	0.74	2
3	Circular	B	Beveled ring, 33.7° bevels*	1	0.0018	2.50	0.0243	0.83	2
8	Rect. Box Concrete	1	30° to 75° wingwall flares	1	0.026	1.0	0.0347	0.81	1, 3
8	Rect. Box Concrete	2	90° and 15° wingwall flares	1	0.061	0.75	0.0400	0.80	1, 3
8	Rect. Box Concrete	3	0° wingwall flares	1	0.061	0.75	0.0423	0.82	1, 3
9	Rect. Box Concrete	1	45° wingwall flare d = 0.043D	2	0.510	0.667	0.0309	0.80	3
9	Rect. Box Concrete	2	18° to 33.7° wingwall flare d = 0.083D	2	0.486	0.667	0.0249	0.83	3
10	Rect. Box Concrete	1	90° headwall w/0.75 in. chamfers	2	0.515	0.667	0.0375	0.79	3
10	Rect. Box Concrete	2	90° headwall w/45° bevels	2	0.495	0.667	0.0314	0.82	3
10	Rect. Box Concrete	3	90° headwall w/33.7° bevels	2	0.486	0.667	0.0252	0.865	3
11	Rect. Box Concrete	1	0.75 in. chamfers; 45° skewed headwall	2	0.545	0.667	0.04505	0.73	3
11	Rect. Box Concrete	2	0.75 in. chamfers; 30° skewed headwall	2	0.533	0.667	0.0425	0.705	3
11	Rect. Box Concrete	3	0.75 in. chamfers; 15° skewed headwall	2	0.522	0.667	0.0402	0.68	3
11	Rect. Box Concrete	4	45° bevels; 10° – 45° skewed headwall	2	0.498	0.667	0.0327	0.75	3
12	Rect. Box 3/4" chamf. Conc.	1	45° non-offset wingwall flares	2	0.497	0.667	0.0339	0.803	3
12	Rect. Box 3/4" chamf. Conc.	2	18.4° non-offset wingwall flares	2	0.493	0.667	0.0361	0.806	3
12	Rect. Box 3/4" chamf. Conc.	3	18.4° non-offset wingwall flares 30° skewed barrel	2	0.495	0.667	0.0386	0.71	3
13	Rect. Box Top Bev. Conc.	1	45° wingwall flares – offset	2	0.497	0.667	0.0302	0.835	3
13	Rect. Box Top Bev. Conc.	2	33.7° wingwall flares – offset	2	0.495	0.667	0.0252	0.881	3
13	Rect. Box Top Bev. Conc.	3	18.4° wingwall flares – offset	2	0.493	0.667	0.0227	0.887	3
55	Circular	1	Smooth tapered inlet throat	2	0.534	0.555	0.0196	0.90	4
55	Circular	2	Rough tapered inlet throat	2	0.519	0.64	0.0210	0.90	4
56	Ellipital Face	1	Tapered inlet-beveled edges	2	0.536	0.622	0.0368	0.83	4
56	Ellipital Face	2	Tapered inlet-square edges	2	0.5035	0.719	0.0478	0.80	4
56	Ellipital Face	3	Tapered inlet-thin edge projecting	2	0.547	0.80	0.0598	0.75	4
57	Rectangular Concrete	1	Tapered inlet throat	2	0.475	0.667	0.0179	0.97	4
58	Rectangular Concrete	1	Side tapered-less favorable edges	2	0.56	0.667	0.0446	0.85	4
58	Rectangular Concrete	2	Side tapered-more favorable edges	2	0.56	0.667	0.0378	0.87	4
59	Rectangular Concrete	1	Slope tapered-less favorable edges	2	0.50	0.667	0.0446	0.65	4
59	Rectangular Concrete	2	Slope tapered-more favorable edges	2	0.50	0.667	0.0378	0.71	4

Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Table A.1, p. A.8. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.

Constants for Inlet Control Equations for Discontinued Charts

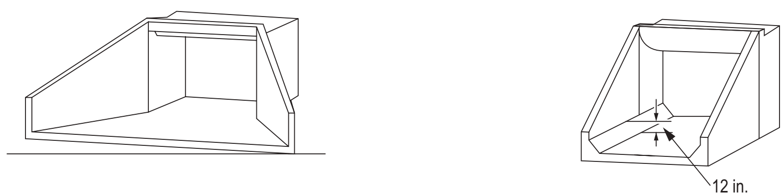
Chart No.	Shape and Material	Nomograph Scale	Inlet Configuration	Equation Form	Unsubmerged *K*	Unsubmerged *M*	Submerged *c*	Submerged *Y*	References
16–19	Boxes CM	2	90° headwall	1	0.0083	2.0	0.0379	0.69	FHWA 1974
16–19	Boxes CM	3	Thick wall projecting	1	0.0145	1.75	0.0419	0.64	FHWA 1974
16–19	Boxes CM	5	Thin wall projecting	1	0.0340	1.5	0.0496	0.57	FHWA 1974
29	Horizontal Ellipse Concrete	1	Square edge w/headwall	1	0.0100	2.0	0.0398	0.67	FHWA 1974
29	Horizontal Ellipse Concrete	2	Groove end w/headwall	1	0.0018	2.5	0.0292	0.74	FHWA 1974
29	Horizontal Ellipse Concrete	3	Groove end projecting	1	0.0045	2.0	0.0317	0.69	FHWA 1974
30	Vertical Ellipse Concrete	1	Square edge w/headwall	1	0.0100	2.0	0.0398	0.67	FHWA 1974
30	Vertical Ellipse Concrete	2	Groove end w/headwall	1	0.0018	2.5	0.0292	0.74	FHWA 1974
30	Vertical Ellipse Concrete	3	Groove end projecting	1	0.0095	2.0	0.0317	0.69	FHWA 1974
34	Pipe Arch 18 in. Corner radius CM	1	90° headwall	1	0.0083	2.0	0.0379	0.69	FHWA 1974
34	Pipe Arch 18 in. Corner radius CM	2	Mitered to slope	1	0.0300	1.0	0.0463	0.75	FHWA 1974
34	Pipe Arch 18 in. Corner radius CM	3	Projecting	1	0.0340	1.5	0.0496	0.57	FHWA 1974
35	Pipe Arch 18 in. Corner radius CM	1	Projecting	1	0.0300	1.5	0.0496	0.57	Bossy 1963
35	Pipe Arch 18 in. Corner radius CM	2	No Bevels	1	0.0088	2.0	0.0368	0.68	Bossy 1963
35	Pipe Arch 18 in. Corner radius CM	3	33.7°Bevels	1	0.0030	2.0	0.0269	0.77	Bossy 1963
36	Pipe Arch 31 in. Corner radius CM	1	Projecting	1	0.0300	1.5	0.0496	0.57	Bossy 1963
36	Pipe Arch 31 in. Corner radius CM	2	No Bevels	1	0.0088	2.0	0.0368	0.68	Bossy 1963
36	Pipe Arch 31 in. Corner radius CM	3	33.7° Bevels	1	0.0030	2.0	0.0269	0.77	Bossy 1963
41–43	Arch CM	1	90° headwall	1	0.0083	2.0	0.0379	0.69	FHWA 1974
41–43	Arch CM	2	Mitered to slope	1	0.0300	1.0	0.0473	0.75	FHWA 1974
41–43	Arch CM	3	Thin wall projecting	1	0.0340	1.5	0.0496	0.57	FHWA 1974

Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Table A.2, p. A.9. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.

Constants for Inlet Control Equations for South Dakota Concrete Box (HY-8 User Manual and Table 11 of FHWA 2006)

Sketch	Wingwall Flare	Top Bevel	Top Radius	Corner Fillet	RCB Inlet Configuration	Equation Form	Unsubmerged *K*	Unsubmerged *M*	Submerged *c*	Submerged *Y*
1	30°	45°	–	–	Single barrel	2	0.44	0.74	0.040	0.48
2	30°	45°	–	6 in.	Multiple barrel (2, 3, and 4 cells)	2	0.47	0.68	0.04	0.62
3	30°	45°	–	–	Single barrel (2:1 to 4:1 span-to-rise ratio)	2	0.48	0.65	0.041	0.57
4	30°	45°	–	–	Multiple barrels (15° skewed headwall)	2	0.69	0.49	0.029	0.95
5	30°	45°	–	–	Multiple barrels (30° to 45° skewed headwall)	2	0.69	0.49	0.027	1.02
6	0°	none	–	–	Single barrel, top edge 90°	2	0.55	0.64	0.047	0.55
7	0°	45°	–	6 in.	Single barrel (0 to 6 in. corner fillets)	2	0.56	0.62	0.045	0.55
8	0°	45°	–	6 in.	Multiple barrels (2, 3, and 4 cells)	2	0.55	0.59	0.038	0.69
9	0°	45°	–	–	Single barrels (2:1 to 4:1 span-to-rise ratio)	2	0.61	0.57	0.041	0.67
10	0°	–	8 in.	6 in.	Single barrel (0 and 6 in. fillets)	2	0.56	0.62	0.038	0.67
11	0°	–	8 in.	12 in.	Single barrel (12 in. corner fillets)	2	0.56	0.62	0.038	0.67
12	0°	–	8 in.	12 in.	Multiple barrels (2, 3, and 4 cells)	2	0.55	0.6	0.023	0.96
13	0°	–	8 in.	12 in.	Single barrel (2:1 to 4:1 span-to-rise ratio)	2	0.61	0.57	0.033	0.79

Sketches are shown in the HY-8 documentation and research report. Since sketches 2 and 8 show fillets, a 6 in. fillet is assumed.

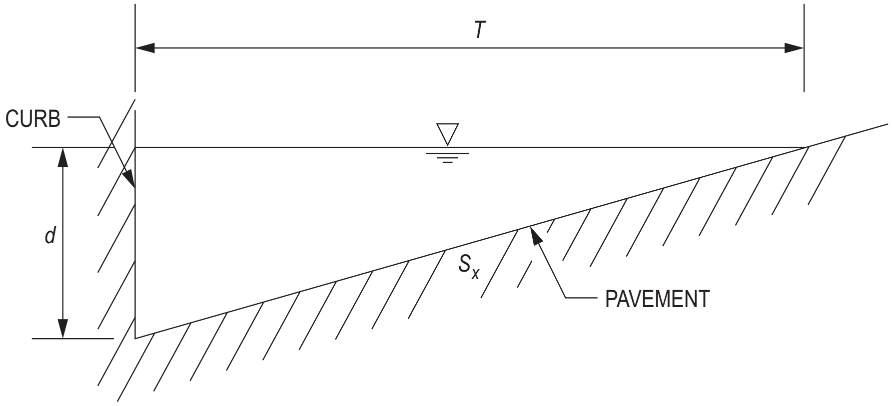


Sketches 1 through 5 have this configuration

Sketches 7 through 13 have this configuration

Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Table A.3, p. A.10. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.

6.4.10.3 Curb and Gutter as Triangular Channels



$$Q = \left(K_u/n\right) S_x^{1.67} S_L^{0.5} T^{2.67}$$

$$Q = \frac{K_u}{n}\frac{S_L^{0.5}}{S_x} d^{8/3}$$

$$T = \left(\frac{Qn}{K_u S_x^{1.67} S_L^{0.5}}\right)^{0.375}$$

where

K_u = 0.56 in USCS units

n = Manning's roughness coefficient

Q = flow rate (ft^3/sec)

T = width of flow, spread (ft)

S_x = cross slope (ft/ft)

S_L = longitudinal slope (ft/ft)

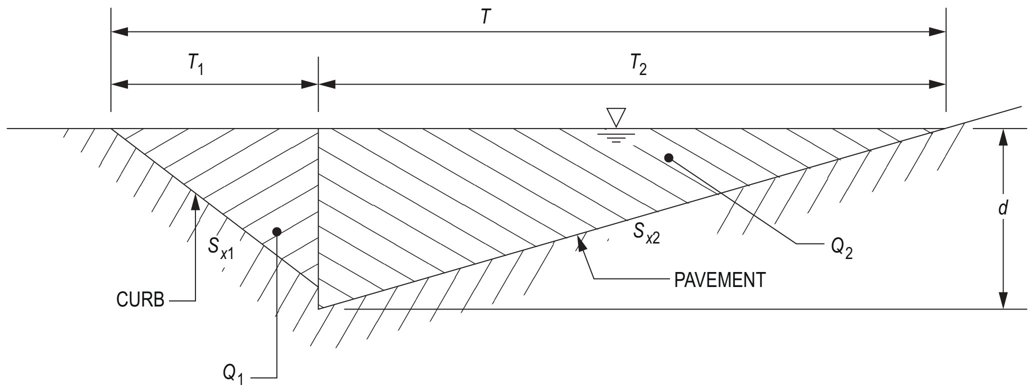
d = depth of flow (ft)

$$v = \frac{Q}{A} = \frac{1.12}{n} S^{0.5} S_x^{0.67} T^{0.67} = \frac{2Q}{T^2 S_x}$$

where

v = velocity (ft/sec)

A = cross-sectional area (ft^2)



$$Q = Q_1 + Q_2 = \left(\frac{0.56}{n}\right)\left(S^{0.5}\right)\left(S_{x1}^{1.67}T_1^{2.67} + S_{x2}^{1.67}T_2^{2.67}\right)$$

$$T = T_1 + T_2 = \left[\frac{Q_1 n}{0.56S^{0.5}S_{x1}^{1.67}}\right]^{0.375} + \left[\frac{Q_2 n}{\left(0.56S^{0.5}S_{x2}\right)^{1.67}}\right]^{0.375}$$

$$v = \frac{Q}{A} = \frac{2Q}{T_1^2 S_{x1} + T_2^2 S_{x2}}$$

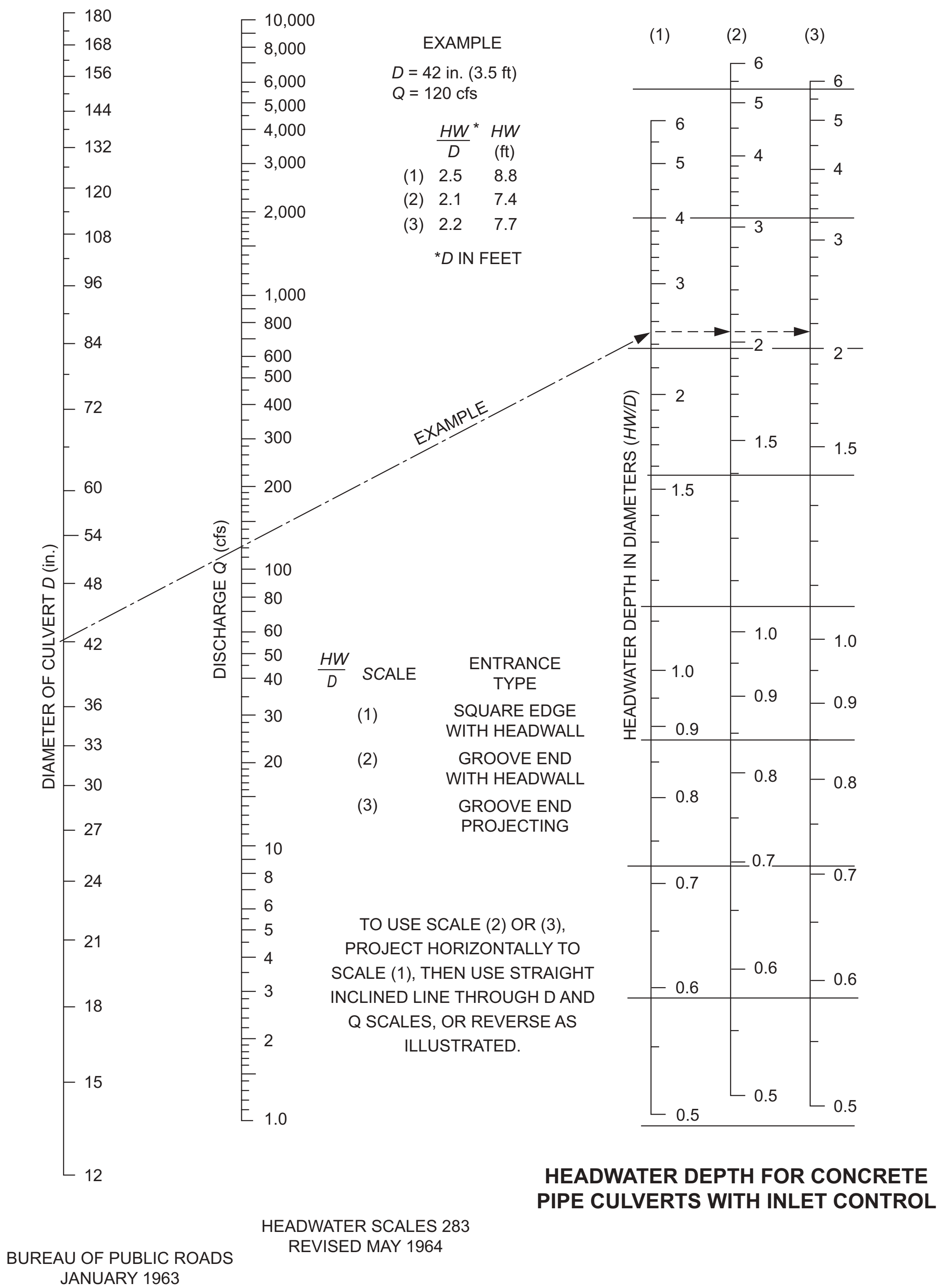
$$d = T_2 S_{x2}$$

Entrance Loss Coefficients
Outlet Control, Full or Partially Full Entrance Head Loss

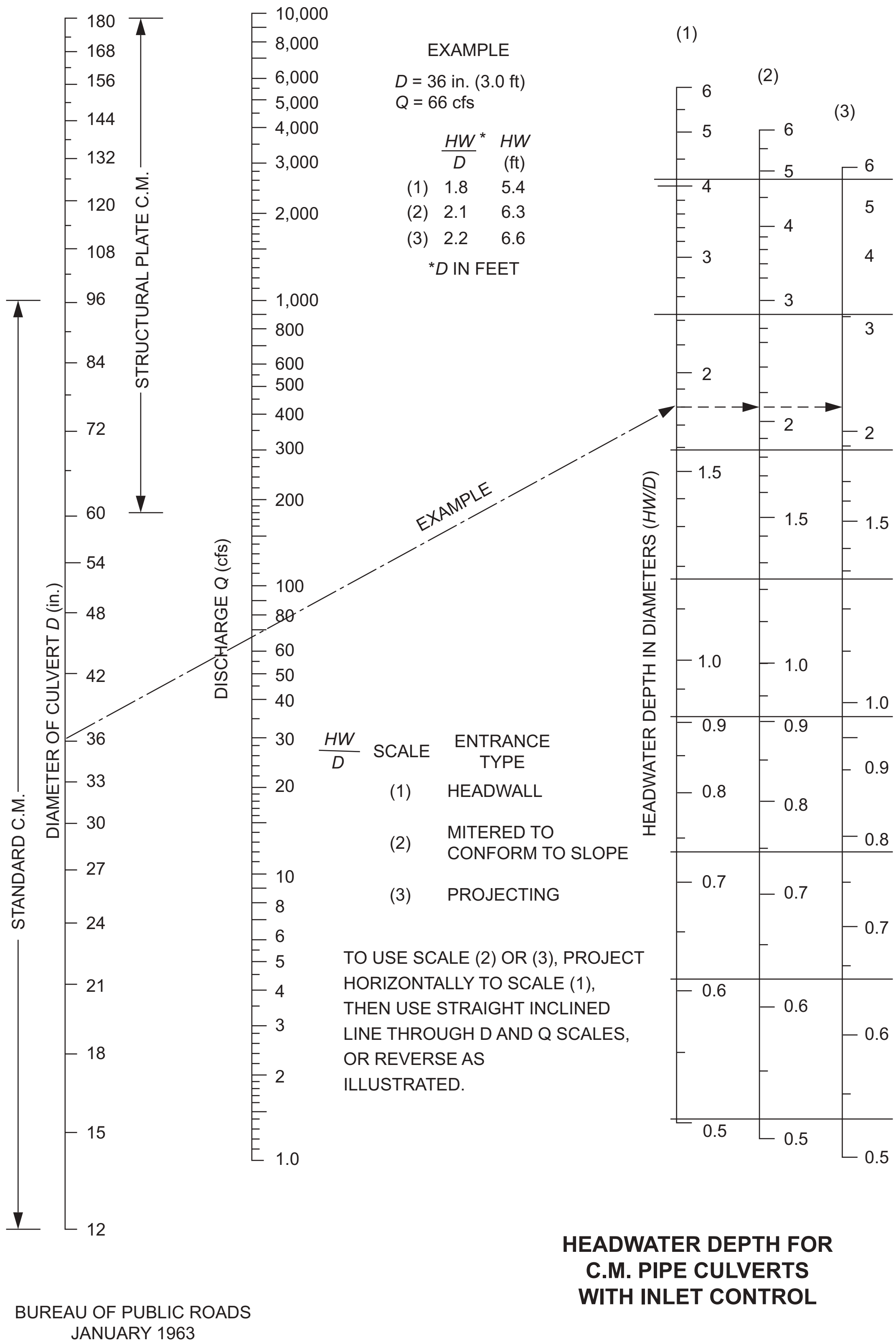
Type of Structure and Design of Entrance	Coefficient k_e
Pipe, Concrete	
Projecting from fill, socket end (groove-end)	0.2
Project from fill, sq. cut end	0.5
Headwall or headwall and wingwalls	
Socket end of pipe (groove-end)	0.2
Square-edge	0.5
Rounded (radius = *D*/12)	0.2
Mitered to conform to fill slope	0.7
*End-Section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side- or slope-tapered inlet	0.2
Pipe, or Pipe-Arch, Corrugated Metal	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls square-edge	0.5
Mitered to conform to fill slope, paved or unpaved slope	0.7
*End section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side- or slope-tapered inlet	0.2
Box, Reinforced Concrete	
Headwall parallel to embankment (no wingwalls)	
Square-edged on 3 edges	0.5
Rounded on 3 edges to radius of *D*/12 or *B*/12 or beveled edges on 3 sides	0.2
Wingwalls at 30° to 75° to barrel	
Square-edges at crown	0.4
Crown edge rounded to radius of *D*/12 or beveled top edge	0.2
Wingwall at 10° to 25° to barrel	
Square-edge at crown	0.5
Wingwalls parallel (extension of sides)	
Square-edged at crown	0.7
Side- or slope-tapered inlet	0.2

*Note: End sections conforming to fill slope, made of either metal or concrete, are the sections commonly available from manufacturers. From limited hydraulic tests, they are equivalent in operation to a headwall in both inlet and outlet control. Some end sections, incorporating a closed taper in their design, have a superior hydraulic performance. These latter sections can be designed using the information given for the beveled inlet.

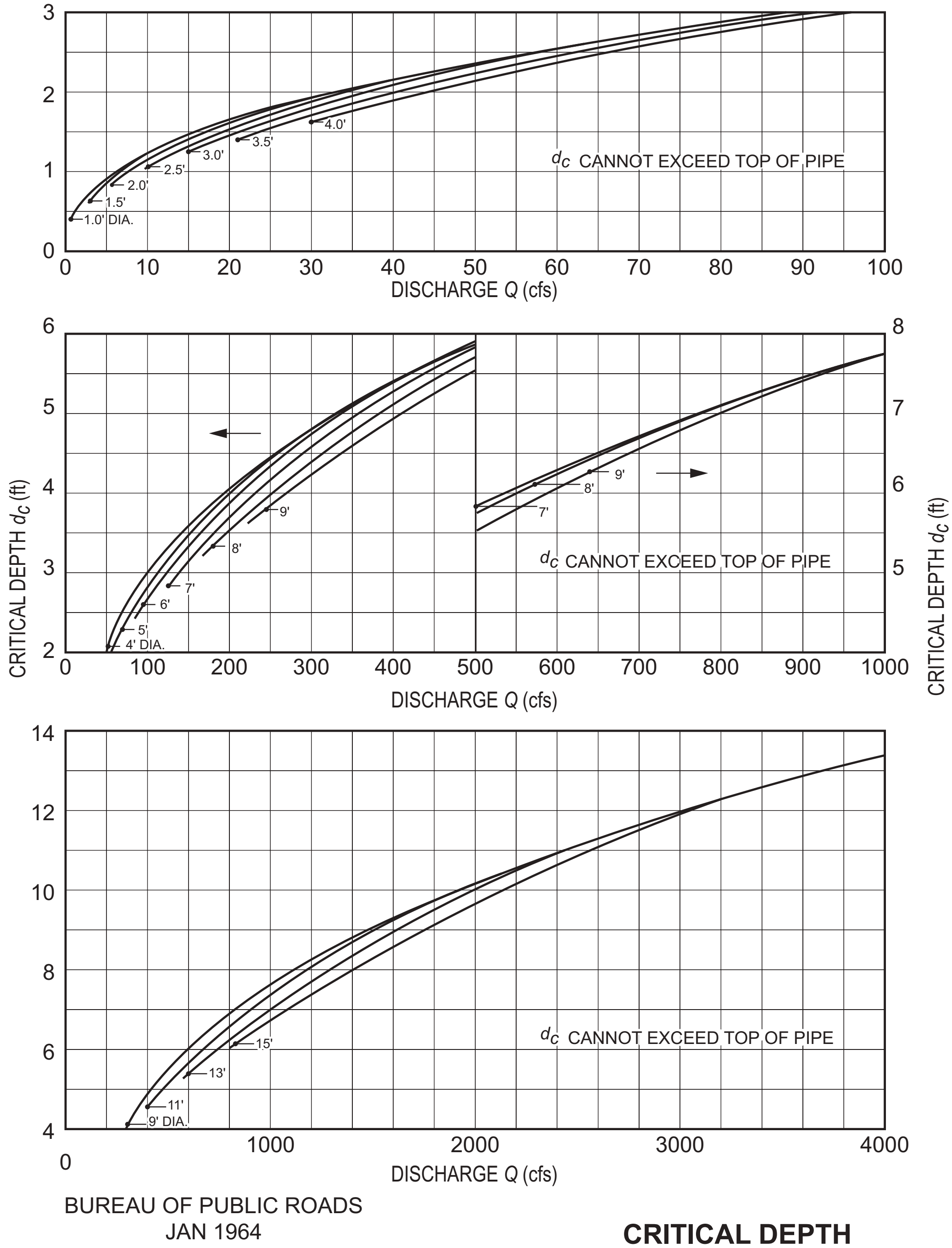
Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Table C.2, p. C.6. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.



Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Chart 1B, p. C.9. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.

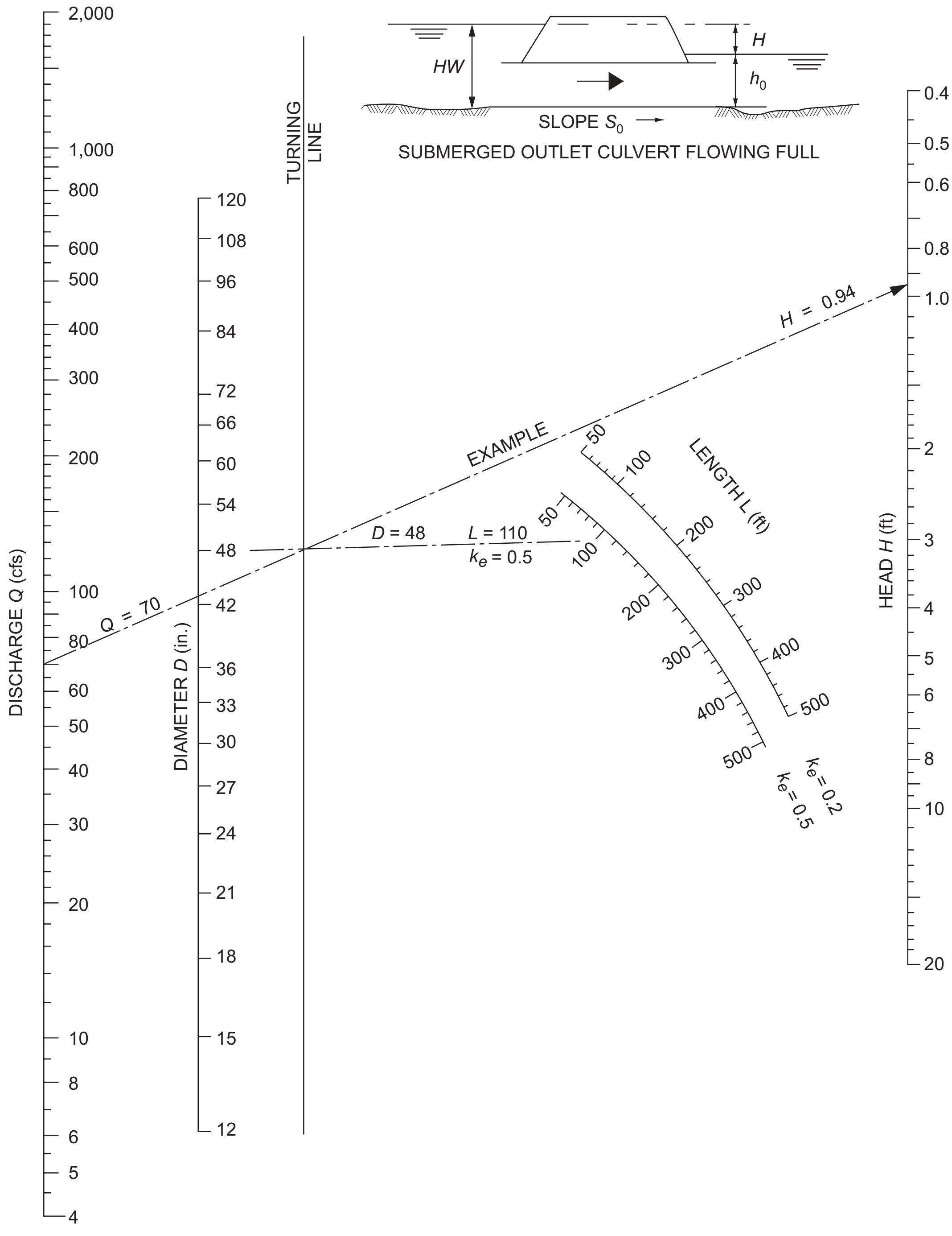


Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Chart 2B, p. C.11. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.



CRITICAL DEPTH CIRCULAR PIPE

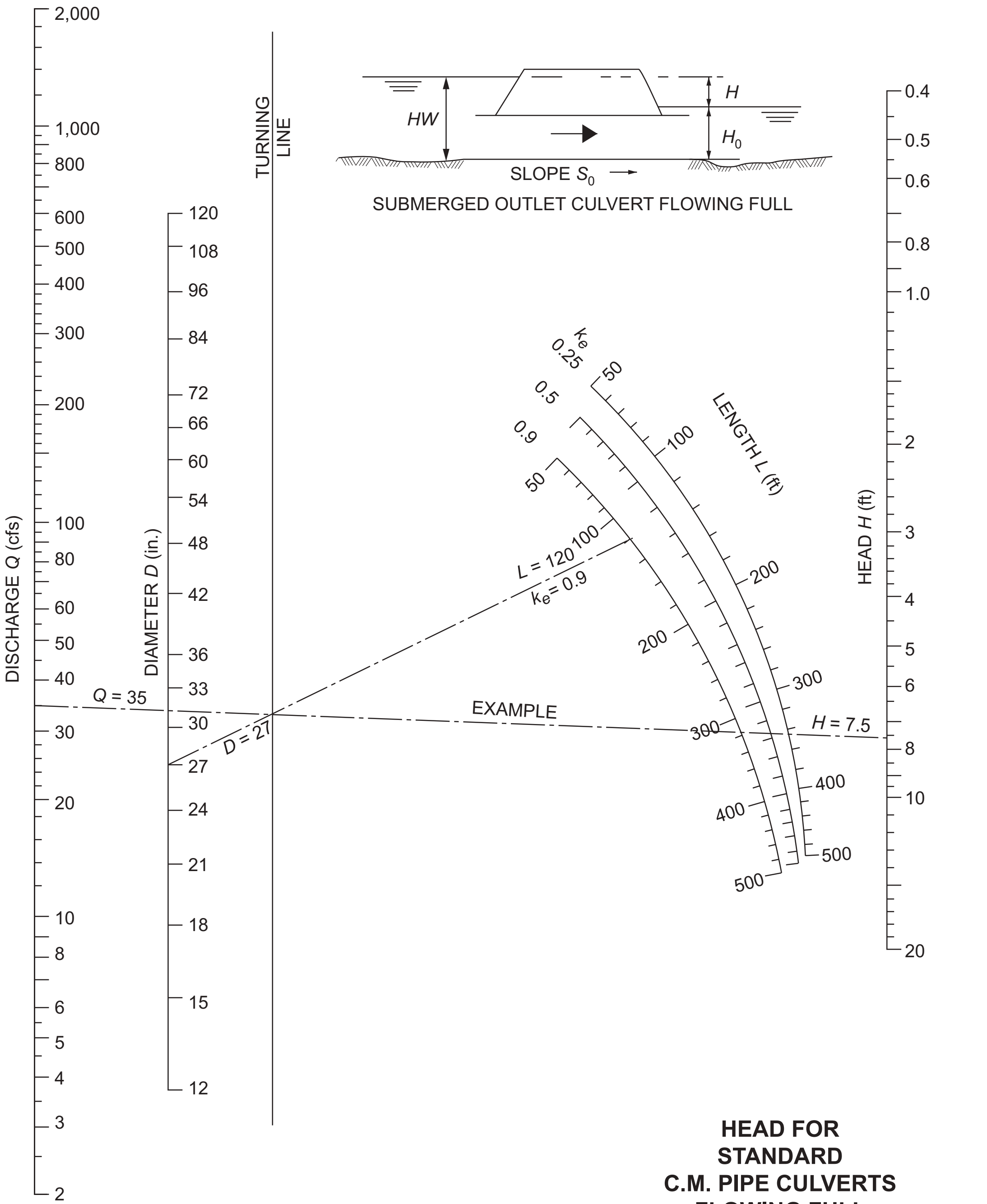
Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Chart 4B, p. C.5. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.



HEAD FOR
CONCRETE PIPE CULVERTS
FLOWING FULL
***n* = 0.012**
WHERE HW/D IS BETWEEN 0.5 TO 3.0

BUREAU OF PUBLIC ROADS JAN. 1963

Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Chart 5B, p. C.17. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.



HEAD FOR STANDARD C.M. PIPE CULVERTS FLOWING FULL $n = 0.024$ WHERE HW/D IS BETWEEN 0.5 TO 3.0

BUREAU OF PUBLIC ROADS JAN. 1963

Source: Federal Highway Administration. *Hydraulic Design of Highway Culverts: Hydraulic Design Series Number 5*. 3rd ed. FHWA-HIF-12-026. Washington, DC: U.S. Department of Transportation, April 2012, Chart 6B, p. C.19. https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf.

6.5 Hydrology

6.5.1 Storm/Flood Frequency Probabilities

6.5.1.1 General Probability

$$p = \frac{n_i}{N}$$

where

p = probability of occurrence of a flood flow of class i (variate i)

n_i = number of items in the ith class

N = total number of items in a series

6.5.1.2 Average Recurrence Interval (ARI)

$$P(x \geq x_T) = p = \frac{1}{T}$$

where

p = probability of a single occurrence in a given storm period

x = magnitude of event

x_T = design level event

T = storm/flood return period (years)

6.5.1.3 Risk or Annual Exceedance Probability (AEP)

$$P(x \geq x_T \text{ at least once in } n \text{ years}) = p = 1 - \left(1 - \frac{1}{T}\right)^n$$

where

p = probability of exceeding a flow or intensity in a given period

n = number of years

6.5.1.4 Reliability (Probability of Nonexceedance)

$$P(x < x_T \text{ each year for } n \text{ years}) = p = \left(1 - \frac{1}{T}\right)^n$$

where

p = probability of *not* exceeding a flow or intensity in a given period

6.5.1.5 Binomial Distribution – Probability of Occurrence

$$f(k;n,p) = {}_nC_k p^k (1-p)^{n-k}$$

where

$f(x)$ = probability of k events in n trials

p = probability of an event in any one trial

k = number of desired events

n = number of trials

$${}_nC_k = C_k^n = \frac{n!}{k!(n-k)!}$$

6.5.1.6 Simplified Flood Frequency Equation

$$x_T = \bar{x} + K_T s$$

where

x_T = estimated event magnitude

$\bar{x}$ = sample mean (observed data)

T = storm/flood return period (years)

K_T = frequency factor (Normal, Extreme Value, Exponential/Pearson)

s = standard deviation of the sample

Frequency Factor for Normal Distribution

Exceedance Probability	Return Period	*K*	Exceedance Probability	Return Period	*K*
0.0001	10,000	3.719	0.450	2.22	0.126
0.0005	2,000	3.291	0.500	2.00	0.000
0.001	1,000	3.090	0.550	1.82	–0.126
0.002	500	2.88	0.600	1.67	–0.253
0.003	333	2.76	0.650	1.54	–0.385
0.004	250	2.65	0.700	1.43	–0.524
0.005	200	2.576	0.750	1.33	–0.674
0.010	100	2.326	0.800	1.25	–0.842
0.025	40	1.960	0.850	1.18	–1.036
0.050	20	1.645	0.900	1.11	–1.282
0.100	10	1.282	0.950	1.053	–1.645
0.150	6.67	1.036	0.975	1.026	–1.960
0.200	5.00	0.842	0.990	1.010	–2.326
0.250	4.00	0.674	0.995	1.005	–2.576
0.300	3.33	0.524	0.999	1.001	–3.090
0.350	2.86	0.385	0.9995	1.0005	–3.291
0.400	2.50	0.253	0.9999	1.0001	–3.719

6.5.2 Runoff Analysis

6.5.2.1 Rational Formula Method

$$Q = CIA$$

where

Q = peak discharge (ft^3/sec)

C = runoff coefficient

I = rainfall intensity from an IDF curve for a duration of t_c (in./hr)

A = watershed area (acres)

t_c = time of concentration, which is the time required for the runoff to travel from the hydraulically most distant point of the watershed to the point of interest (min)

Multiple watersheds may require summation of times to determine an overall time for the system.

Unit conversion from acre-in./hr to ft^3/sec is most often approximated to 1.0.

$$C_w = \frac{A_1 C_1 + A_2 C_2 + \dots A_n C_n}{A_1 + A_2 + \dots A_n}$$

where C_w = weighted/composite runoff coefficient for whole drainage area

Runoff Coefficients for Rational Formula

Type of Drainage Area	Runoff Coefficient, *C**
Business:	
Downtown areas	0.70–0.95
Neighborhood areas	0.50–0.70
Residential:	
Single-family areas	0.30–0.50
Multi-units, detached	0.40–0.60
Multi-units, attached	0.60–0.75
Suburban	0.25–0.40
Apartment dwelling areas	0.50–0.70
Industrial:	
Light areas	0.50–0.80
Heavy areas	0.60–0.90
Parks, cemeteries	0.10–0.25
Playgrounds	0.20–0.40
Railroad yards areas	0.20–0.40
Unimproved areas	0.10–0.30
Lawns:	
Sandy soil, flat, 2%	0.05–0.10
Sandy soil, average, 2–7%	0.10–0.15
Sandy soil, steep, 7%	0.15–0.20
Heavy soil, flat, 2%	0.13–0.17
Heavy soil, average, 2–7%	0.18–0.22
Heavy soil, steep, 7%	0.25–0.35
Streets:	
Asphaltic	0.70–0.95
Concrete	0.80–0.95
Brick	0.70–0.85
Drives and walks	0.75–0.85
Roofs	0.75–0.95

*Higher values are usually appropriate for steeply sloped areas and longer return periods because infiltration and other losses have a proportionally smaller effect on runoff in these cases.

Source: From American Society for Civil Engineering (ASCE). *Design Manual for Storm Drainage*. 3rd ed. Reston, VA: ASCE, 1960. As found in Federal Highway Administration. National Highway Institute. *Urban Drainage Design Manual: Hydraulic Engineering Circular No. 22*. 3rd ed. FHWA-NHI-10-009. Washington, DC: U.S. Department of Transportation, September 2009, Table 3-1, p. 3-6. www.fhwa.dot.gov/engineering/hydraulics/pubs/10009/10009.pdf.

6.5.2.2 NRCS (SCS) Rainfall Runoff Method

Runoff

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S}$$

$$S = \frac{1,000}{CN} - 10$$

$$CN = \frac{1,000}{S + 10}$$

where

Q = runoff depth (in.)

P = precipitation (in.)

S = maximum basin retention (in.)

CN = curve number

Loss

$$\text{Loss (in.)} = P - Q$$

$$= \frac{(S + I_a) - \frac{I_a^2}{P}}{1 - \frac{I_a}{P} + \frac{S}{P}}$$

As P becomes increasingly large,

$$\text{Loss (in.)} = S + I_a$$

where

I_a = initial abstraction (in.)

~ 0.2S

Peak Discharge Method

$$q_p = q_u A_m Q$$

where

q_p = peak flow (ft^3/sec)

q_u = unit peak flow (ft^3/sec/mi^2/in.)

A_m = basin area (mi^2)

Q = runoff depth (in.)

$q_u = 10^{C_0 + C_1 \log t_c + C_2 (\log t_c)^2}$

where

C_0, C_1, C_2 = TR-55 coefficients

t_c = time of concentration (hr)

SCS Hydrologic Soils Group Type Classifications

Group A: Deep sand, deep loess, aggregated silts

Group B: Shallow loess, sandy loam

Group C: Clay loams, shallow sandy loam, soils low in organic content, and soils usually high in clay

Group D: Soils that swell significantly when wet, heavy plastic clays, and certain saline soils

Minimum Infiltration Rates for the Various Soil Groups

Group	Minimum Infiltration Rate (in./hr)
A	0.30–0.45
B	0.15–0.30
C	0.05–0.15
D	0.00–0.05

Ratios for SCS Dimensionless Unit Hydrograph and Mass Curve

Time Ratios t/T_p	Discharge Ratios q/q_p	Mass Curve Ratios Q_a/Q
0	0.000	0.000
0.1	0.030	0.001
0.2	0.100	0.006
0.3	0.190	0.012
0.4	0.310	0.035
0.5	0.470	0.065
0.6	0.660	0.107
0.7	0.820	0.163
0.8	0.930	0.228
0.9	0.990	0.300
1.0	1.000	0.375
1.1	0.990	0.450
1.2	0.930	0.522
1.3	0.860	0.589
1.4	0.780	0.650
1.5	0.680	0.700
1.6	0.560	0.751
1.7	0.460	0.790
1.8	0.390	0.822
1.9	0.330	0.849
2.0	0.280	0.871
2.2	0.207	0.908
2.4	0.147	0.934
2.6	0.107	0.953
2.8	0.077	0.967
3.0	0.055	0.977
3.2	0.040	0.984
3.4	0.029	0.989
3.6	0.021	0.993
3.8	0.015	0.995
4.0	0.011	0.997
4.5	0.005	0.999
5.0	0.000	1.000

Source: Federal Highway Administration. National Highway Institute. *Highway Hydrology: Hydraulic Design Series Number 2*. 2nd ed. FHWA-HIF-02-001. Washington, DC: U. S. Department of Transportation, October 2002, Table 6.10, p. 6-59. https://www.fhwa.dot.gov/engineering/hydraulics/library_arc.cfm?pub_number=2&id=7.

Runoff Curve Numbers (Average Watershed Condition, $I_a = 0.2\ S_R$)

Land Use Description		Curve Numbers for Hydrologic Soil Group			
		A	B	C	D
Fully developed urban areas (vegetation established)					
Lawns, open spaces, parks, golf courses, cemeteries, etc.					
Good condition; grass cover on 75% or more of the area		39	61	74	80
Fair condition; grass cover on 50 to 75% or more of the area		49	69	79	84
Poor condition; grass cover on 50% or less of the area		68	79	86	89
Paved parking lots, roofs, driveways, etc. (excl. right-of-way)					
Streets and roads		98	98	98	98
Paved with curbs and storm sewers (excl. right-of-way)		98	98	98	98
Gravel (incl. right-of-way)		76	85	89	91
Dirt (incl. right-of-way)		72	82	87	89
Paved with open ditches (incl. right-of-way)		83	89	92	93
Average % impervious					
Commercial and business areas	85	89	92	94	95
Industrial districts	72	81	88	91	93
Row houses, town houses, and residential with lot sizes 0.05 ha (1/8 ac) or less	65	77	85	90	92
Residential: average lot size					
0.10 ha (1/4 ac)	38	61	75	83	87
0.14 ha (1/3 ac)	30	57	72	81	86
0.20 ha (1/2 ac)	25	54	70	80	85
0.40 ha (1 ac)	20	51	68	79	84
0.81 ha (2 ac)	12	46	65	77	82
Developing urban areas (no vegetation established)					
Newly graded area		77	86	91	94
Western desert urban areas:					
Natural desert landscaping (pervious area only)		63	77	85	88
Artificial desert landscaping [impervious weed barrier, desert shrub with 25 to 50 mm (1 to 2 in.) sand or gravel mulch and basin borders]		96	96	96	96
Cultivated agricultural land					
Fallow					
Straight row or bare soil		77	86	91	94
Conservation tillage Poor		76	85	90	93
Conservation tillage Good		74	83	88	90

Runoff Curve Numbers (Average Watershed Condition, $I_a = 0.2\ S_R$) (cont'd)

Land Use Description	Treatment or Practice	Hydrologic Condition	Curve Numbers for Hydrologic Soil Group			
			A	B	C	D
Row crops	Straight row	Poor	72	81	88	91
	Straight row	Good	67	78	85	89
	Conservation tillage	Poor	71	80	87	90
	Conservation tillage	Good	64	75	82	85
	Contoured	Poor	70	79	84	88
	Contoured	Good	65	75	82	86
	Contoured and conservation tillage	Poor	69	78	83	87
	Contoured and conservation tillage	Good	64	74	81	85
	Contoured and terraces	Poor	66	74	80	82
	Contoured and terraces	Good	62	71	78	81
	Contoured and terraces and conservation tillage	Poor	65	73	79	81
	Contoured and terraces and conservation tillage	Good	61	70	77	80
Small grain	Straight row	Poor	65	76	84	88
	Straight row	Good	63	75	83	87
	Conservation tillage	Poor	64	75	83	86
	Conservation tillage	Good	60	72	80	84
	Contoured	Poor	63	74	82	85
	Contoured	Good	61	73	81	84
	Contoured and conservation tillage	Poor	62	73	81	84
	Contoured and conservation tillage	Good	60	72	80	83
	Contoured and terraces	Poor	61	72	79	82
	Contoured and terraces	Good	59	70	78	81
	Contoured and terraces and conservation tillage	Poor	60	71	78	81
	Contoured and terraces and conservation tillage	Good	58	69	77	80
Close-seeded legumes rotations meadows[e]	Straight row	Poor	66	77	85	89
	Straight row	Good	58	72	81	85
	Contoured	Poor	64	75	83	85
	Contoured	Good	55	69	78	83
	Contoured and terraces	Poor	63	73	80	83
	Contoured and terraces	Good	51	67	76	80
Noncultivated agricultural land						
Pasture or range	No mechanical treatment	Poor	68	79	86	89
	No mechanical treatment	Fair	49	69	79	84
	No mechanical treatment	Good	39	61	74	80
	Contoured	Poor	47	67	81	88
	Contoured	Fair	25	59	75	83
	Contoured	Good	6	35	70	79
Meadow		–	30	58	71	78
Forestland–grass or orchards–evergreen deciduous		Poor	55	73	82	86
		Fair	44	65	76	82
		Good	32	58	72	79
Brush		Poor	48	67	77	83
		Fair	35	56	70	77
		Good	30	48	65	73
Woods		Poor	45	66	77	83
		Fair	36	60	73	79
		Good	25	55	70	77

Runoff Curve Numbers (Average Watershed Condition, $I_a = 0.2\ S_R$) (cont'd)

Land Use Description	Treatment or Practice	Hydrologic Condition	Curve Numbers for Hydrologic Soil Group			
			A	B	C	D
Farmsteads		–	59	74	82	86
Forest-range herbaceous		Poor Fair Good		80 71 62	87 81 74	93 89 85
Forest-range oak-aspen		Poor Fair Good		66 48 30	74 57 41	79 63 48

Source: Federal Highway Administration. National Highway Institute. *Highway Hydrology: Hydraulic Design Series Number 2*. 2nd ed. FHWA-HIF-02-001. Washington, DC: U.S. Department of Transportation, October 2002, Table 5-4, pp. 5-22 through 5-24. https://www.fhwa.dot.gov/engineering/hydraulics/library_arc.cfm?pub_number=2&id=7.

Coefficients for SCS Peak Discharge Method

Rainfall Type	I_a/P	C_0	C_1	C_2
I	0.10	2.30550	–0.51429	–0.11750
	0.20	2.23537	–0.50387	–0.08929
	0.25	2.18219	–0.48488	–0.06589
	0.30	2.10624	–0.45695	–0.02835
	0.35	2.00303	–0.40769	–0.01983
	0.40	1.87733	–0.32274	0.05754
	0.45	1.76312	–0.15644	0.00453
	0.50	1.67889	–0.06930	0.0
IA	0.10	2.03250	–0.31583	–0.13748
	0.20	1.91978	–0.28215	–0.07020
	0.25	1.83842	–0.25543	–0.02597
	0.30	1.72657	–0.19826	0.02633
	0.50	1.63417	–0.09100	0.0
II	0.10	2.55323	–0.61512	–0.16403
	0.30	2.46532	–0.62257	–0.11657
	0.35	2.41896	–0.61594	–0.08820
	0.40	2.36409	–0.59857	–0.05621
	0.45	2.29238	–0.57005	–0.02281
	0.50	2.20282	–0.51599	–0.01259
III	0.10	2.47317	–0.51848	–0.17083
	0.30	2.39628	–0.51202	–0.13245
	0.35	2.35477	–0.49735	–0.11985
	0.40	2.30726	–0.46541	–0.11094
	0.45	2.24876	–0.41314	–0.11508
	0.50	2.17772	–0.36803	–0.09525

Source: Federal Highway Administration. National Highway Institute. *Highway Hydrology: Hydraulic Design Series Number 2*. 2nd ed. FHWA-HIF-02-001. Washington, DC: U.S. Department of Transportation, October 2002, Table 5-5, p. 5-28. https://www.fhwa.dot.gov/engineering/hydraulics/library_arc.cfm?pub_number=2&id=7.

I_a/P for Selected Rainfall Depths and Curve Numbers

Rainfall		Runoff Curve Number (*CN*)											
mm	in.	40	45	50	55	60	65	70	75	80	85	90	95
10	0.39	*	*	*	*	*	*	*	*	*	*	*	0.27
20	0.79	*	*	*	*	*	*	*	*	*	0.45	0.28	0.13
30	1.18	*	*	*	*	*	*	*	*	0.42	0.30	0.19	+
40	1.57	*	*	*	*	*	*	*	0.42	0.32	0.22	0.14	+
50	1.97	*	*	*	*	*	*	0.44	0.34	0.25	0.18	0.11	+
60	2.36	*	*	*	*	*	0.46	0.36	0.28	0.21	0.15	+	+
70	2.76	*	*	*	*	0.48	0.39	0.31	0.24	0.18	0.13	+	+
80	3.15	*	*	*	*	0.42	0.34	0.27	0.21	0.16	0.11	+	+
90	3.54	*	*	*	0.46	0.38	0.30	0.24	0.19	0.14	0.10	+	+
100	3.94	*	*	*	0.42	0.34	0.27	0.22	0.17	0.13	+	+	+
110	4.33	*	*	0.46	0.38	0.31	0.25	0.20	0.15	0.12	+	+	+
120	4.72	*	*	0.42	0.35	0.28	0.23	0.18	0.14	0.11	+	+	+
130	5.12	*	0.48	0.39	0.32	0.26	0.21	0.17	0.13	0.10	+	+	+
140	5.51	*	0.44	0.36	0.30	0.24	0.20	0.16	0.12	+	+	+	+
150	5.91	*	0.41	0.34	0.28	0.23	0.18	0.15	0.11	+	+	+	+
160	6.30	0.48	0.39	0.32	0.26	0.21	0.17	0.14	0.11	+	+	+	+
170	6.69	0.45	0.37	0.30	0.24	0.20	0.16	0.13	0.10	+	+	+	+
180	7.09	0.42	0.34	0.28	0.23	0.19	0.15	0.12	+	+	+	+	+
190	7.48	0.40	0.33	0.27	0.22	0.18	0.14	0.11	+	+	+	+	+
200	7.87	0.38	0.31	0.25	0.21	0.17	0.14	0.11	+	+	+	+	+
210	8.27	0.36	0.30	0.24	0.20	0.16	0.13	0.10	+	+	+	+	+
220	8.66	0.35	0.28	0.23	0.19	0.15	0.12	0.10	+	+	+	+	+
230	9.06	0.33	0.27	0.22	0.18	0.15	0.12	+	+	+	+	+	+
240	9.45	0.32	0.26	0.21	0.17	0.14	0.11	+	+	+	+	+	+
250	9.84	0.30	0.25	0.20	0.17	0.14	0.11	+	+	+	+	+	+
260	10.24	0.29	0.24	0.20	0.16	0.13	0.11	+	+	+	+	+	+
270	10.63	0.28	0.23	0.19	0.15	0.13	0.10	+	+	+	+	+	+
280	11.02	0.27	0.22	0.18	0.15	0.12	0.10	+	+	+	+	+	+
290	11.42	0.26	0.21	0.18	0.14	0.12	+	+	+	+	+	+	+
300	11.81	0.25	0.21	0.17	0.14	0.11	+	+	+	+	+	+	+
310	12.20	0.25	0.20	0.16	0.13	0.11	+	+	+	+	+	+	+
320	12.60	0.24	0.19	0.16	0.13	0.11	+	+	+	+	+	+	+
330	12.99	0.23	0.19	0.15	0.13	0.10	+	+	+	+	+	+	+
340	13.39	0.22	0.18	0.15	0.12	0.10	+	+	+	+	+	+	+
350	13.78	0.22	0.18	0.15	0.12	0.10	+	+	+	+	+	+	+

I_a/P for Selected Rainfall Depths and Curve Numbers (cont'd)

Rainfall		Runoff Curve Number (*CN*)											
mm	in.	40	45	50	55	60	65	70	75	80	85	90	95
360	14.17	0.21	0.17	0.14	0.12	+	+	+	+	+	+	+	+
370	14.57	0.21	0.17	0.14	0.11	+	+	+	+	+	+	+	+
380	14.96	0.20	0.16	0.13	0.11	+	+	+	+	+	+	+	+
390	15.35	0.20	0.16	0.13	0.11	+	+	+	+	+	+	+	+
400	15.75	0.19	0.16	0.13	0.10	+	+	+	+	+	+	+	+
* signifies that $I_a/P = 0.50$ should be used + signifies that $I_a/P = 0.10$ should be used													

Source: Federal Highway Administration. National Highway Institute. *Urban Drainage Design Manual: Hydraulic Engineering Circular 22*. 3rd ed. FHWA-HIF-10-009. Washington, DC: U.S. Department of Transportation, September 2009, revised August 2013, Table 3-8, p. 3-24. www.fhwa.dot.gov/engineering/hydraulics/pubs/10009/10009.pdf.

6.5.2.3 Kinematic-Wave Runoff Method

$$Q = \frac{1.486}{n} R^{2/3} S_0^{1/2} A$$ for very shallow flow at depth h using Manning's equation

$$q = \frac{1.486}{N} S_0^{1/2} h^{5/3}$$

$$q = \alpha A^m$$ when $S_f = S_0$

$$\alpha = \frac{1.486}{N} S_0^{1/2}$$ where $m = 5/3$ for wide flow plane

where

Q = discharge (cfs)

q = discharge per unit width (cfs/ft)

n = Manning's roughness coefficient (sec/ft$^{1/3}$)

N = effective roughness parameter for overland flow (sec/ft$^{1/3}$)

R = hydraulic radius (ft)

S_0 = slope of plane (ft/ft)

S_f = energy gradient or friction slope

A = cross-sectional area of channel (ft^2)

h = mean depth of flow (ft)

α = kinematic wave routing parameter for a particular cross-sectional shape, slope, and roughness (ft^2/sec)/ft^m

m = kinematic wave routing parameter for a particular cross-sectional shape, slope, and roughness (ft^2/sec)/ft^m

Relations for Estimating α and *m* Based on Physical Characteristics of Channel Segments

Type of segment	α	*m*	Definition of channel parameter P_c
Wide rectangular or trapezoidal cross section	$\frac{1.49\, S_0^{1/2}}{n\left(P_c\right)^{2/3}}$	1.67	Width of conduit at about mean depth of flow
Circular pipe	$\frac{1.49}{n}\frac{P_c^{2/3}}{4}S_0^{1/2}$	1.0	Diameter of pipe
Triangular cross section	$\frac{1.41\, S_0^{1/2}}{n\left(P_c\right)^{1/3}}$	1.33	Width at 1-ft depth

Source: Miller, Jeffery E. Basic Concepts of Kinematic-Waver Models: Paper 1302. Washington DC: U.S. Geological Survey, 1984, p. 27. https://pubs.usgs.gov/pp/1302/report.pdf.

Time to Equilibrium

$$t_c = \frac{nL}{1.49\, S_0^{1/2} h^{2/3}}$$

where

t_c = time (sec)

L = plane length (ft)

S_0 = slope of plane (ft/ft)

Recommended Manning's Roughness Coefficients for Overland Flow

Cover or treatment	Residue Rate (tons/acre)	Value Recommended	Range
Concrete or asphalt		0.011	0.010–0.013
Bare sand		0.01	0.010–0.016
Graveled surface		0.02	0.012–0.03
Bare clay loam (eroded)		0.02	0.012–0.033
Fallow, no residue		0.05	0.006–0.16
Chisel plow	< 1/4 1/4–1 1–3 > 3	0.07 0.18 0.30 0.40	0.006–0.17 0.07–0.34 0.19–0.47 0.34–0.46
Disk/harrow	< 1/4 1/4–1 1–3 > 3	0.08 0.16 0.25 0.30	0.008–0.41 0.10–0.25 0.14–0.53 –
No till	< 1/4 1/4–1 1–3	0.04 0.07 0.30	0.03–0.07 0.01–0.13 0.16–0.47
Moldboard plow (fall)		0.06	0.02–0.10
Colter		0.10	0.05–0.13
Range (natural)		0.13	0.01–0.32
Range (clipped)		0.10	0.02–0.24
Grass (bluegrass sod)		0.45	0.39–0.63
Short grass prairie		0.15	0.10–0.20
Dense grass[1]		0.24	0.17–0.30
Bermuda grass[1]		0.41	0.30–0.48

[1]Weeping lovegrass, bluegrass, buffalo grass, blue gamma grass, native grass mix (OK), alfalfa, lespedeza (from Palmer, 1946)

6.5.2.4 USGS Regression Method – Peak Flow

Rural Equations

$$RQ_T = aA^bB^cC^d$$

where

RQ_T = T-year rural peak flow (ft^3/sec)

a = regression constant

b, c, d = regression coefficients

A, B, C = basin characteristics

Urban Equations

$$UQ2 = 2.35\ A_s^{0.41} SL^{0.17} \left(RI2+3\right)^{2.04} \left(ST+8\right)^{-0.65} \left(13-BDF\right)^{-0.32}\ IA_s^{0.15} RQ2^{0.47}$$

$$UQ5 = 2.70\ A_s^{0.35} SL^{0.16} \left(RI2+3\right)^{1.86} \left(ST+8\right)^{-0.59} \left(13-BDF\right)^{-0.31}\ IA_s^{0.11} RQ5^{0.54}$$

$$UQ10 = 2.99\ A_s^{0.32} SL^{0.15} \left(RI2+3\right)^{1.75} \left(ST+8\right)^{-0.57} \left(13-BDF\right)^{-0.30}\ IA_s^{0.09} RQ10^{0.58}$$

$$UQ25 = 2.78\ A_s^{0.31} SL^{0.15} \left(RI2+3\right)^{1.76} \left(ST+8\right)^{-0.55} \left(13-BDF\right)^{-0.29}\ IA_s^{0.07} RQ25^{0.60}$$

$$UQ50 = 2.67\ A_s^{0.29} SL^{0.15} \left(RI2+3\right)^{1.74} \left(ST+8\right)^{-0.53} \left(13-BDF\right)^{-0.28}\ IA_s^{0.06} RQ50^{0.62}$$

$$UQ100 = 2.50\ A_s^{0.29} SL^{0.15} \left(RI2+3\right)^{1.76} \left(ST+8\right)^{-0.52} \left(13-BDF\right)^{-0.28}\ IA_s^{0.06} RQ100^{0.63}$$

$$UQ500 = 2.27\ A_s^{0.29} SL^{0.16} \left(RI2+3\right)^{1.86} \left(ST+8\right)^{-0.54} \left(13-BDF\right)^{-0.27}\ IA_s^{0.05} RQ500^{0.63}$$

where

UQT = urban peak discharge for T-year recurrence interval (ft^3/sec)

A_s = contributing drainage area (mi^2)

SL = main channel slope (measured between points that are 10% and 85% of main channel length upstream of site) (ft/mi)

$RI2$ = rainfall amount for 2-hr, 2-year recurrence (in.)

ST = basin storage (percentage of basin occupied by lakes, reservoirs, swamps, and wetlands) (percent)

BDF = basin development factor (a measure of the hydraulic efficiency of the basin)

IA = percentage of basin occupied by impervious surfaces

RQT = T-year rural peak flow

Source: Federal Highway Administration. National Highway Institute. *Urban Drainage Design Manual: Hydraulic Engineering Circular 22*. 3rd ed. FHWA-HIF-10-009. Washington, DC: U.S. Department of Transportation, September 2009, revised August 2013, Table 3-5, p. 3-17. www.fhwa.dot.gov/engineering/hydraulics/pubs/10009/10009.pdf.

6.5.3 Rainfall Intensity, Duration, and Frequency

6.5.3.1 Point Precipitation

$$i = \frac{c}{T_d^e + f} \qquad \text{Equation 1}$$

$$i = \frac{cT^m}{T_d + f} \qquad \text{Equation 2}$$

$$i = \frac{cT^m}{T_d^e + f} \qquad \text{Equation 3}$$

where

i = design rainfall intensity (in./hr)

T_d = duration of event (min)

m = rainfall coefficient

T = return period (years)

c, e, f = return-period coefficients

6.5.4 Time of Concentration

6.5.4.1 SCS Lag Formula

$$t_L = 0.000526\, L^{0.8}\left(\frac{1{,}000}{CN} - 9\right)^{0.7} S^{-0.5}$$

where

t_L = time lag (hr)

L = watershed length (ft)

CN = SCS curve number

S = watershed slope (%)

$$t_c = \frac{5}{3} t_L$$

where

t_c = time of concentration (hr)

6.5.4.2 Sheet Flow Formula

$$T_{ti} = \frac{K_u}{I^{0.4}}\left(\frac{nL}{\sqrt{S}}\right)^{0.6}$$

where

T_{ti} = sheet flow travel time (min)

n = roughness coefficient

L = flow length (ft or m)

I = rainfall intensity (in./hr or mm/hr)

S = surface slope (ft/ft or m/m)

K_u = empirical coefficient equal to 6.92 (0.933 in USCS units)

6.5.4.3 Inlet Flow Formula

Time of concentration for a particular inlet to a storm sewer:

$$t = t_i + t_s$$

where

t = time of concentration

t_s = time of sewer flow

The inlet time can be estimated by

$$t_i = C(L/S\, i^2)^{1/3}$$

where

t_i = time of overland flow (min)

L = distance of overland flow (ft)

S = slope of land (ft/ft)

i = rainfall intensity (in./hr)

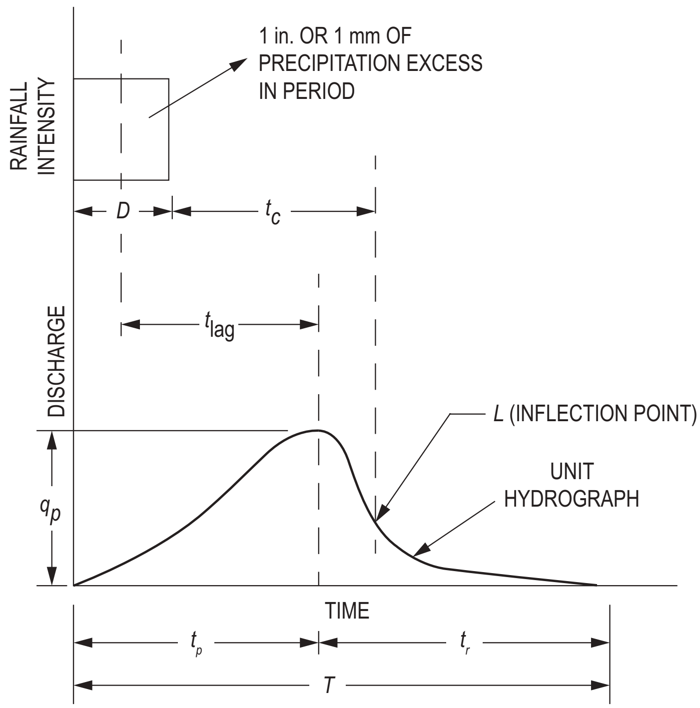
C = coefficient

6.5.5 Hydrograph Development and Applications, Including Synthetic Hydrographs

6.5.5.1 Unit Hydrograph

A unit hydrograph is the direct runoff hydrograph that would result from one unit of runoff occurring uniformly in space and time over a specified period of time.

Graphical Representation of a Unit Hydrograph



$$Q = \frac{1}{2} q_p T = \frac{1}{2} q_p \left(t_p + t_r \right)$$

where

q_p = peak discharge (flow)

t_p = time to peak

t_r = time to recession

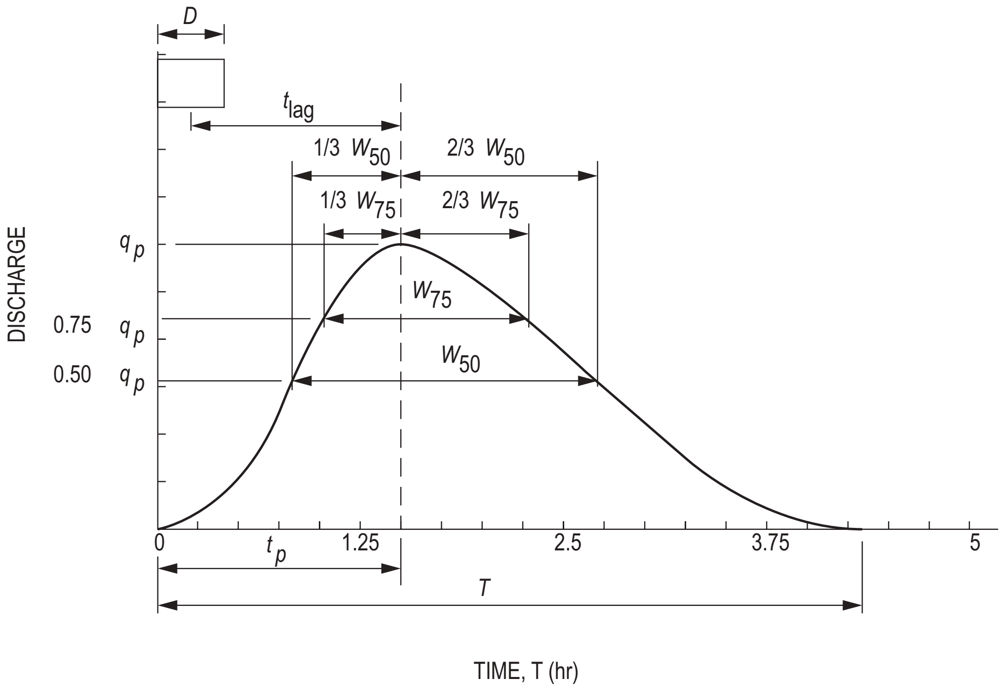
t_c = time of concentration

t_{lag} = lag time

T = time duration of unit hydrograph

D = duration of unit excess rainfall

Snyder Synthetic Unit Hydrograph



Source: Federal Highway Administration. National Highway Institute. *Urban Drainage Design Manual: Hydraulic Engineering Circular 22*. 3rd ed. FHWA-HIF-10-009. Washington, DC: U.S. Department of Transportation, September 2009, revised August 2013, Table 3-16, p. 3-26. www.fhwa.dot.gov/engineering/hydraulics/pubs/10009/10009.pdf.

where

D = duration of unit excess rainfall (hr)

t_{lag} = lag time from the centroid of the unit rainfall excess to the peak of the unit hydrograph (hr)

t_p = lag time from $T = 0$ to peak of the unit hydrograph (hr)

$t_p = C_t \left(LL_c\right)^{0.3}$

W_{50}, W_{75} = time width of unit hydrograph at discharge equal to 50% and 75% (hr)

L = stream distance from the outlet to the upstream limits of the basin (mi)

L_c = stream distance from the outlet to a point opposite the basin centroid (mi)

A = drainage area (mi^2)

T = time duration of the unit hydrograph (hr)

$T = 3 + \frac{t_p}{8}$

C_t = coefficient representing the slope of the basin

C_p = coefficient indicating the storage capacity

q_p = unit peak flow for standard duration t_D (ft^3/sec/in.)

$q_p = \frac{C_p A}{t_p} \times 640$

t_D = standard duration of rainfall excess (hr)

$t_D = \frac{t_p}{5.5}$

t_{pR} = lag time from midpoint of duration t_r to the peak of the unit hydrograph (hr)

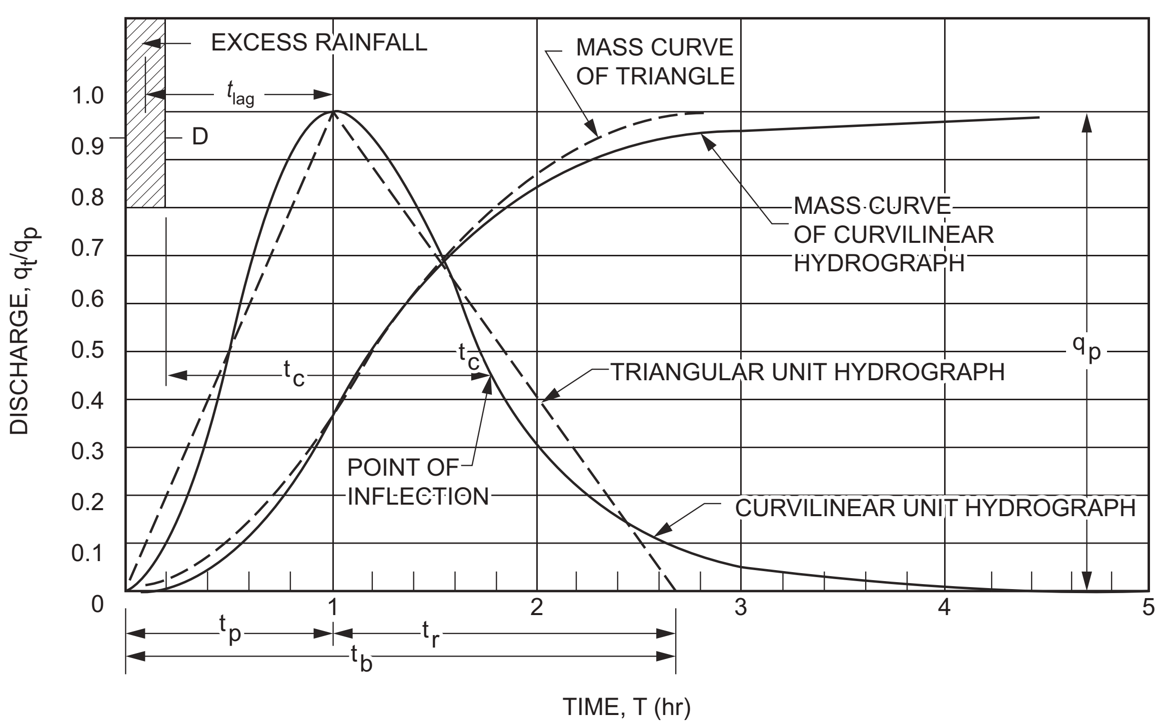
$t_{pR} = t_p + 0.25(t_r - t_D)$

t_r = duration of rainfall excess other than standard duration adopted in the study (hr)

q_{pR} = peak flow for duration t_r (ft^3/sec)

$q_{pR} = q_p \frac{t_p}{t_{pR}}$

SCS (NRCS) Unit Hydrograph



Source: Federal Highway Administration. National Highway Institute. *Urban Drainage Design Manual: Hydraulic Engineering Circular 22*. 3rd ed. FHWA-HIF-10-009. Washington, DC: U.S. Department of Transportation, September 2009, revised August 2013, Table 3-7, p. 3-32. www.fhwa.dot.gov/engineering/hydraulics/pubs/10009/10009.pdf.

$$q_p = \frac{K_p A_m Q_D}{t_p}$$

where

q_p = peak flow (ft^3/sec)

A_m = drainage area (mi^2)

Q_D = volume of direct runoff = 1 for unit hydrograph (in.)

t_p = time to peak (hr)

K_p = 484 = peaking constant

Time to peak:

$$t_p = \frac{2}{3} t_c$$

$$q_p = \frac{\alpha' K_p A_k Q_D}{t_c}$$

where

α' = 1.5 (USCS units)

t_c = time of concentration (hr)w

SCS Lag Method:

$$t_L = \frac{L^{0.8}\left(S+1\right)^{0.7}}{1{,}900Y^{0.5}}$$

$$t_c = \frac{5}{3} t_L$$

$$t_c = \frac{L^{0.8}\left(S+1\right)^{0.7}}{1{,}140Y^{0.5}}$$

$$t_c = \frac{L}{3{,}600v}$$

where

t_L = time lag from the center of mass of the rainfall excess to peak discharge (hr)

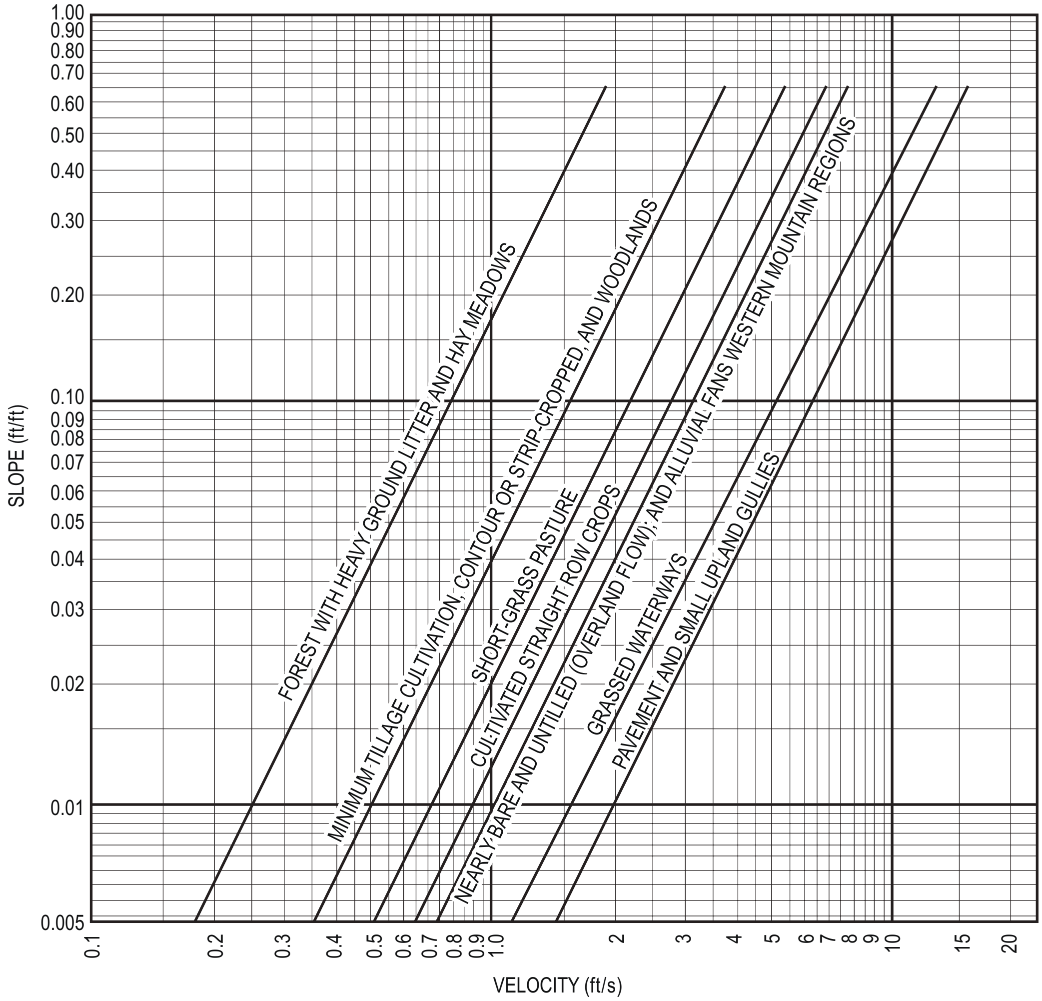
t_c = time of concentration (hr)

Y = watershed slope (%)

S = potential maximum retention (in.)

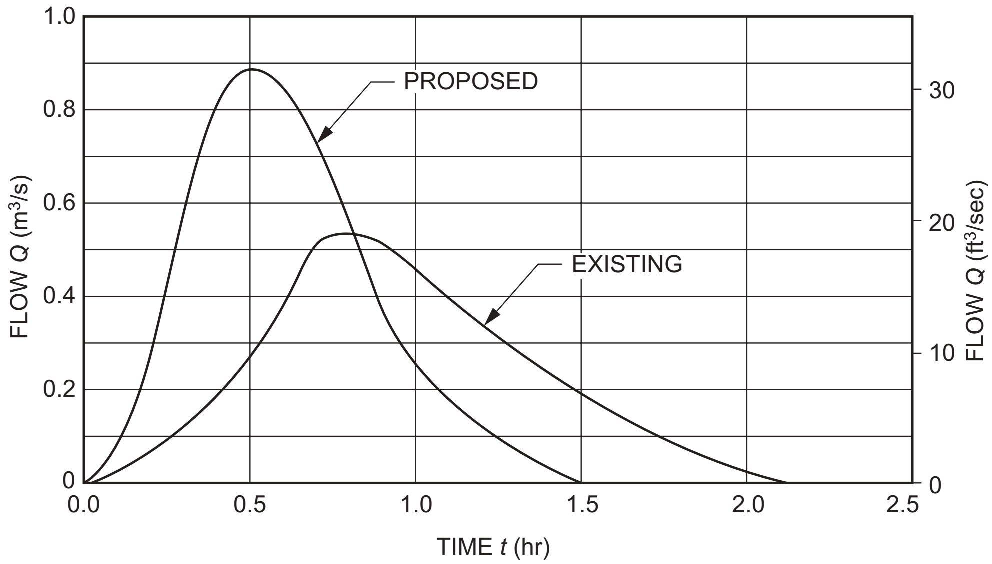
L = length (ft)

v = velocity (ft/sec)



Velocity Versus Slope for Shallow Concentrated Flow

Source: Natural Resources Conservation Service. *National Engineering Handbook: Part 630 Hydrology*. 210-VI-NEH. Washington, DC: U.S. Department of Agriculture, May 2010, Fig. 15-4, p. 15-8. https://directives.sc.egov.usda.gov/OpenNonWebContent.aspx?content=27002.wba.



USGS Nationwide Urban Hydrograph for Existing (Unimproved) and Proposed (Improved) Conditions

Source: Federal Highway Administration. National Highway Institute. *Urban Drainage Design Manual: Hydraulic Engineering Circular 22*. 3rd ed. FHWA-HIF-10-009. Washington, DC: U.S. Department of Transportation, September 2009, revised August 2013, Table 3-9, p. 3-39. www.fhwa.dot.gov/engineering/hydraulics/pubs/10009/10009.pdf.

Clark Unit Hydrograph

$$\frac{A}{A_c} = 1.414\left(\frac{t}{t_c}\right)^{1.5} \qquad \text{for } 0 \le \frac{t}{t_c} \le 0.5$$

$$\frac{A}{A_c} = 1 - 1.414\left(1 - \frac{t}{t_c}\right)^{1.5} \qquad \text{for } 0.5 \le \frac{t}{t_c} \le 1.0$$

$$\frac{I_{t1} + I_{t2}}{2} - \frac{O_{t1} + O_{t2}}{2} = \frac{R\left(O_{t2} - O_{t1}\right)}{\Delta t}$$

$$O_{t2} = CI_{t2} + \left(1 - C\right)O_{t1} \qquad \text{when } I_{t1} = I_{t2}$$

$$C = \frac{2\Delta t}{2R + \Delta t}$$

Linear Reservoir:

$$S_t = RO_t$$

where

A = contributing area at time t (area)

A_c = total watershed area (area)

t = time (time)

t_c = time of concentration of the watershed area (time)

C = dimensionless

I_t = unit inflow into reservoir at time t (flow)

O_t = unit outflow from the linear reservoir at time t (flow)

S_t = storage in the linear reservoir at time t (volume)

R = linear reservoir storage coefficient or attenuation constant (time)

6.5.5.2 Hydrograph Estimate of Stream Flow

$$Q_t = Q_0 K^t$$

where

Q_t = discharge, t time units after Q_0

Q_0 = initial discharge at the start of recession ($t = 0$)

K = recession constant

6.5.6 Rainfall Gauging Stations

6.5.6.1 Precipitation Gauge Analysis

$$\overline{P} = \sum_{i=1}^{n} \left(\frac{A_i}{A}\right) P_i$$

where

A = total area of entire watershed (length^2)

A_i = area in watershed associated with gauge i (length^2)

P_i = precipitation at gauge i (length)

n = number of intervals

6.5.6.2 Thiessen Polygon Method

Weight is assigned to each station in proportion to its representative area defined by a polygon. Polygons are formed by:

1. Stations are plotted on a map of the area, drawn to scale.
2. Adjoining stations are connected by dashed lines.
3. Perpendicular bisectors are constructed on each dashed line.
4. Bisectors and boundaries form polygons around each station. Each polygon is representative of the effective area for the station within the polygon.
5. The area of each polygon is determined and multiplied by the rainfall value for the station within.
6. The sum of the products divided by the total drainage area provides the weighted average precipitation.

6.5.6.3 Isohyetal Method

Weight is assigned to each station in proportion to its representative area between isohyets (contours) within a total catchment area.

General procedure is as follows:

1. Stations and rainfall values are plotted on a map to a suitable scale.
2. Contours of equal precipitation (isohyets) are drawn. The accuracy of the method depends on the construction of the isohyets and their intervals.

3. Areas between successive isohyets are computed and multiplied by the numerical average of two contour (isohyets) values.
4. The sum of the previous calculation divided by the drainage area provides the weighted average precipitation.

6.5.6.4 National Weather Service IDF Curve Creation

$$P = \frac{T_d + 20}{100}$$

$$i = \frac{60P}{T_d}$$

where

P = precipitation depth (in.)

i = design rainfall intensity (in./hr)

T_d = duration of event (min)

6.5.6.5 Distance Weighting

$$P = \frac{\sum_{i=1}^{4}\left(\frac{P_i}{L_i^2}\right)}{\sum_{i=1}^{4}\left(\frac{1}{L_i^2}\right)}$$

where

P = precipitation (in.)

L = distance between index stations and station X_i

i = each index station

6.5.6.6 Estimation and Analysis for Missing Data

Arithmetic/Station Average Method

$$P_x = \frac{1}{n}\sum_{i=1}^{n} P_i$$

where

P_x = missing point of rainfall (length)

n = number of gauges

P_i = catch at gauge i (length)

Normal-Ratio Method

$$P_x = \sum_{i=1}^{n} W_i P_i$$

where

W_i = weight for the rainfall depth P_i at gauge i

$W_i = \dfrac{A_x}{nA_i}$

A_i = average annual catch at gauge i (length)

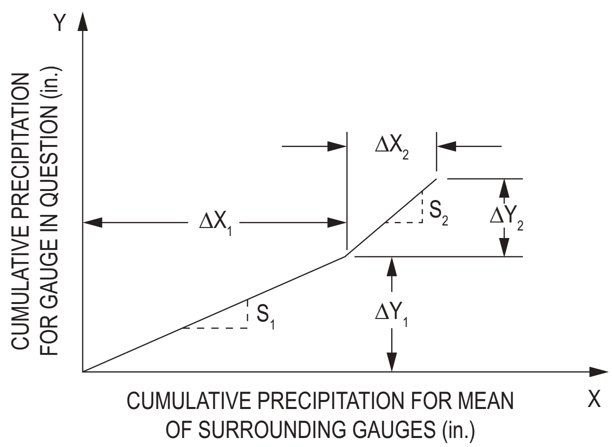
A_x = average annual catch at station x (length)

n = number of stations

or

$$\frac{P_x}{A_x} = \frac{1}{n} \sum_{i=1}^{n} \frac{P_i}{A_i}$$

Regression Method (Double-Mass Curve)



$S_i = \dfrac{\Delta Y_i}{\Delta X_i}$ when plotting the change in cumulative catch of a questionable gauge (Y_i) against the cumulative catch of surrounding gauges that are not in question (X_i).

where

S_i = slope of the section

ΔY_i = change in cumulative catch between end points for gauge Y

ΔX_i = change in cumulative catch for the sum of the regulated gauges between end points

and

$$y_1 = Y_i \frac{S_2}{S_1}$$

where

$\frac{S_2}{S_1}$ = adjustment factor

y_1 = adjusted series for gauge i

Y_i = measured value at gauge i

6.5.7 Stream Gauging

The following methods are used to determine mean velocities using in-stream velocity measurements.

Two-Point Method: $\overline{V} = \dfrac{V_{0.2} + V_{0.8}}{2}$

Six-Tenths-Depth Method: $\overline{V} = V_{0.6}$

Three-Point Method: $\overline{V} = \dfrac{V_{0.2} + 2V_{0.6} + V_{0.8}}{4}$

Five-Point Method: $\overline{V} = \dfrac{V_{\text{surf}} + 3V_{0.2} + 3V_{0.6} + 2V_{0.8} + V_{\text{bed}}}{10}$

Six-Point Method: $\overline{V} = \dfrac{V_{\text{surf}} + 2V_{0.2} + 2V_{0.4} + 2V_{0.6} + 2V_{0.8} + V_{\text{bed}}}{10}$

Two-Tenths-Depth Method: $\overline{V} = V_{0.2}$

Surface-Velocity Method: $\overline{V} = 0.87V_{\text{surf}}$

where

V = velocity in stream (ft/sec)

V_i = velocity measured at point in stream (ft/sec)

6.5.8 Depletions (e.g., Evaporation, Detention, Percolation, and Diversions)

6.5.8.1 Surface Water System Hydrologic Budget

$$P + Q_{\text{in}} - Q_{\text{out}} + Q_g - E_s - T_s - I = \Delta S_s$$

where

P = precipitation

Q_{in} = surface water flow into system

Q_{out} = surface water flow out of the system

Q_g = groundwater flow into the stream

E_s = surface evaporation

T_s = transpiration

I = infiltration

ΔS = change in water storage of surface water system

6.5.8.2 Consumptive Use – Blaney-Criddle Method

$$U = \sum K_t K_c t_m \frac{p}{100}$$

where

U = consumptive use (in./month)

K_t = climate coefficient related to mean monthly temperature

K_t = $0.0173t_m - 0.314$

($K_t = 0.30$ at $t_m < 36°\text{F}$)

K_C = growth storage coefficient

t_m = mean monthly temperature (°F)

p = monthly percentage of annual daytime hours (%)

6.5.8.3 Evaporation

Pan Method

$$E_L = K_p E_p$$

where

E_L = evaporation from water body (length/time)

E_p = evaporation from pan (length/time)

K_p = pan coefficient

Typical Values of Pan Coefficient, K_p

Types of Pan	Average Value	Range
Class A land pan	0.70	0.60–0.80
Colorado sunken pan	0.78	0.75–0.86
USGS floating pan	0.80	0.70–0.82

Aerodynamic Method

$$E_a = M(e_s - e)u_z$$

where

E_a = evaporation (length/time)

M = mass-transfer coefficient (1/psi)

e_s = saturation vapor pressure (psi)

e = actual vapor pressure of air at z elevation (psi) = $RH\left(e_z^0\right)$

e_z^0 = saturation vapor pressure at air temperature at z elevation (psi)

RH = relative humidity (decimal form)

u_z = wind velocity (length/time)

Evaporation Bulk Mass Transfer

$$M = 0.622\frac{\rho_a C_E}{\rho_w P}$$

where

M = mass-transfer coefficient (1/psi)

ρ_w = density of water (lbf-sec^2/ft^4)

ρ_a = density of air (lbf-sec^2/ft^4)

C_E = bulk evaporation coefficient

P = atmospheric pressure at z elevation (psi)

6.5.8.4 Porosity

$$\eta = \frac{\text{volume of voids}}{\text{total volume}}$$

where η = porosity, $0.25 < \eta < 0.40$

6.5.8.5 Soil Moisture Content

$$\theta = \frac{\text{volume of water}}{\text{total volume}}$$

where θ = soil moisture content

6.5.8.6 Infiltration

Richards Equation

$$\frac{d\theta}{dt} = \frac{\partial}{\partial z}\left(K \frac{dh}{d\theta}\frac{d\theta}{dz}\right) - \frac{dK}{dz}$$

where

isotropic conditions and one-dimensional vertical flow

$\frac{\partial\theta}{\partial t}$ = water flux

θ = soil moisture content

K = hydraulic conductivity (ft/sec)

h = pressure head on soil medium (ft)

z = distance measured positively downward from the surface (ft)

Horton Model

$$f = \left(f_0 - f_c\right)e^{-kt} + f_c$$

where

f = infiltration rate at a given time (in./hr)

f_0 = initial infiltration capacity (in./hr)

f_c = ultimate infiltration capacity or apparent saturated conductivity (in./hr)

k = factor representing the rate of decrease in the capacity (1/hr)

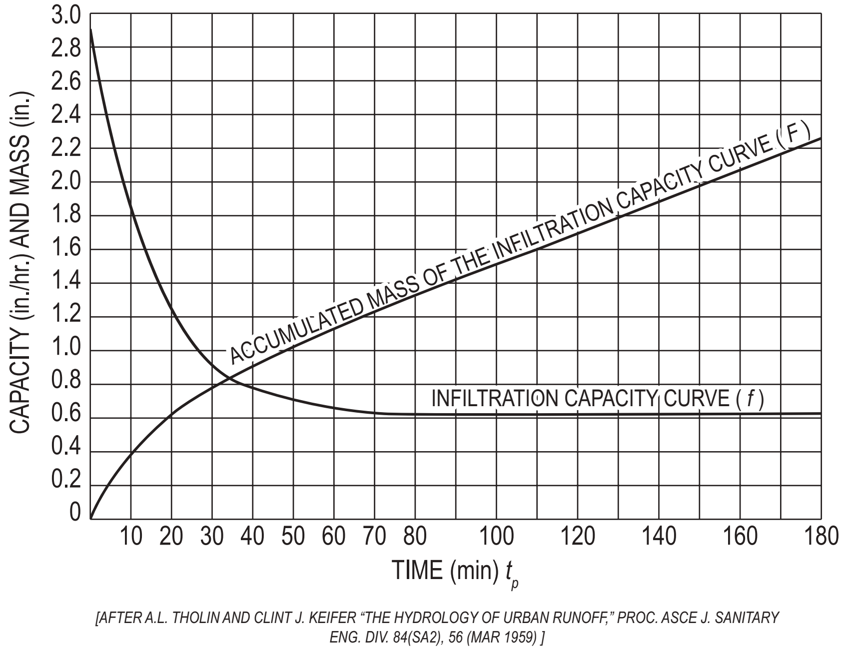
t = time elapsed for infiltration (hr)

$$F = f_c t_p + \frac{f_0 - f_c}{k}\left(1 - e^{-kt_p}\right)$$

where

F = accumulated infiltration at a given time t_p (in.)

t_p = time from beginning of infiltration (hr) ($t_p < t$)



Representative Horton Infiltration Capacity Curves

Source: After the American Society of Civil Engineers. "The Hydrology of Urban Runoff" by A. L. Tholin and Clint J. Keifer. *Journal of the Sanitary Engineering Division*. Reston, VA: American Society of Civil Engineers, 1959, Vol. 85, Issue 2, pp. 47-106.

Green-Ampt Equation

$$f = K\frac{Z + h_0 + S_z}{Z}$$

where

f = infiltration rate at a given time (in./hr, typ.)

K = hydraulic conductivity of saturated soil (in./hr, typ.)

h_0 = depth of ponded water above or pressure head at surface (ft)

S_z = suction (capillary) head at the wetting front (ft)

Z = depth from surface to wetting front (ft)

$f(t) = i$ for $t \le t_p$

$f(t) = K + K\frac{S_z \Delta\theta}{F}$ for $t > t_p$

where

f = infiltration rate at a given time (in./hr, typ.)

t_p = time when water begins to pond (hr)

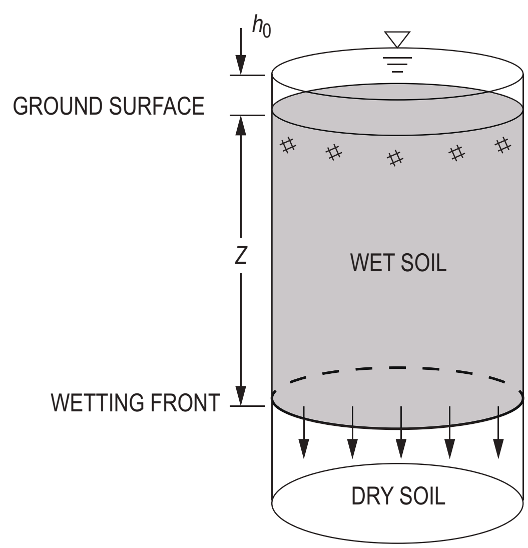
i = precipitation rate (in./hr)

K = hydraulic conductivity of saturated soil (in./hr, typ.)

S_z = suction (capillary) head at the wetting front (ft)

F = cumulative amount of water that has infiltrated (ft)

$\Delta\theta$ = initial soil water deficit = $\eta - \theta_i$



Green-Ampt Infiltration into a Column

Source: Republished with permission of McGraw-Hill, from *Applied Hydrology* by Chow, V. T., D. R. Maidment & L. Mays. 1988, Fig. 4.3.2, p. 112; permission conveyed through Copyright Clearance Center, Inc.

where

θ_i = initial soil moisture content

η = porosity

F = $Z\,\Delta\theta$

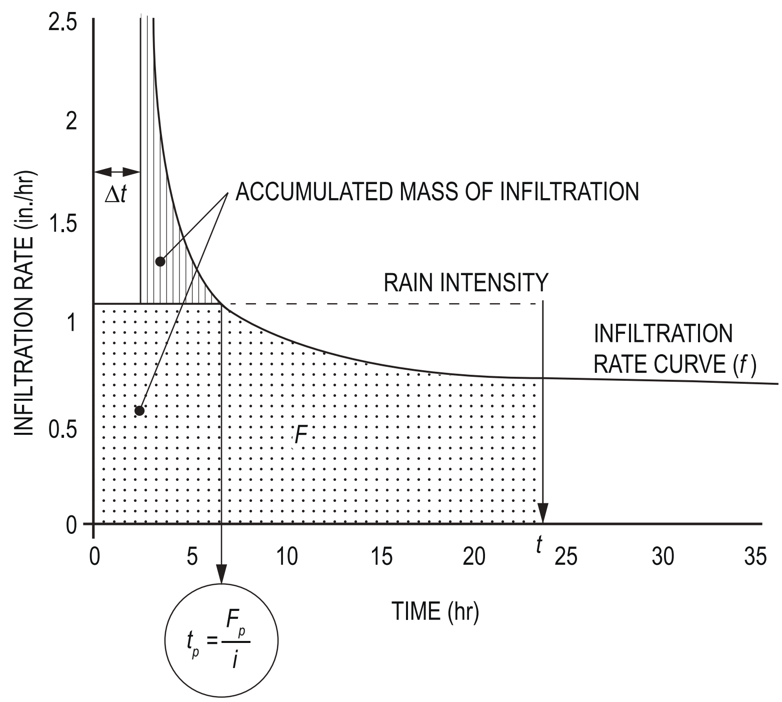
$$F_p = \frac{S_z K\,\Delta\theta}{i - K} \qquad \text{for } t = t_p$$

$$t_p = \frac{F_p}{i}$$

where

F_p = amount of water that infiltrates before ponding (in.)

t_p = time when water begins to pond (hr)



Representative Green-Ampt Infiltration Capacity Curve

6.5.9 Stormwater Management (e.g., Detention Ponds, Retention Ponds, Infiltration Systems, and Swales)

6.5.9.1 Detention and Retention

Rational Method

$$V_{in} = i\Sigma ACt$$
$$V_{out} = Q_0 t$$

where

Q_0 = outflow rate (cfs)

A = drainage area (acres)

C = rational method runoff coefficient

t = time (min)

i = average rainfall intensity (in./hr)

Storage volume required:

$$V_s = \text{maximum } V_{in} - V_{out}$$

Routing Equation

$$\frac{1}{2}\left(I_1 + I_2\right)\Delta t + \left(S_1 - \frac{1}{2}O_1\,\Delta t\right) = \left(S_2 + \frac{1}{2}O_2\,\Delta t\right)$$

where

I = inflow (volume/time)

O = outflow (volume/time)

S = storage (volume/time)

Δt = time interval (time)

Modified Puls Routing Method

$$\left(I_1 + I_2\right) + \left(\frac{2S_1}{\Delta t} - O_1\right) = \frac{2S_2}{\Delta t} + O_2$$

6.5.9.2 Erosion

Estimation of Wave Heights Based on Surface Winds

Estimated Wind Speed at 30 feet above Water Surface:

$$U = U_z\left(\frac{10}{z}\right)^{1/7}$$

where

U = 1-hour wind speed at 30 ft above water (miles/hr)

U_z = 1-hour wind speed at height z, where $z < 20$ m (miles/hr)

z = height above water where wind speed measured (m)

Duration-Averaged Wind Speed:

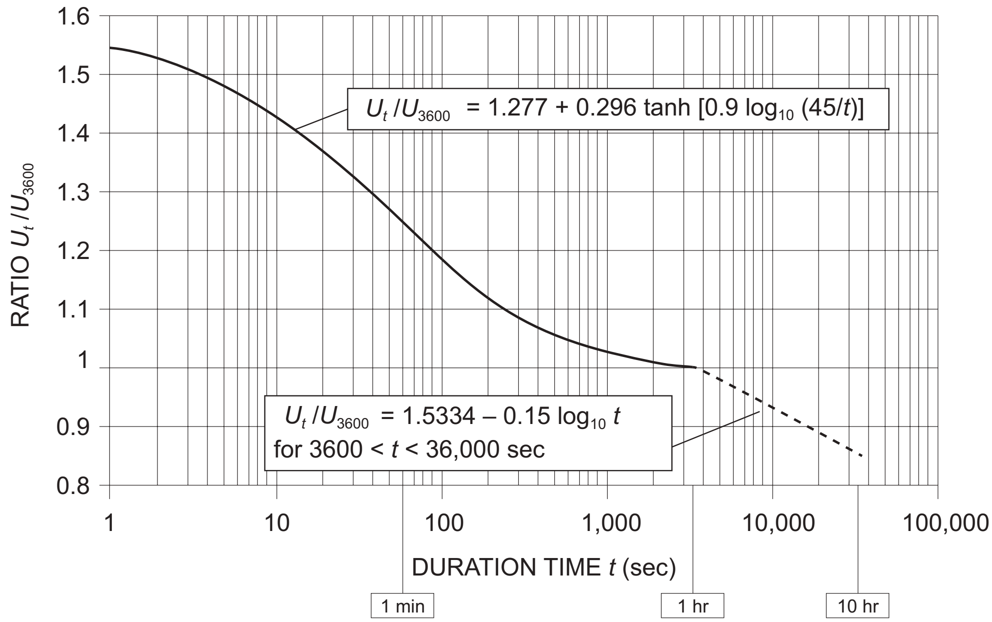
$$\frac{U_t}{U_{3,600}} = 1.277 + 0.296 \times \tanh\left(0.9 \times \log_{10}\frac{45}{t}\right) \qquad \text{for } t < 3,600 \text{ sec}$$

$$\frac{U_t}{U_{3,600}} = 1.5334 - 0.15 \times \log_{10} t \qquad \text{for } 3,600 \text{ sec} < t < 36,000 \text{ sec}$$

where

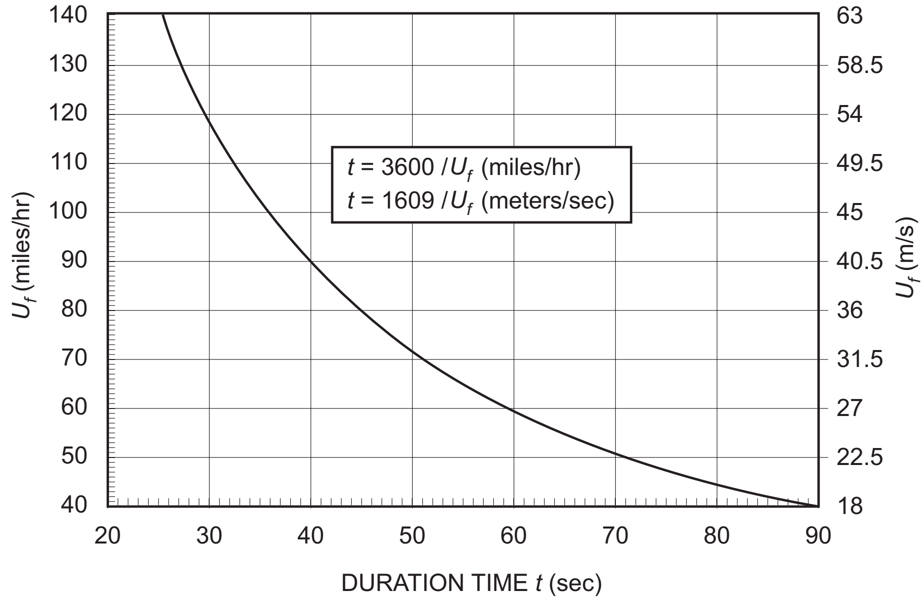
U_t = average wind speed over time t (miles/hr)

$U_{3,600}$ = 1-hour wind speed (miles/hr)



Ratio of Wind Speed of Any Duration U_t to the 1-hr Wind Speed $U_{3,600}$

Source: U.S. Army Corp of Engineers. *Coastal Engineering Manual*. EM 1110-2-1100 (Part II). Washington, DC: U.S. Department of the Army, Change 4, September 2015, Table II-2-1, p. II-2-4. https://www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1100_Part-02.pdf?ver=2016-02-11-153511-290.



Duration of the Fastest-Mile Wind Speed U_f as a Function of Wind Speed (for Open Terrain Conditions)

Source: U.S. Army Corp of Engineers. *Coastal Engineering Manual*. EM 1110-2-1100 (Part II). Washington, DC: U.S. Department of the Army, Change 4, September 2015, Table II-2-2, p. II-2-5. https://www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1100_Part-02.pdf?ver=2016-02-11-153511-290.

Wind Stress Factor:

$$U_A = 0.71U^{1.23}$$

where U_A = wind stress factor or adjusted wind speed on wave (miles/hr)

Deep Water Wave Fetch:

$$\frac{gH_{mo}}{U_A^2} = 0.0016\left(\frac{gF}{U_A^2}\right)^{\frac{1}{2}}$$

$$\frac{gT_p}{U_A} = 0.286\left(\frac{gF}{U_A^2}\right)^{\frac{1}{3}}$$

$$\frac{gt_{\min}}{U_A} = 68.8\left(\frac{gF}{U_A^2}\right)^{\frac{2}{3}} \text{ in fetch limited conditions}$$

where

H_{mo} = spectrally based wave height (ft)

T_p = period of the peak of the wave spectrum (sec)

F = fetch (ft)

t = fetch duration (hr)

g = acceleration due to gravity (ft/sec^2)

Shallow Water Wave Fetch:

$$\frac{gH}{U_A^2} = 0.283 \tanh\left[0.530\left(\frac{gd}{U_A^2}\right)^{\frac{3}{4}}\right] \tanh\left\{\frac{0.00565\left(\frac{gF}{U_A^2}\right)^{\frac{1}{2}}}{\tanh\left[0.530\left(\frac{gd}{U_A^2}\right)^{\frac{3}{4}}\right]}\right\}$$

Revised Universal Soil Loss Equation

$$A = R\,K\,L\,S\,C\,P$$

where

A = predicted average soil loss (tons/acre-yr)

R = climatic erosivity $\left(\frac{\text{hundreds of ton-in.}}{\text{acre-yr}}\right)$

K = soil erodibility $\left(\frac{\text{ton-acre}}{\text{hundreds of acre-ft-ton-in.}}\right)$

L = slope length (ft)

S = slope steepness

C = crop and cover management factor

P = conservation practice factor

Soil Erodibility Factor *K* in tons/acre

Textural class	Organic matter content, %	
	0.5	2
Fine sand	0.16	0.14
Very fine sand	0.42	0.36
Loamy sand	0.12	0.10
Loamy very fine sand	0.44	0.38
Sandy loam	0.27	0.24
Very fine sandy loam	0.47	0.41
Silt loam	0.48	0.42
Clay loam	0.28	0.25
Silty clay loam	0.37	0.32
Silty clay	0.25	0.23

Source: Schwab, G., et al. (1981) *Soil and Water Conservation Engineering*. John Wiley Inc., New York.

Values of the Topographic Factor, *LS*, for Specific Combinations of Slope Length and Steepness[1]

Percent slope	Slope Length (ft)											
	25	50	75	100	150	200	300	400	500	600	800	1,000
0.2	0.060	0.069	0.075	0.080	0.086	0.092	0.099	0.105	0.110	0.114	0.121	0.126
0.5	0.073	0.083	0.090	0.096	0.104	0.110	0.119	0.126	0.132	0.137	0.145	0.152
0.8	0.086	0.098	0.107	0.113	0.123	0.130	0.141	0.149	0.156	0.162	0.171	0.179
2	0.133	0.163	0.185	0.201	0.227	0.248	0.280	0.305	0.326	0.344	0.376	0.402
3	0.190	0.233	0.264	0.287	0.325	0.354	0.400	0.437	0.466	0.492	0.536	0.573
4	0.230	0.303	0.357	0.400	0.471	0.528	0.621	0.697	0.762	0.820	0.920	1.01
5	0.268	0.379	0.464	0.536	0.656	0.758	0.928	1.07	1.20	1.31	1.52	1.69
6	0.336	0.476	0.583	0.673	0.824	0.952	1.17	1.35	1.50	1.65	1.90	2.13
8	0.496	0.701	0.859	0.992	1.21	1.41	1.72	1.98	2.22	2.43	2.81	3.14
10	0.685	0.968	1.19	1.37	1.68	1.94	2.37	2.74	3.06	3.36	3.87	4.33
12	0.903	1.28	1.56	1.80	2.21	2.55	3.13	3.61	4.04	4.42	5.11	5.71
14	1.15	1.62	1.99	2.30	2.81	3.25	3.98	4.59	5.13	5.62	6.49	7.26
16	1.42	2.01	2.46	2.84	3.48	4.01	4.92	5.68	6.35	6.95	8.03	8.98
18	1.72	2.43	2.97	3.43	4.21	3.86	5.95	6.87	7.68	8.41	9.71	10.9
20	2.04	2.88	3.53	4.08	5.00	5.77	7.07	8.16	9.12	10.0	11.5	12.9

[1] $LS = (\lambda/72.6)^m \left(65.41 \sin^2\theta + 4.56 \sin\theta + 0.065\right)$

where

λ = slope length (ft)

m = 0.2 for gradients < 1 percent, 0.3 for 1 to 3 percent slopes, 0.4 for 3.5 to 4.5 percent slopes, 0.5 for 5 percent slopes and steeper

θ = angle of slope

(For other combinations of length and gradient, interpolate between adjacent values)

Source: U.S. Department of Agriculture. *Predicting Rainfall Erosion Losses: A Guide to Conservation Planning*. Agricultural Handbook Number 537. Washington, DC: USDA, 1978, Table 3, p. 12. https://www.ars.usda.gov/ARSUserFiles/50201000/USLEDatabase/ah_537.pdf.

Mulch Factors and Length Limits for Construction Slopes[1]

Type of Mulch	Mulch Rate (tons/acre)	Land Slope	Factor *C*	Length limit[2] (ft)
None	0	all	1.0	–
Straw or hay, tied down by anchoring and tacking equipment[3]	1.0	1–5%	0.20	200
	1.0	6–10%	0.20	100
	1.5	1–5%	0.12	300
	1.5	6–10%	0.12	150
	2.0	1–5%	0.06	400
	2.0	6–10%	0.06	200
	2.0	11–15%	0.07	150
	2.0	16–20%	0.11	100
	2.0	21–25%	0.14	75
	2.0	26–33%	0.17	50
	2.0	34–50%	0.20	35
Crushed stone, 0.25 to 1.5 in.	135	< 16%	0.05	200
	135	16–20%	0.05	150
	135	21–33%	0.05	100
	135	34–50%	0.05	75
	240	< 21%	0.02	300
	240	21–33%	0.02	200
	240	34–50%	0.02	150
Wood chips	7	< 16%	0.08	75
	7	16–20%	0.08	50
	12	< 16%	0.05	150
	12	16–20%	0.05	100
	12	21–33%	0.05	75
	25	< 16%	0.02	200
	25	16–20%	0.02	150
	25	21–33%	0.02	100
	25	34–50%	0.02	75

[1]From Meyer and Ports (24). Developed by an interagency workshop group on the basis of field experience and limited research data.

[2]Maximum slope length for which the specified mulch rate is considered effective. When this limit is exceeded, either a higher application rate or mechanical shortening of the effective slope length is required.

[3]When the straw or hay mulch is not anchored to the soil, C values on moderate or steep slopes of soils having K values greater than 0.30 should be taken at double the values given in this table.

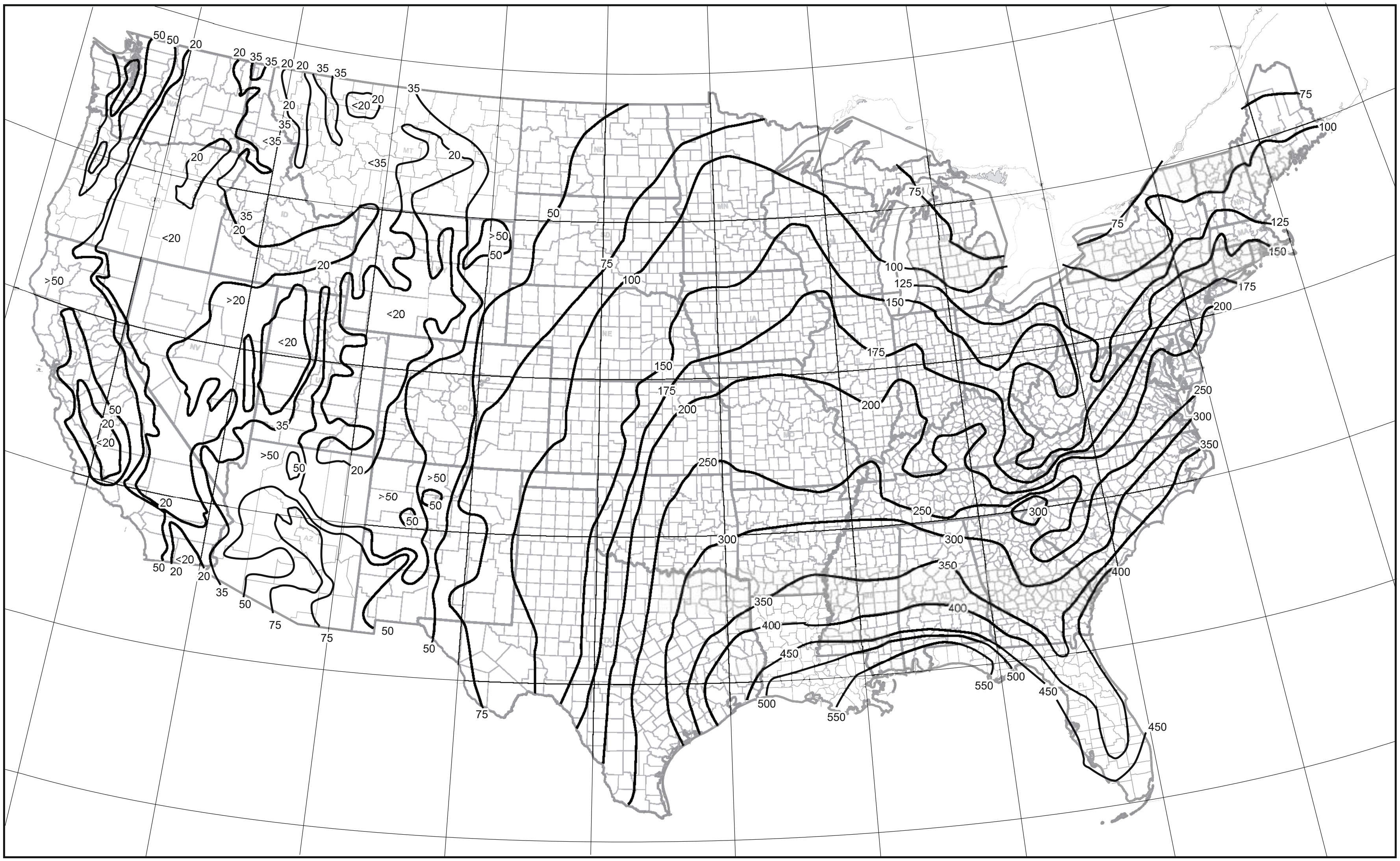
Source: U.S. Department of Agriculture. *Predicting Rainfall Erosion Losses: A Guide to Conservation Planning*. Agricultural Handbook Number 537. Washington, DC: USDA, 1978, Table 9, p. 31. https://www.ars.usda.gov/ARSUserFiles/64080530/RUSLE/AH_537.pdf.

Factor C for Permanent Pasture, Range, and Idle Land[1]

Vegetative canopy		Cover that contacts the soil surface						
Type and Height[2]	Percent Cover[3]	Type[4]	Percent Ground Cover					
			0	20	40	60	80	95+
No appreciable canopy		G	0.45	0.20	0.10	0.042	0.013	0.003
		W	0.45	0.24	0.15	0.091	0.043	0.011
Tall weeds or short brush with average drop fall height of 20 in.	25	G	0.36	0.17	0.09	0.038	0.013	0.003
		W	0.36	0.20	0.13	0.083	0.041	0.011
	50	G	0.26	0.13	0.07	0.035	0.012	0.003
		W	0.26	0.16	0.11	0.076	0.039	0.011
	75	G	0.17	0.10	0.06	0.032	0.011	0.003
		W	0.17	0.12	0.09	0.068	0.038	0.011
Appreciable brush or bushes, with average drop fall height of 6.5 ft	25	G	0.40	0.18	0.09	0.040	0.013	0.003
		W	0.40	0.22	0.14	0.087	0.042	0.011
	50	G	0.34	0.16	0.08	0.038	0.012	0.003
		W	0.34	0.19	0.13	0.082	0.041	0.011
	75	G	0.28	0.14	0.08	0.036	0.012	0.003
		W	0.28	0.17	0.12	0.078	0.040	0.011
Trees, but no appreciable low brush. Average drop fall height of 13 ft	25	G	0.42	0.19	0.10	0.041	0.013	0.003
		W	0.42	0.23	0.14	0.089	0.042	0.011
	50	G	0.39	0.18	0.09	0.040	0.013	0.003
		W	0.39	0.21	0.14	0.087	0.042	0.011
	75	G	0.36	0.17	0.09	0.039	0.012	0.003
		W	0.36	0.20	0.13	0.084	0.041	0.011

[1] The listed *C* values assume that the vegetation and mulch are randomly distributed over the entire area.
[2] Canopy height is measured as the average fall height of water drops falling from the canopy to the ground. Canopy effect is inversely proportional to drop fall height and is negligible if fall height exceeds 33 ft.
[3] Portion of total-area surface that would be hidden from view by canopy in a vertical projection (a bird's-eye view).
[4] G: Cover at surface is grass, grass-like plants, decaying compacted duff, or litter at least 2 in. deep.
W: Cover at surface is mostly broadleaf herbaceous plants (as weeds with little lateral-root network near the surface) or undecayed residues or both.

Source: U.S. Department of Agriculture. *Predicting Rainfall Erosion Losses: A Guide to Conservation Planning*. Agricultural Handbook Number 537. Washington, DC: USDA, 1978, Table 10, p. 32. https://www.ars.usda.gov/ARSUserFiles/64080530/RUSLE/AH_537.pdf.



Average Annual Values of the Rainfall Erosion Index

Source: U.S. Department of Agriculture. *Predicting Rainfall Erosion Losses: A Guide to Conservation Planning*. Agricultural Handbook Number 537. Washington, DC: USDA, 1978, Fig. 1, p. 5a.
https://www.ars.usda.gov/ARSUserFiles/64080530/RUSLE/AH_537.pdf.

6.6 Groundwater and Wells

6.6.1 Groundwater System and Hydrologic Budget

$$I + G_{\text{in}} - G_{\text{out}} - Q_g - E_g - T_g = \Delta S_g$$

where

G_{in} = groundwater flow into system

G_{out} = groundwater flow out of the system

Q_g = groundwater flow drawn out

E_g = evaporation

T_g = transpiration

I = infiltration

ΔS_g = change in water storage of groundwater system

6.6.2 Aquifers

6.6.2.1 Darcy's Law

$$Q = -KA\frac{dh}{dx}$$

where

Q = discharge rate (ft^3/sec or m^3/s)

K = hydraulic conductivity (ft/sec or m/s)

h = hydraulic head (ft or m)

A = cross-sectional area of flow (ft^2 or m^2)

$$q = -K\frac{dh}{dx}$$

where

q = specific discharge (also called Darcy velocity or superficial velocity)

$$v = \frac{q}{S_y} = \frac{-K}{S_y}\frac{dh}{dx}$$

where

v = average seepage velocity

S_y = effective porosity

Transmissivity, T: The product of hydraulic conductivity and thickness, b, of the aquifer (L^2T^{-1}):

$$T = Kb$$

Storativity or *storage coefficient of an aquifer, S*: The volume of water taken into or released from storage per unit surface area per unit change in potentiometric (piezometric) head.

6.6.2.2 Hydraulic Conductivity

$$K = \frac{\gamma}{\mu}k$$

where

K = hydraulic conductivity (ft/sec)

k = permeability of a formation (ft^2)

μ = dynamic viscosity ($lbf\text{-}sec/ft^2$)

γ = specific weight of water ion (lbf/ft^3)

6.6.2.3 Seepage Velocity

$$V_v = \frac{K}{\eta}\frac{\Delta h}{L}$$

where

V_v = seepage velocity (ft/sec)

K = hydraulic conductivity (ft/sec)

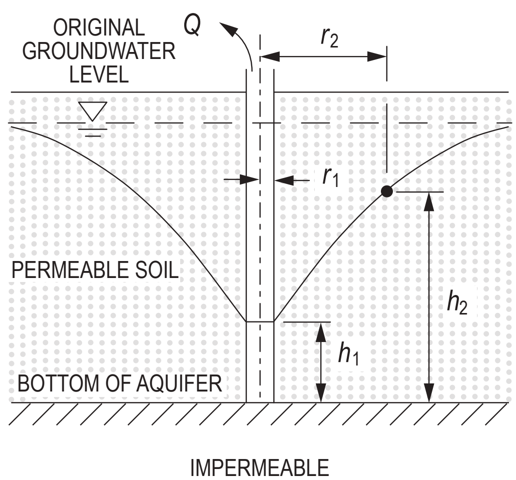
Δh = change in head (ft)

L = length of flow path (ft)

η = porosity

6.6.3 Groundwater Flow

6.6.3.1 Unconfined Aquifers



Dupuit's Formula

$$Q = \frac{\pi K\left(h_2^2 - h_1^2\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$

where

Q = flow rate of water drawn from well (ft^3/sec)

K = coefficient of permeability of soil or hydraulic conductivity (ft/sec)

h_1 = height of water surface above bottom of aquifer at perimeter of well (ft)

h_2 = height of water surface above bottom of aquifer at distance r_2 from well centerline (ft)

r_1 = radius of water surface at perimeter of well, i.e., radius of well (ft)

r_2 = radius of water surface whose height is h_2 above bottom of aquifer (ft)

Aquifer Capacity and Yield

$$q = \frac{Q}{D_w}$$

where

q = specific capacity or specific discharge (ft^2/sec)

Q = well discharge rate (ft^3/sec)

D_w = depth of well drawdown (ft)

$S = S_y + S_s b$

S = storativity or coefficient of storage, common range of 0.1–0.3

where

$S_y >> S_s b$

S_y = specific yield

S_s = specific storage of aquifer (ft^{-1})

b = saturated aquifer thickness (ft)

$n = S_y + S_r$

where

n = porosity

S_y = specific yield

S_r = specific retention

$$S_y = \frac{V_{yield}}{V_{total}}$$

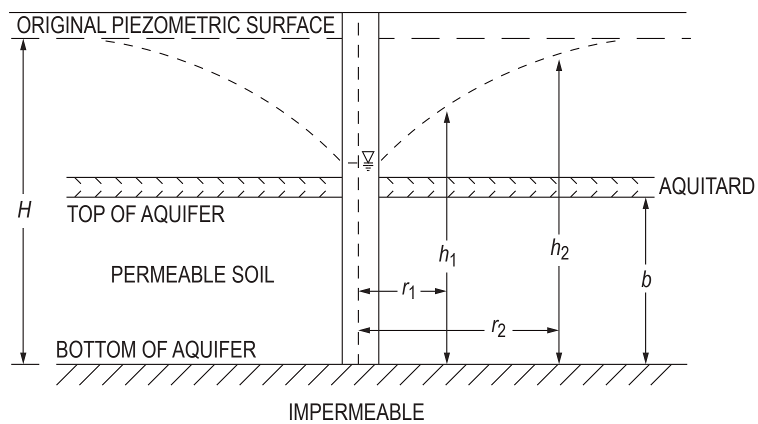
where

S_y = specific yield

V_{yield} = volume of water removed in unit area with unit drawdown (ft^2-ft)

V_{total} = volume of bearing formation (ft^3)

6.6.3.2 Confined Aquifer



Uniform Flow (Thiem Equation)

$$Q = \frac{2\pi T(h_2 - h_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

where

T = Kb = transmissivity (ft^2/sec)

b = thickness of confined aquifer (ft)

h_1, h_2 = heights of piezometric surface above bottom of aquifer (ft)

r_1, r_2 = radii from pumping well (ft)

Cooper-Jacob Equations

$$s = \frac{Q}{4\pi T}\ln\frac{2.25Tt}{r_w^2 S} = \frac{Q}{4\pi T}\left(-0.5772 - \ln\left(\frac{r^2 S}{4Tt}\right)\right)$$

$$s = \frac{2.303Q}{4\pi T}\log\left(\frac{2.25Tt}{r^2 S}\right)$$

where

T = aquifer transmissivity (ft^2/sec)

Q = constant discharge rate (ft^3/sec)

S = storativity or coefficient of storage, common range of 5×10^{-5} through 5×10^{-3}

s = drawdown (ft)

t = time (sec, typ.)

r_w = well radius (ft)

$$T = \frac{2.3Q}{4\pi} \frac{\Delta(\log t)}{\Delta s}$$

$$S = \frac{2.25Tt_o}{r^2}$$

where

Q and r are constant.

Δs = change of drawdown for a given change in log t

$\Delta(\log t)$ = change in time (note: $\Delta(\log t) = 1$ for one log cycle)

t_o = intercept of the straight line with zero drawdown

Aquifer Capacity and Yield

$$q = \frac{Q}{b}$$

where

q = specific capacity or specific discharge (ft^2/sec)

Q = well discharge rate (ft^3/sec)

b = saturated layer thickness (ft)

Multiple Aquifer Layers

$$K_x = \sum_{i=1}^{n} \left[\frac{K_i\, b_i}{b} \right]$$

where

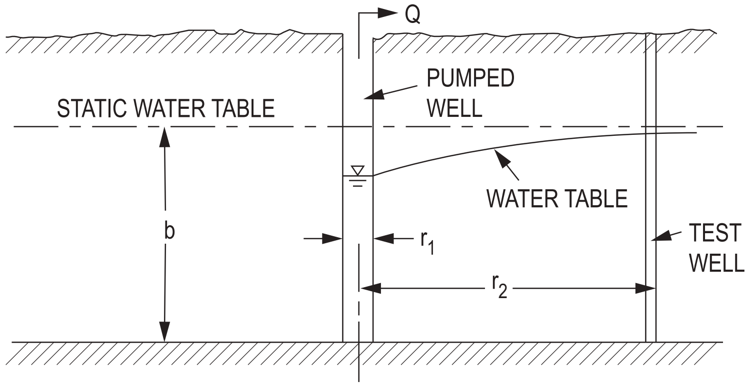
K_x = equivalent hydraulic conductivity of composite section per a unit width i (ft/sec)

K = coefficient of permeability or hydraulic conductivity per a unit width i (ft/sec)

b = individual layer thickness (ft)

6.6.4 Well Analysis–Steady State

6.6.4.1 Time of Travel



$$t = \frac{\pi D \eta}{Q}\left(r_2^2 - r_1^2\right)$$

where

t = time of travel from r_1 to r_2 (sec)

r_1 = radial diameter at pumped well (ft)

r_2 = radial distance at the boundary from which the time of travel is to be computed (ft)

D = thickness of the confined aquifer b or average saturated thickness between radial measuring points r_1 and r_2 (ft)

η = porosity

Q = constant discharge rate (ft^3/sec)

6.6.4.2 Well Performance

Well Loss:

$$s_t = s_a + s_w$$

where

s_t = well drawdown (ft)

s_a = aquifer loss (ft)

s_w = well loss (ft) = CQ^n

Specific Capacity of Well:

$$q = \frac{Q}{s_t}$$

Well Efficiency:

$$E\ (\%) = \frac{s_a}{s_t} \times 100$$

$$E\ (\%) = \left(1 - \frac{s_w}{s_t}\right) \times 100$$

6.7 Water Quality

6.7.1 Mass Conservation and Continuity

Mass Balance:

$$\frac{dM}{dt} = \frac{dM_{\text{in}}}{dt} + \frac{dM_{\text{out}}}{dt} \pm r$$

$$M = CV$$

Continuity equation = $Q = vA$

where

M = mass

M_{in} = mass in

M_{out} = mass out

r = reaction rate = kC^n

k = reaction rate constant $\left(\frac{1}{(\text{concentration units})^{n-1} \times \text{time}}\right)$

n = order of reaction

C = concentration (mass/volume)

Q = flow rate

V = volume

v = velocity

A = cross-sectional area of flow

Concentration to Mass per Day Conversion:

M (lb/day) = C (mg/L) × Q (MGD) × 8.34 (lb/Mgal•mg/L)

where

MGD = million gallons per day

Mgal = million gallons

6.7.2 Advection-Dispersion Reactions

Solute/contaminant movement in porous media for one-dimensional flow

6.7.2.1 Advection

$$J_{\text{adv}} = n_e \, v \, C$$

where

J_{adv} = advective mass flux (mass/L^2/T)

n_e = effective porosity

v = average linear fluid velocity (L/T)

C = solute/contaminant concentration (mass/L^3)

$v = -\frac{K}{n_e}$

K = hydraulic conductivity (L/T)

6.7.2.2 Diffusion—Fick's Law

$$J_{\text{diff}} = D_{\text{mol}}\, dC/dx$$

where

J_{diff} = diffusive mass flux (mass/L^3/T)

D_{mol} = molecular diffusion coefficient in water (L^2/T)

x = distance (L)

6.7.2.3 Dispersion

$$J_{\text{disp}} = D_h\, dC/dx$$

where

J_{disp} = dispersion mass flux (mass/L^3/T)

D_h = hydrodynamic dispersion coefficient (L^2/T)

x = distance (L)

$$D_h = D_e + D_{\text{mech}}$$

where

D_e = effective diffusion coefficient (L^2/T)

D_{mech} = mechanical dispersion coefficient (L^2/T)

$$D_{\text{mech}} = \alpha v$$

where

α = dynamic dispersivity (L)

v = average linear fluid velocity (L/T)

6.7.3 Biochemical Oxygen Demand

6.7.3.1 BOD Testing/Sampling

$$\text{BOD}\left(\text{mg/L}\right) = \frac{D_1 - D_2}{P}$$

When the dilution water is seeded:

$$\text{BOD}\left(\text{mg/L}\right) = \frac{\left(D_1 - D_2\right) - \left(B_1 - B_2\right)f}{P}$$

where

D_1 = dissolved oxygen of diluted sample immediately after preparation (mg/L)

D_2 = dissolved oxygen of diluted sample after 5-day incubation at 20°C (mg/L)

B_1 = dissolved oxygen of seed control before incubation (mg/L)

B_2 = dissolved oxygen of seed control after incubation (mg/L)

f = fraction of seeded dilution water volume in sample to volume of seeded dilution water in seed control

P = fraction of wastewater sample volume to total combined volume

$$\text{BOD}_t = \left(DO_{b,t} - DO_{s,t}\right) \times \text{dilution factor}$$

where

$DO_{b,t}$ = dissolved oxygen concentration in blank after t days in incubation (mg/L)

$DO_{s,t}$ = dissolved oxygen concentration in sample after t days of incubation (mg/L)

$$\text{Dilution factor} = \frac{\text{volume of bottle or diluted sample}}{\text{volume of undiluted sample}}$$

$$\text{Volume of seed, undiluted (mL)} = \frac{\text{allowable depletion} \times \text{volume of bottle}}{\text{estimated BOD of sample}}$$

where

Allowable depletion (mg/L)

Volume of bottle (mL)

Estimated BOD of sample (mg/L)

6.7.3.2 BOD Exertion

$$\text{BOD}_t = L_0\left(1 - e^{-kt}\right)$$

where

BOD_t = amount of BOD exerted at time t (mg/L)

k = BOD decay rate constant, base e (day^{-1})

k = $2.30K$

K = rate constant, base 10 (day^{-1})

L_0 = ultimate BOD (mg/L)

t = time (days)

Typical Values for the BOD Rate Constant

Sample	*K* (20°C) (day^{-1})	*k* (20°C) (day^{-1})
Raw sewage	0.15–0.30	0.35–0.70
Well-treated sewage	0.05–0.10	0.12–0.23
Polluted river water	0.05–0.10	0.12–0.23

Kinetic Temperature Corrections:

$$k_T = k_{20} (\theta)^{T-20}$$

Reaeration $\theta = 1.024$

where

T = temperature of interest (°C)

k_T = BOD rate constant at the temperature of interest (day^{-1})

k_{20} = BOD rate constant determined at 20°C (day^{-1})

θ = temperature coefficient

$$BOD_r = UBOD\left(e^{-k_1 t}\right)$$

where

BOD_r = amount of waste remaining at time t (days) expressed in oxygen equivalents (mg/L)

k_1 = first-order reaction rate constant (day^{-1})

UBOD = total or ultimate carbonaceous BOD (mg/L)

t = time (day)

Thus the BOD exerted up to time t is given by

$$BOD_t = UBOD - BOD_r = UBOD - UBOD\left(e^{-k_1 t}\right) = UBOD\left(1 - e^{-k_1 t}\right)$$

6.7.4 Oxygen Dynamics (Microbial Kinetics)

6.7.4.1 Stream Modeling–Streeter Phelps

$$D = \frac{k_d L_a}{k_r - k_d}\left[e^{-k_d t} - e^{-k_r t}\right] + D_a e^{-k_r t}$$

$$t_c = \frac{1}{k_r - k_d} \ln\left[\frac{k_r}{k_d}\left(1 - D_a \frac{(k_r - k_d)}{k_d L_a}\right)\right]$$

$$DO = DO_{sat} - D$$

where

D = dissolved oxygen deficit (mg/L)

DO = dissolved oxygen concentration (mg/L)

D_a = initial dissolved oxygen deficit in mixing zone (mg/L)

DO_{sat} = saturated dissolved oxygen concentration (mg/L)

k_d = deoxygenation rate constant, base e (day^{-1})

k_r = reaeration rate constant, base e (day^{-1})

L_a = initial ultimate BOD in mixing zone (mg/L)

t = time (days)

t_c = time at which minimum dissolved oxygen occurs (days)

Source: Davis, MacKenzie and David Cornwell. *Introduction to Environmental Engineering*. 4th ed. New York: McGraw-Hill, 2008, 380 & 382.

6.7.4.2 Oxygen Saturation

$$DO_{sat} = K_H \times P_{O_2}$$

where

K_H = Henry's Law constant (moles/L•atm)

P_{O_2} = partial pressure of oxygen (atm)

Saturation Values of Dissolved Oxygen in Freshwater Exposed to a Saturated Atmosphere Containing 20.9% Oxygen Under a Pressure of 101.325 kPa[a]

Temperature (°C)	Dissolved Oxygen (mg/L)	Saturated Vapor Pressure (kPa)
0	14.62	0.6108
1	14.23	0.6566
2	13.84	0.7055
3	13.48	0.7575
4	13.13	0.8129
5	12.80	0.8719
6	12.48	0.9347
7	12.17	1.0013
8	11.87	1.0722
9	11.59	1.1474
10	11.33	1.2272
11	11.08	1.3119
12	10.83	1.4017
13	10.60	1.4969
14	10.37	1.5977
15	10.15	1.7044
16	9.95	1.8173
17	9.74	1.9367
18	9.54	2.0630
19	9.35	2.1964
20	9.17	2.3373
21	8.99	2.4861
22	8.83	2.6430
23	8.68	2.8086
24	8.53	2.9831
25	8.38	3.1671
26	8.22	3.3608
27	8.07	3.5649
28	7.92	3.7796
29	7.77	4.0055
30	7.63	4.2430
31	7.51	4.4927
32	7.42	4.7551
33	7.28	5.0307
34	7.17	5.3200
35	7.07	5.6236
36	6.96	5.9422
37	6.86	6.2762
38	6.75	6.6264

[a] For other barometric pressures, the solubilities vary approximately in proportion to the ratios of these pressures to the standard pressures.

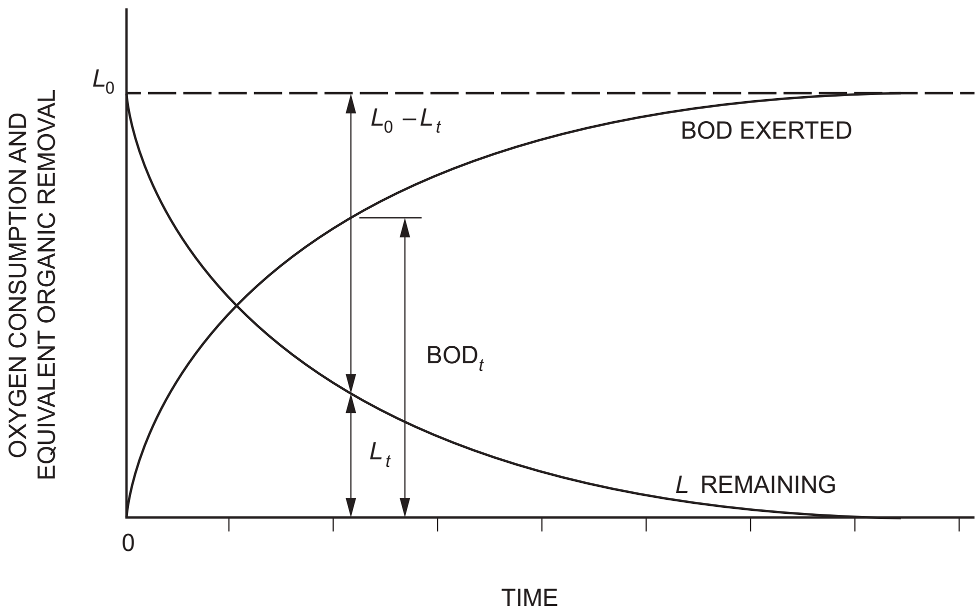
Source: Calculated by G.C. Whipple and M.C. Whipple from measurements of C.J.J. Fox. *Journal of the American Chemical Society*, Vol. 33, 1911, p. 362.

Dissolved-Oxygen Concentration in Water as a Function of Temperature and Salinity (Barometric Pressure = 760 mm Hg[a])

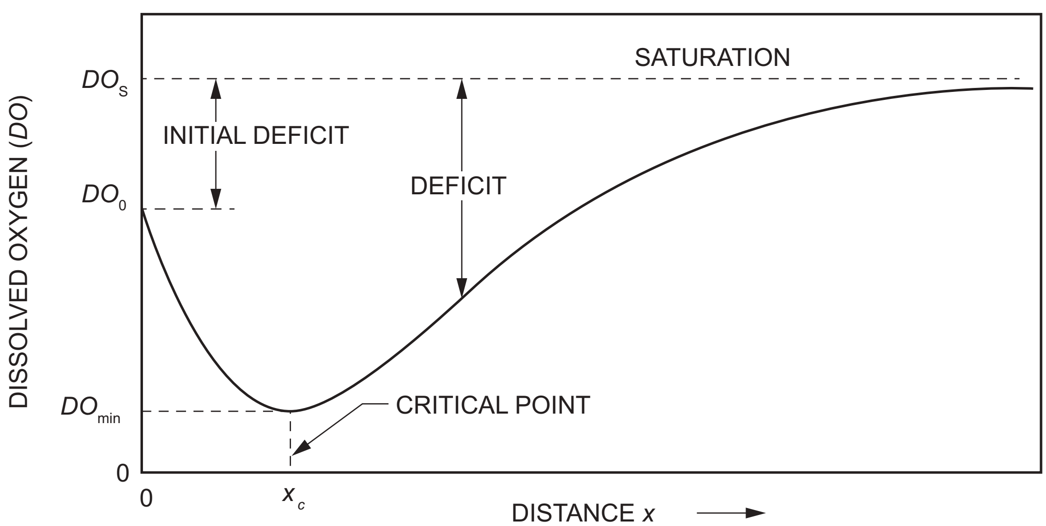
Dissolved-Oxygen Concentration (mg/L)										
Temp. °C	Salinity (parts per thousand)									
	0	5	10	15	20	25	30	35	40	45
0	14.60	14.11	13.64	13.18	12.74	12.31	11.90	11.50	11.11	10.74
1	14.20	13.73	13.27	12.83	12.40	11.98	11.58	11.20	10.83	10.46
2	13.81	13.36	12.91	12.49	12.07	11.67	11.29	10.91	10.55	10.20
3	13.45	13.00	12.58	12.16	11.76	11.38	11.00	10.64	10.29	9.95
4	13.09	12.67	12.25	11.85	11.47	11.09	10.73	10.38	10.04	9.71
5	12.76	12.34	11.94	11.56	11.18	10.82	10.47	10.13	9.80	9.48
6	12.44	12.04	11.65	11.27	10.91	10.56	10.22	9.89	9.57	9.27
7	12.13	11.74	11.37	11.00	10.65	10.31	9.98	9.66	9.35	9.06
8	11.83	11.46	11.09	10.74	10.40	10.07	9.75	9.44	9.14	8.85
9	11.55	11.19	10.83	10.49	10.16	9.84	9.53	9.23	8.94	8.66
10	11.28	10.92	10.58	10.25	9.93	9.62	9.32	9.03	8.75	8.47
11	11.02	10.67	10.34	10.02	9.71	9.41	9.12	8.83	8.56	8.30
12	10.77	10.43	10.11	9.80	9.50	9.21	8.92	8.65	8.38	8.12
13	10.53	10.20	9.89	9.59	9.30	9.01	8.74	8.47	8.21	7.96
14	10.29	9.98	9.68	9.38	9.10	8.82	8.55	8.30	8.04	7.80
15	10.07	9.77	9.47	9.19	8.91	8.64	8.38	8.13	7.88	7.65
16	9.86	9.56	9.28	9.00	8.73	8.47	8.21	7.97	7.73	7.50
17	9.65	9.36	9.09	8.82	8.55	8.30	8.05	7.81	7.58	7.36
18	9.45	9.17	8.90	8.64	8.39	8.14	7.90	7.66	7.44	7.22
19	9.26	8.99	8.73	8.47	8.22	7.98	7.75	7.52	7.30	7.09
20	9.08	8.81	8.56	8.31	8.07	7.83	7.60	7.38	7.17	6.96
21	8.90	8.64	8.39	8.15	7.91	7.69	7.46	7.25	7.04	6.84
22	8.73	8.48	8.23	8.00	7.77	7.54	7.33	1.12	6.91	6.72
23	8.56	8.32	8.08	7.85	7.63	7.41	7.20	6.99	6.79	6.60
24	8.40	8.16	7.93	7.71	7.49	7.28	7.07	6.87	6.68	6.49
25	8.24	8.01	7.79	7.57	7.36	7.15	6.95	6.75	6.56	6.38
26	8.09	7.87	7.65	7.44	7.23	7.03	6.83	6.64	6.46	6.28
27	7.95	7.73	7.51	7.31	7.10	6.91	6.72	6.53	6.35	6.17
28	7.81	7.59	7.38	7.18	6.98	6.79	6.61	6.42	6.25	6.08
29	7.67	7.46	7.26	7.06	6.87	6.68	6.50	6.32	6.15	5.98
30	7.54	7.33	7.14	6.94	6.75	6.57	6.39	6.22	6.05	5.89
31	7.41	7.21	7.02	6.83	6.65	6.47	6.29	6.12	5.96	5.80
32	7.29	7.09	6.90	6.72	6.54	6.36	6.19	6.03	5.87	5.71
33	7.17	6.98	6.79	6.61	6.44	6.26	6.10	5.94	5.78	5.63
34	7.05	6.86	6.68	6.51	6.33	6.17	6.01	5.85	5.69	5.54
35	6.93	6.75	6.58	6.40	6.24	6.07	5.92	5.76	5.61	5.46
36	6.82	6.65	6.47	6.31	6.14	5.98	5.83	5.68	5.53	5.39
37	6.72	6.54	6.37	6.21	6.05	5.89	5.74	5.59	5.45	5.31
38	6.61	6.44	6.28	6.12	5.96	5.81	5.66	5.51	5.37	5.24
39	6.51	6.34	6.18	6.03	5.87	5.72	5.58	5.44	5.30	5.16
40	6.41	6.25	6.09	5.94	5.79	5.64	5.50	5.36	5.22	5.09

Source: Colt, J., "Computation of Dissolved Gas Concentrations in Water as Functions of Temperature, Salinity and Pressure," American Fisheries Society Special Publication 14, 1984, American Fisheries Society. Used with permission.

Oxygen Consumption



Oxygen Deficit After Mixing



$$D = DO_{stream} - DO$$

where

D = oxygen deficit (mg/L)

DO_{stream} = saturation concentration of dissolved oxygen at the temperature of the stream after mixing (mg/L)

DO = actual concentration of dissolved oxygen in stream (mg/L)

6.7.4.3 Initial Deficit

$$D_a = DO_{\text{stream}} - \frac{Q_w DO_w + Q_{\text{stream}} DO_{\text{stream}}}{Q_w + Q_{\text{stream}}}$$

where

D_a = initial deficit after stream and waste have mixed (mg/L)

DO_{stream} = dissolved oxygen content in stream (mg/L)

DO_w = dissolved oxygen content in waste (mg/L)

6.7.4.4 BOD Depletion Over Time

$$BOD_t = L_0 - L_t = L_0\left(1 - \mathrm{e}^{-kt}\right)$$

where L_0 = ultimate BOD (mg/L)

Oxygen Demand in Stream Over Time

$$L_t = L_0 e^{-kt}$$

where

L_t = oxygen equivalent of organic compounds at time t (mg/L)

L_0 = oxygen equivalent of organic compounds at time $t = 0$ (mg/L)

k = reaction rate constant, base e (day^{-1})

or

$$L_t = L_0\left(1 - 10^{-Kt}\right)$$

where K = reaction rate constant, base 10 (day^{-1})

6.7.4.5 Deoxygenation Rate

k_d = deoxygenation rate constant, lab based, base e (day^{-1})

= 2.303 K_d

~ 0.23 – 0.70 day^{-1} at 20°C in municipal wastewater

~ 0.14 – 0.28 day^{-1} at 20°C in treated effluent

K_d = deoxygenation rate coefficient, lab based, base 10 (day^{-1})

Logarithmic Method

$$\frac{0.85S}{2.303t} = k_d\left(L_a - y_t\right)$$

where

k_d = deoxygenation rate coefficient, base e (day^{-1})

S = strength factor = BOD intercept on y-axis of line at 5 days

L_a = first stage ultimate BOD (mg/L)

y = BOD exerted in time t (mg/L)

6.7.4.6 Stream Reaeration Rate

$$K_r = \frac{13.0v^{0.5}}{H^{1.5}} \quad \text{(O'Connor and Dobbins Method)}$$

$$K_r = \frac{11.57v^{0.969}}{H^{1.673}} \quad \text{(Churchill et al. Method)}$$

$$K_r = \frac{7.63v}{H^{1.33}} \quad \text{(Langbein and Durum Method)}$$

where

K_r = reaeration rate coefficient, base e (day^{-1})

v = average velocity (ft/sec)

H = average depth of stream (ft)

6.7.4.7 Stream Oxygen Deficit

$$D_t = \frac{K_d L_a}{K_r - K_d}\left(10^{-K_d t} - 10^{-K_r t}\right) + D_a \times 10^{-K_r t}$$

where

D_t = dissolved oxygen deficit (mg/L)

K_d = lab deoxygenation rate coefficient, base 10 (day^{-1})

K_r = reaeration rate coefficient, base 10 (day^{-1})

L_a = first stage ultimate BOD (mg/L)

D_a = initial deficit after river and wastewater have mixed (mg/L)

L_t = BOD remaining at time t (mg/L)

t = time of travel of wastewater discharge downstream (day)

Critical Point/Deficit

$$D_c = \left(\frac{K_d}{K_r} L_a\right) 10^{-K_d t_c}$$

Time to Critical Deficit

$$t_c = \frac{1}{K_r - K_d} \ln\left[\frac{Kr}{K_d}\left(1 - D_a \frac{Kr - K_d}{K_d La}\right)\right]$$

where

t_c = time of critical deficit (days)

L_a = maximum demand allowable for discharge within the stream (mg/L)

D_a = initial deficit after stream and waste have mixed (mg/L)

Maximum Allowable Oxygen Demand

$$L_a = D_c\left(\frac{K_2}{K_1}\right)\left[1 + \frac{K_r}{K_2 - K_r}\left(1 - \frac{D_a}{D_c}\right)^{0.418}\right]$$

6.7.5 Monod Kinetics–Substrate Limited Growth

Continuous flow systems where growth is limited by one substrate (chemostat):

$$\mu = \frac{Y k_m S}{K_s + S} - k_d = \mu_{\max}\frac{S}{K_s + S} - k_d$$

6.7.5.1 Multiple Limiting Substrates

$$\frac{\mu}{\mu_{\max}} = \left[\mu_1\left(S_1\right)\right]\left[\mu_2\left(S_2\right)\right]\left[\mu_3\left(S_3\right)\right]\ldots\left[\mu_n\left(S_n\right)\right]$$

where $\mu_i = \dfrac{S_i}{K_{s_i} + S_i}$ for i = 1 to n

6.7.5.2 Nonsteady State Continuous Flow

$$\frac{dx}{dt} = Dx_0 + \left(\mu - k_d - D\right)x$$

6.7.5.3 Steady State Continuous Flow

$$\mu = D \quad \text{with } k_d << \mu$$

6.7.5.4 Product Production at Steady State, Single Substrate Limiting

$$X_1 = Y_{P/S}\left(S_o - S_i\right)$$

where

k_d = microbial death rate or endogenous decay rate constant (time^{-1})

k_m = maximum growth rate constant (time^{-1})

K_s = saturation constant or half-velocity constant = concentration at $\frac{\mu_{\max}}{2}$

S = concentration of substrate in solution (mass/unit volume)

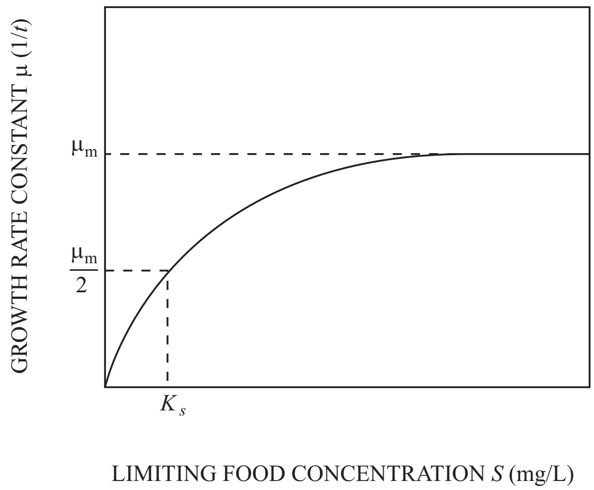
Y = yield coefficient [(mass/L product)/(mass/L food used)]

μ = specific growth rate (time^{-1})

$\mu_{\max}$ = maximum specific growth rate (time^{-1}) = $Y k_m$

6.7.5.5 Monod Growth Rate Constant as a Function of Limiting Concentration

Monod growth rate constant as a function of limiting food concentration.



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where

X_1 = product (mg/L)

V_r = volume (L)

D = dilution rate (hr^{-1}) = flow f /reactor volume V_r

f = flow rate (L/hr)

μ_i = growth rate with one or multiple limiting substrates (hr^{-1})

S_i = substrate i concentration (mass/unit volume)

S_0 = initial substrate concentration (mass/unit volume)

$Y_{P/S}$ = product yield per unit of substrate (mass/mass)

p = product concentration (mass/unit volume)

x = cell concentration (mass/unit volume)

x_0 = initial cell concentration (mass/unit volume)

t = time (time)

Steady-State Reactor Parameters (Constant Density Systems)

Comparison of Steady-State Retention Times (θ) for Decay Reactions of Different Order [a]

		Equations for Mean Retention Times (θ)		
Reaction Order	**r**	**Ideal Batch**	**Ideal Plug Flow**	**Ideal CMFR**
Zero[b]	$-k$	$\frac{(C_0 - C_t)}{k}$	$\frac{(C_0 - C_t)}{k}$	$\frac{(C_0 - C_t)}{k}$
First	$-kC$	$\frac{\ln(C_0/C_t)}{k}$	$\frac{\ln(C_0/C_t)}{k}$	$\frac{(C_0/C_t) - 1}{k}$
Second	$-kC^2$	$\frac{(C_0/C_t) - 1}{kC_0}$	$\frac{(C_0/C_t) - 1}{kC_0}$	$\frac{(C_0/C_t) - 1}{kC_t}$

[a] C_0 = initial concentration or influent concentration; C_t = final condition or effluent concentration.

[b] Expressions are valid for $k\theta \le C_0$; otherwise $C_t = 0$.

Comparison of Steady-State Performance for Decay Reactions of Different Order [a]

		Equations for C_t		
Reaction Order	**r**	**Ideal Batch**	**Ideal Plug Flow**	**Ideal CMFR**
Zero[b] $t \le C_0/k$	$-k$	$C_0 - kt$	$C_0 - k\theta$	$C_0 - k\theta$
$t > C_0/k$		0		
First	$-kC$	$C_0[\exp(-kt)]$	$C_0[\exp(-k\theta)]$	$\frac{C_0}{1 + k\theta}$
Second	$-kC^2$	$\frac{C_0}{1 + ktC_0}$	$\frac{C_0}{1 + k\theta C_0}$	$\frac{(4k\theta C_0 + 1)^{1/2} - 1}{2k\theta}$

[a] C_0 = initial concentration or influent concentration; C_t = final condition or effluent concentration.

[b] Time conditions are for ideal batch reactor only.

Davis, M.L., and S.J. Masten, *Principles of Environmental Engineering and Science*, 2nd ed., McGraw-Hill, 2004.

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where CMFR = completely mixed fluid reactor

6.7.6 Total Maximum Daily Load (TMDL)

$$TMDL = \sum WLA + \sum LA + \sum MOS$$

where

WLA = sum of waste load allocations (point sources) (mass/day)

LA = sum of load allocations (nonpoint sources and background) (mass/day)

MOS = margin of safety (mass/day)

6.7.7 Biological Contaminants (Partition Coefficients)

6.7.7.1 Bioconcentration, Bioaccumulation, and Biomagnification Factors

The amount of a chemical that accumulates in aquatic organisms:

$$BCF = C_{\text{org}} / C$$

where

BCF = bioconcentration factor (L/kg)

C_{org} = equilibrium concentration in organism (mg/kg or ppm)

C = concentration in water (mg/L or ppm)

6.7.7.2 Bioaccumulation Factor

The amount of a chemical that accumulates in organisms with respect to solids:

$$BAF = C_{\text{org}} / C_s$$

where

BAF = bioaccumulation factor

C_{org} = equilibrium concentration in organism (g/kg)

C_s = concentration in surrounding medium, such as sediment or soils (g/kg)

6.7.7.3 Biomagnification Factor

The amount of a chemical that accumulates in predator organisms:

$$BMF = C_{\text{pred}} / C_{\text{prey}}$$

where

BMF = biomagnification factor

C_{pred} = steady state concentration in predator organism (mg/kg)

C_{prey} = steady state concentration in prey organisms (mg/kg)

6.7.7.4 Octanol-Water Partition Coefficient

The ratio of a chemical's concentration in the octanol phase to its concentration in the aqueous phase of a two-phase octanol-water system:

$$K_{ow} = C_O / C_w$$

where

C_O = concentration of chemical in octanol phase (mg/L or μg/L)

C_w = concentration of chemical in aqueous phase (mg/L or μg/L)

6.7.7.5 Soil-Water Partition Coefficient

$$K_{sw} = K_p$$

$$K_{sw} = \frac{X}{C}$$

where

K_{sw} = soil-water partition coefficient

X = concentration of chemical in soil (ppb or μg/kg)

C = concentration of chemical in water (ppb or μg/kg)

$$K_{sw} = K_{oc} f_{oc}$$

where f_{oc} = fraction of organic carbon in the soil (dimensionless)

6.7.7.6 Organic Carbon Partition Coefficient

$$K_{oc} = C_{soil} / C_{water}$$

where

K_{oc} = organic carbon partition coefficient

C_{soil} = concentration of chemical in organic carbon component of soil $\left(\frac{\mu\text{g adsorbed}}{\text{kg organic } C} \text{ or ppb}\right)$

C_{water} = concentration of chemical in water (ppb or μg/kg)

6.7.7.7 Retardation Factor R

$$R = 1 + \left(\frac{\rho}{n_e}\right) K_d$$

where

ρ = bulk soil density (lbf-sec^2/ft^4)

n_e = effective porosity

K_d = distribution coefficient

6.7.8 Risk Calculation

6.7.8.1 No-Effect Level (Reference Dose)

$$\text{RfD} = \frac{\text{NOAEL or LOAEL}}{\text{uncertainty factor}}$$

where

RfD = reference dose (mg/kg of body weight per day)

NOAEL = no-observed-adverse-effect level

LOAEL = lowest-observed-effect level

6.7.8.2 Drinking Water Equivalent Level (DWEL)

$$\text{DWEL} = \frac{\text{RfD} \times \text{body weight}}{\text{drinking water consumption}}$$

where

DWEL = drinking water equivalent level (mg/L)

RfD = reference dose (mg/kg of body weight per day)

body weight = (kg)

drinking water consumption = (L/day)

6.7.8.3 Toxic Unit Acute (TU_a)

$TU_a = 100/\text{LC}_{50}$

where LC_{50} = median lethal concentration

6.7.8.4 Toxic Unit Chronic (TU_c)

$TU_c = 100/\text{NOEC}$

where NOEC = no-observed-effect concentration (mg/L)

6.7.8.5 Probability of Risk from Carcinogenic Substances

Linearized Multistage Model (EPA)

$$p = \frac{d(SF)}{m(t)} = CDI \times SF$$

where

p = probability of lifetime risk associated with carcinogenic substance

d = dose (mg)

CDI = chronic daily intake (mg/kg•day)

SF = slope factor for contaminant (kg•day/mg)

m = mass of organism (kg)

t = time (days)

Chronic Daily Intake for Ingested Contaminated Water

$$CDI = C\frac{(IR)(EF)(ED)}{(BW)(AT)}$$

where

CDI = chronic daily intake (mg/kg•day)

C = average ingested chemical concentration in air/water over period (mg/L)

IR = intake/ingestion rate (L/day)

EF = exposure frequency (days/year)

ED = exposure duration (years)*

BW = body weight of person (kg)

AT = averaging time (period over which exposure is averaged) (days)**

* Frequent *ED* Values:

- average lifetime = 70 years
- national upper bound time (90th percentile) at one residence = 30 years
- national median time (50th percentile) at one residence = 9 years

** *AT* is frequently calculated as the *ED* * 365 days/yr for noncarcinogens

Common Probability Distributions Table

Distribution	Probability Density Function (PDF), $p(X)$	Cumulative Density Function (CDF), $P(X \le x)$	Range	Mean μ or $\overline{X}$	Standard Deviation σ or S
Normal	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$ or $\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ where $z = \frac{X-\mu}{\sigma}$	$\int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} dx$	$-\infty \le x \le \infty$	μ	σ
Lognormal $y = \ln x$	$\frac{1}{\sigma_y\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2}$	$\int_{-\infty}^{y} \frac{1}{\sigma_y\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2} dy$	$-\infty \le y \le \infty$ $0 \le x \le \infty$	μ_y	σ_y
Extreme value: Type I (Gumbel) $y = \frac{x-\mu}{\beta}$ Type III	$\frac{1}{\alpha} e^{-\left(y+e^{-y}\right)}$ $\alpha x^{\alpha-1} \beta^{-\alpha} e^{-(x/\beta)\alpha}$	e^{-e-y} $1 - e^{-(x/\beta)\alpha}$	$-\infty \le x \le \infty$ $x \ge 0$	$\beta + 0.577\alpha$ $\alpha = \frac{\sqrt{6}S_x}{\pi}$ $\beta\Gamma(1+1/\alpha)$	1.283α $\beta\left[\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha)\right]^{1/2}$
Log-Pearson Type III $y = \ln x$	$p_0(1+y/\alpha)^c e^{-cy/\alpha}$ where p_0 = prob. at the mode $= \frac{N}{\alpha} \frac{c^{c+1}}{e^c\Gamma(c+1)}$	$\int_{-\infty}^{y} p_0(1+y/\alpha)^c e^{-cy/\alpha} dy$ (known as incomplete gamma function)	$-\infty \le y \le \infty$ $0 \le x \le \infty$	$(c+1)\frac{\alpha}{c}$	$\sqrt{c+1}\frac{\alpha}{c}$

Γ is the gamma function; $\Gamma(n) = (n-1)!$.

α and β are evaluated from relations under mean and standard deviation.

c and α are evaluated from relations under mean and standard deviation.

6.8 Wastewater Collection and Treatment

6.8.1 Wastewater Collection Systems

6.8.1.1 Wet Wells

$$t_r = \frac{\overline{V}}{D - Q}$$

$$t_f = \frac{V}{Q}$$

where

t_r = pump running time (min)

t_f = filling time with pump off (min)

V = storage volume of wet well (gal)

D = pump discharge (gal/min)

Q = inflow (gal/min)

Total cycle time (t) is

$$t = t_r + t_f = \frac{V}{D - Q} + \frac{V}{Q}$$

For peak (maximum) flow rate, the volume of a wet well can be calculated as

$$V = TQ/4$$

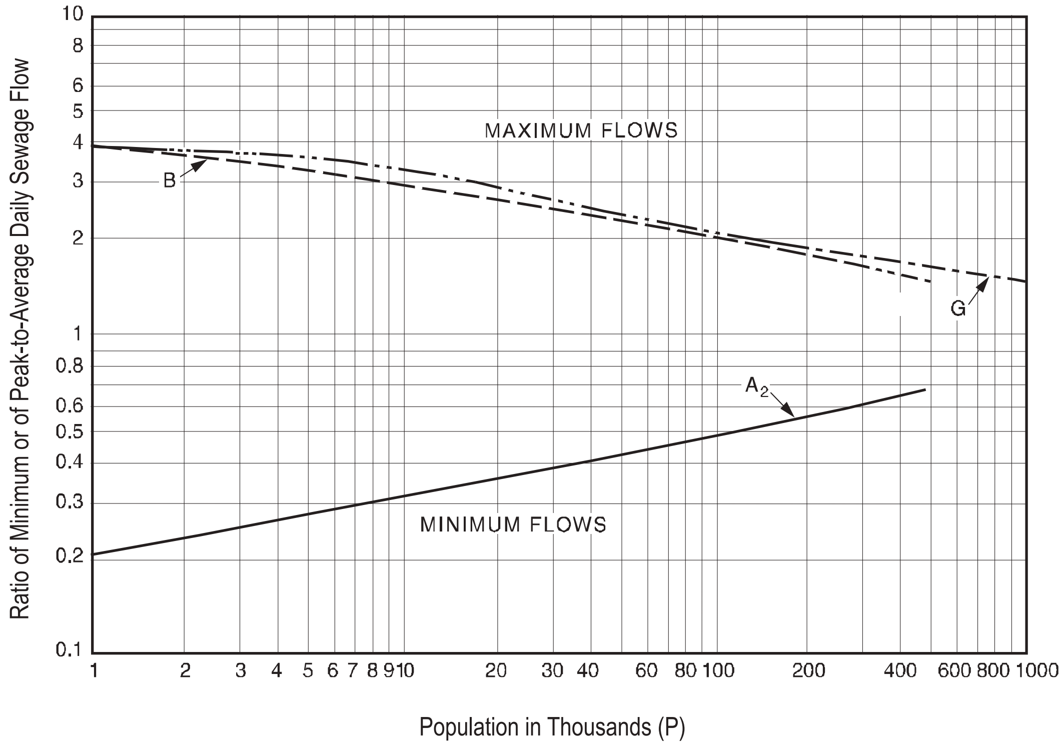
where

V = storage volume of wet well (gal)

T = pump cycle time (min)

Q = peak flow (gal/min)

6.8.2 Wastewater Flow Rates



Curve A_2: $\frac{P^{0.2}}{5}$

Curve B: $\frac{14}{4+\sqrt{P}}+1$

Curve G: $\frac{18+\sqrt{P}}{4+\sqrt{P}}$

Sewage Flow Ratio Curves

Source: American Society of Civil Engineers (ASCE). *Design and Construction of Sanitary and Storm Sewers*. Reston, VA: ASCE, 1970, Fig. 4, pg. 33.

6.8.3 Wastewater Testing

6.8.3.1 BOD Test Solution and Seeding Procedures

When the dilution of water is not seeded:

$$\text{BOD}\left(\text{mg/L}\right)=\frac{D_1-D_2}{P}$$

When the dilution of water is seeded:

$$\text{BOD}\left(\text{mg/L}\right)=\frac{\left(D_1-D_2\right)-\left(B_1-B_2\right)f}{P}$$

D_1 = dissolved oxygen of diluted sample immediately after preparation (mg/L)

D_2 = dissolved oxygen of diluted sample after 5-day incubation at 20°C (mg/L)

B_1 = dissolved oxygen of seed control before incubation (mg/L)

B_2 = dissolved oxygen of seed control after incubation (mg/L)

f = fraction of seeded dilution water volume in sample to volume of seeded dilution water in seed control

P = fraction of wastewater sample volume to total combined volume

6.8.3.2 First Order Reactions

$$\ln \frac{c}{c_0} = -kt$$

where

c = concentration of constituent

c_0 = concentration of liquid bulk

t = time (days)

k = first order reaction rate constant (day^{-1})

6.8.3.3 Kinetic Temperature Corrections

$$\frac{k_2}{k_1} = \theta^{(T_2 - T_1)}$$

where

θ = temperature coefficient, unitless

T = temperature (°C)

$$k_T = k_{20}\,(\theta)^{T-20}$$

BOD (k):	$\theta = 1.135$	(T = 4–20°C)
	$\theta = 1.056$	(T = 21–30°C)
Reaeration (k_r)	$\theta = 1.024$	
Biotowers	$\theta = 1.035$	
Trickling Filters	$\theta = 1.072$	

6.8.3.4 Absorbance

$$A = \log \frac{I_0}{I}$$

where

A = absorbance (absorbance unit/centimeter or a.u./cm)

I_0 = initial detector reading for the blank (i.e., distilled water) after passing through a solution of known depth

I = final detector reading after passing through solution containing constituents of interest

6.8.3.5 Transmittance

$$T = \frac{I}{I_a} \times 100$$

where T = transmittance (%)

6.8.4 Preliminary Treatment

6.8.4.1 Coarse Screening/Bar Rack Head Loss (Bernoulli Method)

$$h_L = \frac{1}{C}\left(\frac{V^2 - v^2}{2g}\right)$$

where

h_L = head loss through bar rack (ft)

C = discharge friction coefficient

= frequently 0.7 for clean screen and 0.6 for clogged screen

V = velocity of flow through the openings (ft/sec)

v = velocity in upstream channel (ft/sec)

g = acceleration due to gravity (ft/sec^2)

$$V = \sqrt{1.4 h_L\, g + v^2}$$

6.8.4.2 Bar Rack Head Loss–Kirschmer Method (Clean Racks)

$$h_L = \beta\left(\frac{w}{b}\right)^{1.33} \frac{v^2}{2g} \sin\theta$$

$$h_L = \beta\left(\frac{w}{b}\right)^{1.33} h_v \sin\theta$$

where

h_L = head loss (ft)

β = bar shape factor

w = maximum cross-sectional width of bars facing upstream flow (ft)

b = minimum clear spacing of bars (ft)

v = velocity upstream of bars (ft/sec)

g = acceleration due to gravity (ft/sec^2)

h_v = upstream head (ft)

θ = angle of the bar screen with horizontal (degrees)

Bar Shape Factors

Bar Type	β
Sharp-edged rectangular	2.42
Rectangular with semicircular face	1.83
Circular	1.79
Rectangular with semicircular upstream and downstream faces	1.67
Tear shape	0.76

6.8.4.3 Fine Screening Head Loss

$$h_L = \frac{1}{C \times 2g}\left(\frac{Q}{A}\right)^2$$

where

h_L = head loss (ft)

Q = discharge through screen (ft^3/sec)

A = effective open area of submerged screen (ft^2)

g = acceleration due to gravity (ft/sec^2)

C = discharge friction coefficient

$$A = \frac{Q}{\sqrt{2h_L Cg}}$$

$$C = \frac{1}{2h_L g}\left(\frac{Q}{A}\right)^2$$

6.8.5 Primary Treatment

6.8.5.1 Mixing

Refer to Drinking Water Distribution and Treatment – Mixing and Flocculation.

6.8.5.2 Clarifiers

Hydraulic Loading Rate

$v_0 = Q/A$

where

v_0 = clarifier's hydraulic loading rate or surface overflow rate (gal/ft^2-day)

Q = flow rate (gal/day)

A = surface area, plan view (ft^2)

and

$\theta = V/Q$

= hydraulic retention/residence time (HRT) (days)

V = volume of clarifier (gal)

Typical Primary Clarifier Efficiency Removal

	Overflow Rates			
	1,200 (gpd/ft^2) 48.9 (m/day)	1,000 (gpd/ft^2) 40.7 (m/day)	800 (gpd/ft^2) 32.6 (m/day)	600 (gpd/ft^2) 24.4 (m/day)
Suspended Solids	54%	58%	64%	68%
BOD_5	30%	32%	34%	36%

Design Criteria for Primary Clarifiers

Overflow Rate: Average (gpd/ft^2)	Overflow Rate: Average (m^3/m^2•day)	Overflow Rate: Peak (gpd/ft^2)	Overflow Rate: Peak (m^3/m^2•day)	Hydraulic Residence Time (hr)	Depth (ft)
800–1,200	32–49	1,200–2,000	50–80	2	10–12
400–800	16–33			2	

Scour Velocity

$$v_c = \left[\frac{8\beta(s-1)gd}{f}\right]^{0.5}$$

where

v_c = critical velocity that will produce scour (ft/sec)

β = constant for type of material being scoured

s = specific gravity of particles

g = acceleration due to gravity = 32.2 ft/sec^2

d = diameter of particles (in.)

f = Darcy-Weisbach friction factor

6.8.5.3 Activated Sludge Treatment

Activated Sludge Reactions

Biomass destruction:

$$C_5H_7NO_2 + 5O_2 \rightarrow 4CO_2 + H_2O + NH_4HCO_3$$

Nitrification of released ammonia nitrogen:

$$NH_4^+ + 2O_2 \rightarrow NO_3^- + 2H^+ + H_2O$$

Overall equation with complete nitrification:

$$C_5H_7NO_2 + 7O_2 \rightarrow 5CO_2 + 3H_2O + HNO_3$$

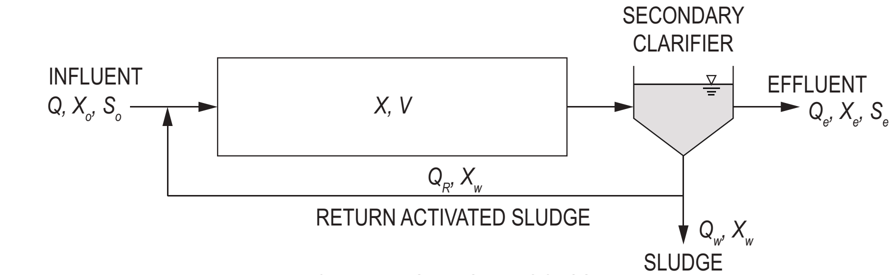
Using nitrate nitrogen as electron acceptor (denitrification):

$$C_5H_7NO_2 + 4NO_3^- + H_2O \rightarrow NH_4^+ + 5HCO_3^- + 2NO_2$$

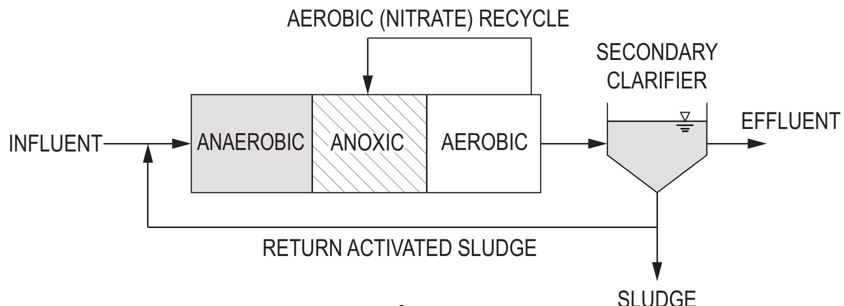
With complete nitrification/denitrification:

$$2C_5H_7NO_2 + 11.5O_2 \rightarrow 10CO_2 + 7H_2O + 2N_2$$

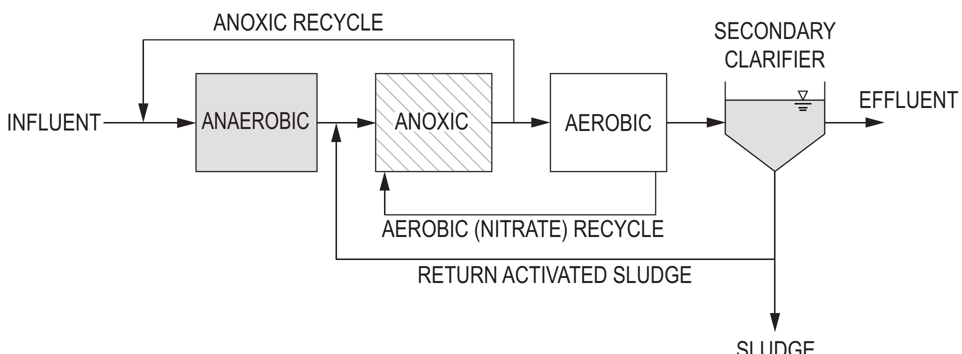
ACTIVATED SLUDGE PROCESS SCHEMATICS



ACTIVATED SLUDGE PROCESS



A^2O, UCT



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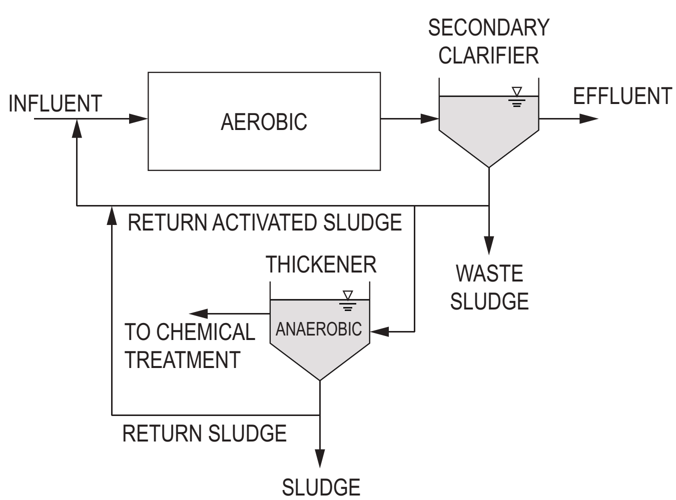

BIOCHEMICAL SIDESTREAM

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Mixed Liquor Suspended Solids (MLSS)

$$X = \frac{\theta_c Y\left(S_0 - S_e\right)}{\theta\left(1 + k_d \theta_c\right)}$$

where

X = mixed liquor suspended solids (MLSS) concentration (mg/L)

S_0 = influent BOD or COD concentration (ppm or mg/L)

S_e = effluent BOD or COD concentration (ppm or mg/L)

θ = hydraulic retention/residence time = HRT (days)

θ_c = solids residence time, cell residence time, sludge age (days)

k_d = endogenous decay coefficient (day^{-1})

Y = synthesis yield coefficient, biomass produced/BOD consumed (dimensionless)

= typical range 0.4–1.2

Solids Loading (System)

$$SL = 8.34\ Q\ X$$

where

SL = solids loading (lb/day)

Q = volumetric flow rate (Mgal/day or MGD)

X = suspended solids concentration (ppm or mg/L)

unit conversion = 8.34 (lb/Mgal•mg/L)

Organic Loading Rate

$$F/M = \frac{S_0 Q_0}{VX} = \frac{S_0}{\theta X} = \frac{\text{BOD}}{\text{MLSS}}$$

where

F/M = organic loading rate or food-to-microorganism ratio (day^{-1})

Q_0 = influent flow rate (Mgal/day or MGD)

S_0 = influent BOD or COD concentration (ppm or mg/L)

V = volume of unit (Mgal)

X = suspended solids concentration in unit (ppm or mg/L)

θ = hydraulic retention/residence time (days)

Organic loading rate, volumetric (mg/L•day) = $\dfrac{Q_0 S_0}{V}$

Process Efficiency

$$E(\%) = \frac{S_0 - S_e}{S_0} \times 100$$

Design and Operational Parameters for Activated-Sludge Treatment of Municipal Wastewater

Type of Process	Mean cell residence time (θ_c, day)	Food-to-mass ratio [(kg BOD_5/(day•kg MLSS)]	Volumetric loading ($kgBOD_5/m^3$)	Hydraulic residence time in aeration basin (θ, hr)	Mixed liquor suspended solids (MLSS, mg/L)	Recycle ratio (Q_r /Q)	Flow regime*	BOD_5 removal efficiency (%)	Air supplied (m^3/kg BOD_5)
Tapered aeration	5−15	0.2−0.4	0.3−0.6	4−8	1,500−3,000	0.25−0.5	PF	85−95	45−90
Conventional	4−15	0.2−0.4	0.3−0.6	4−8	1,500−3,000	0.25−0.5	PF	85−95	45−90
Step aeration	4−15	0.2−0.4	0.6−1.0	3−5	2,000−3,500	0.25−0.75	PF	85−95	45−90
Completely mixed	4−15	0.2−0.4	0.8−2.0	3−5	3,000−6,000	0.25−1.0	CM	85−95	45−90
Contact stabilization	4−15	0.2−0.6	1.0−1.2			0.25−1.0			45−90
Contact basin				0.5−1.0	1,000−3,000		PF	80−90	
Stabilization basin				4−6	4,000−10,000		PF		
High-rate aeration	4−15	0.4−1.5	1.6−16	0.5−2.0	4,000−10,000	1.0−5.0	CM	75−90	25−45
Pure oxygen	8−20	0.2−1.0	1.6−4	1−3	6,000−8,000	0.25−0.5	CM	85−95	
Extended aeration	20−30	0.05−0.15	0.16−0.40	18−24	3,000−6,000	0.75−1.50	CM	75−90	90−125

*PF = plug flow, CM = completely mixed.

Solids

$$\text{TSS} = \frac{\left(1{,}000\frac{\text{mg}}{\text{g}}\right)\left(1{,}000\frac{\text{mL}}{\text{L}}\right)}{\text{SVI}}\left(\frac{\text{mL}}{\text{g}}\right)$$

where

TSS = total suspended solids (mg/L)

SVI = sludge volume index (mL/g)

Solids Residence/Retention Time (Refer to Activated Sludge Process Schematics)

$$\theta_c = \frac{VX}{Q_w X_w + Q_e X_e} = \frac{VX}{(Q - Q_w)X_e + Q_w X_w}$$

where

θ_c = solids residence time, cell residence time, sludge age (days)

V = volume of reactor (Mgal)

Q = wastewater flow rate (MGD)

X = reactor suspended solids concentration (ppm or mg/L)

X_w = waste suspended solids concentration (ppm or mg/L)

Q_w = waste sludge flow rate (MGD)

X_e = effluent suspended solids concentration (ppm or mg/L)

Q_e = effluent sludge flow rate (MGD)

Rate of Biomass Growth

$$r_g = -Yr_{su} - k_d X = Y\frac{kXS}{K_s + S} - k_d X$$

where

r_g = biomass growth/production rate (mg/L•day)

Y = synthesis yield coefficient, biomass produced/BOD consumed (dimensionless)

r_{su} = substrate utilization rate (mg/L)

X = suspended solids concentration in unit (ppm or mg/L)

K_s = half-velocity constant (mg/L)

k_d = endogenous decay coefficient (day^{-1})

k = maximum rate of substrate utilization (day^{-1})

K_s = half-velocity constant (mg/L)

Sludge Production

$$R = Q_R / Q$$

where

R = recycle ratio (dimensionless)

Q = influent flow rate (MGD)

Q_R = returned suspended solids flow rate (MGD)

$$V_s = \frac{M}{P_s S_s g_w}$$

where

V_s = sludge volume produced (ft^3)

M = sludge production rate, dry weight basis (lbf)

g_w = specific weight of water (lbf/ft^3)

S_s = specific gravity of sludge slurry (dimensionless)

P_s = percent solids expressed as decimal (dimensionless)

Sludge Volume Index

$$\text{SVI} = \frac{SV \times 1,000}{MLSS}$$

where

SVI = sludge volume index (mL/g)

SV = volume occupied by 1,000 mL of MLSS after 30 min of settling (mL/L)

$MLSS$ = mixed liquor suspended solids (mg/L)

unit conversion = 1,000 (mg/g)

Sludge Density Index

$$SDI = \frac{100}{SVI}$$

where SDI = Sludge density index (g/mL)

Substrate Utilization Rate

$$r_{su} = \frac{kSX}{K_s + S}$$

where r_{su} = substrate utilization rate (mg/L)

Specific Substrate Utilization Rate

$$U = \frac{r_{su}}{X} = \frac{Q(S_0 - S_e)}{VX} = \frac{S_0 - S_e}{\tau X}$$

$$U = \frac{(F/M)E}{100}$$

where

U = specific substrate utilization rate (day^{-1})

r_{su} = substrate utilization rate (mg/L)

X = volatile suspended solids concentration in unit (ppm or mg/L)

Q = wastewater flow rate (Mgal/day or MGD)

S_0 = influent soluble BOD or COD concentration (ppm or mg/L)

S_e = effluent soluble BOD or COD concentration (ppm or mg/L)

V = volume of unit (Mgal)

τ = hydraulic detention time (days) = $\frac{V}{Q}$

Secondary/Final Clarifiers

Secondary Clarifier Flow:

$$Q = Q_0 + Q_R$$

where

Q_0 = influent flow rate (gal/day)

Q_R = return activated sludge (RAS) rate to clarifier (gal/day)

The surface overflow rate for activated sludge system final clarifiers is based only on influent wastewater flow (Q_0) and does not include return activated sludge flow (Q_R).

The solids loading rate for activated sludge system final clarifiers is based on the influent wastewater flow (Q_0) plus the return activated sludge flow (Q_R). Solids loading rate criteria typically assumes the return/recycle sludge flow rate (Q_R) is 100% of the average design flow at the design mixed liquor suspended solids (MLSS) concentration.

Weir Loadings

$$WOR = Q_H/L$$

where

WOR = weir loading/overflow rate (gal/day-ft)

Q_H = peak hourly flow (gal/day)

L = weir length (ft)

Flow ≤ 1 MGD: weir overflow rates should not exceed 10,000 gpd/ft

Flow > 1 MGD: weir overflow rates should not exceed 15,000 gpd/ft

Solids Loading Rate

$$SLR = \frac{8.34\,Q\,X}{A}$$

where

SLR = solids loading rate (lb/day-ft^2)

Q = volumetric flow rate (Mgal/day or MGD)

X = suspended solids concentration (ppm or mg/L)

unit conversion = 8.34 (lb/Mgal•mg/L)

A = surface area, plan view (ft^2)

Settling

Refer to Drinking Water Distribution and Treatment – Settling and Sedimentation.

$$v_h = Q/A_x$$

where

v_h = horizontal velocity = approach velocity (ft/sec)

Q = flow rate (ft^3/sec)

A_x = cross-sectional area (ft^2)

Design Criteria for Secondary Clarifiers

Process Location	Overflow Rate				Solids Loading Rate				Hydraulic Residence Time (hr)	Depth (ft)
	Average		Peak		Average		Peak			
	(gpd/ft^2)	(m^3/m^2•day)	(gpd/ft^2)	(m^3/m^2•day)	(lb/ft^2-day)	(kg/m^2•hr)	(lb/ft^2-hr)	(kg/m^2•hr)		
Clarifier following fixed film reactors	400–800	16–33							2	
Clarifier following air-activated sludge reactors										
All configurations EXCEPT extended aeration	400–700	16–28	1,000–1,200	40–64	19–29	4–6	38	8	2	12–15
Extended aeration	200–400	8–16	600–800	24–32	5–24	1–5	34	7	2	12–15
Clarifier following chemical flocculation reactors	800–1,200								2	

Return Activated Sludge

Returned Solids:

$$Q_R = \frac{QX}{X_R - X} = \frac{SV \times Q}{1{,}000 - SV}$$

where

Q_R = return activated sludge wastewater flow rate (MGD)

Q = wastewater flow rate (MGD)

X = suspended solids concentration in unit (ppm or mg/L)

X_R = returned suspended solids concentration (ppm or mg/L)

SV = wet settled sludge (mL/L)

$$R = \frac{1 - \left(\frac{\tau}{\theta_c}\right)}{\left(\frac{X_R}{X}\right) - 1}$$

$$R = \frac{Q_R}{Q} = \frac{X}{X_R - X}$$

where R = recycling ratio (dimensionless)

Steady State Mass Balance Around Secondary Clarifier

$$(Q_o + Q_R)X = Q_e X_e + Q_R X_R + Q_w X_w$$

Methanol Addition

$$C_m = 2.47(NO_3\text{-}N) + 1.53(NO_2\text{-}N) + 0.87(DO)$$

where

C_m = methanol required (mg/L)

NO_3-N = nitrate nitrogen removed (mg/L)

NO_2-N = nitrite nitrogen removed (mg/L)

DO = dissolved oxygen removed (mg/L)

Biomass produced:

$$C_b = 0.53(NO_3\text{-}N) + 0.32(NO_2\text{-}N) + 0.19(DO)$$

where C_b = biomass production (mg/L)

6.8.5.4 Dissolved Air Flotation

$$\frac{A}{S} = \frac{1.3 s_a \left(f P - 1 \right)}{S_a}$$

where

A/S = air to solids ratio [mL (air)/mg (solids)]

s_a = air solubility (mL/L)

f = fraction of air dissolved at pressure P, usually 0.5

P = pressure (atm)

$P = \dfrac{p + 101.35}{101.35}$ (SI units)

$P = \dfrac{p + 14.7}{14.7}$ (U.S. customary units)

p = gauge pressure (kPa or lbf/ft^2 gauge)

S_a = influent suspended solids (g/m^3 or mg/L)

Temp. (°C)	s_a (mL/L)
0	29.2
10	22.8
20	18.7
30	15.7

6.8.5.5 Facultative Ponds

Typical Design Parameters

BOD Loading Total System ≤ 35 pounds BOD_5/(acre-day)

Minimum = 3 ponds

Depth = 3−8 ft

Minimum t = 90−120 days

BOD_5 for Mixed Lagoons in Series

$$\frac{S}{S_0} = \frac{1}{1 + k_p \theta}$$

where

S_0 = inlet total BOD_5

S = outlet total BOD_5

θ = fresh-feed residence time

k_p = kinetic constant ($time^{-1}$)

6.8.5.6 Biotowers/Trickling Filters

Fixed-Film Equation Without Recycle

$$\frac{S_e}{S_0} = e^{-kD/q^n}$$

Organic loading rate per surface area (lb/day-ft^2):

$$\frac{Q_0 S_0 (8.34)}{A_M}$$

where

Q_0 = influent flow rate (Mgal/day or MGD)

S_0 = influent BOD or COD concentration (ppm or mg/L)

S_e = effluent BOD or COD concentration (ppm or mg/L)

A_M = surface area of media fixed-film reactor (ft^2)

unit conversion = 8.34 (lb/Mgal•mg/L)

Fixed-Film Equation with Recycle

$$\frac{S_e}{S_a} = \frac{e^{-kD/q^n}}{(1+R) - R\left(e^{-kD/q^n}\right)}$$

where

S_o = influent BOD or COD concentration (ppm or mg/L)

S_e = effluent BOD or COD concentration (ppm or mg/L)

R = recycle ratio = Q_R/Q_0

Q_R = recycle flow rate

S_a = $\dfrac{S_o + RS_e}{1+R}$

D = depth of biotower media (m)

q = hydraulic loading [m^3/(m^2•min)]

q = $(Q_0 + RQ_0)/\mathrm{A}_{\mathrm{plan}}$ (with recycle)

k = treatability constant; functions of wastewater and medium (min^{-1}); range 0.01–0.1; for municipal wastewater and modular plastic media 0.06 min^{-1} at 20°C

k_T = $k_{20}(1.035)^{T-20}$

n = coefficient relating to media characteristics; modular plastic, n = 0.5

National Research Council (NRC) Trickling Filter Performance

For a single-stage or first-stage rock filter, the equation is

$$E_1 = \frac{100}{1 + 0.0561\sqrt{\frac{W}{VF}}}$$

where

E_1 = efficiency of BOD removal for process at 20°C, including recirculation and sedimentation (%)

W = BOD loading to filter (lb/day)

V = volume of filter media (thousands of ft^3)

F = recirculation factor

The recirculation factor is calculated using

$$F = \frac{1 + \frac{R}{I}}{\left(1 + 0.1\frac{R}{I}\right)^2}$$

where

R = recirculated flow

I = raw inflow

6.8.5.7 Blowers

Power Requirements

$$P_W = \frac{WRT_1}{Cne}\left[\left(\frac{P_2}{P_1}\right)^{0.283} - 1\right]$$

where

P_W = power requirement (hp)

C = 550 ft-lbf/(sec-hp)

W = weight of flow of air (lbf/sec)

R = engineering gas constant for air = 53.3 ft-lbf/(lbf air-°R)

T_1 = absolute inlet temperature (°R)

P_1 = absolute inlet pressure (lbf/in^2)

P_2 = absolute outlet pressure (lbf/in^2)

n = $(k – 1)/k$ = 0.283 for air

e = efficiency (usually $0.70 < e < 0.90$)

Source: Metcalf and Eddy. *Wastewater Engineering: Treatment, Disposal, and Reuse*. 3rd ed. New York: McGraw-Hill, 1991, p. 565.

6.8.6 Nitrification/Denitrification

6.8.6.1 Overall Nitrification Reaction

$$NH_4^+ + 1.863\ O_2 + 0.098\ CO_2 \rightarrow 0.0196\ C_5H_7NO_2 + 0.98\ NO_3^- + 0.0941\ H_2O + 1.98\ H^+$$

6.8.6.2 Biological Denitrification Reaction

$$NO_3^- \rightarrow NO_2^- \rightarrow NO \rightarrow N_2O \rightarrow N_2$$

Wastewater:

$$C_{10}H_{19}O_3N + 10\ NO_3^- \rightarrow 5\ N_2 + 10\ CO_2 + 3\ H_2O + NH_3 + 10\ OH^-$$

Methanol:

$$5\ CH_3OH + 6\ NO_3^- \rightarrow 3\ N_2 + 5\ CO_2 + 7\ H_2O + 6\ OH^-$$

Acetate:

$$5\ CH_3COOH + 8\ NO_3^- \rightarrow 4\ N_2 + 10\ CO_2 + 6\ H_2O + 8\ OH^-$$

6.8.6.3 Rate of Denitrification

$$U'_{DN} = U_{DN} \times 1.09^{(T-20)}(1 - DO)$$

where

U'_{DN} = overall denitrification rate

U_{DN} = specific denitrification rate (lb NO_3-N/lb MLVSS day)

T = wastewater temperature (°C)

DO = dissolved oxygen in the wastewater (mg/L)

6.8.7 Phosphorus Removal

6.8.7.1 Biological Phosphorus Removal (BPR) Process

Type of BPR Process	BOD/P Ratio (g BOD/g P)	COD/P Ratio (g COD/g P)	SRT (day)
Phoredox, VIP	15–20	26–34	>8.0
A^2O, UCT	20–25	34–43	7–15
Bardenpho	>25	>43	15–25

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6.8.7.2 Chemical Phosphorus Removal

Phosphorus removal equations:

1. Ferric chloride
   $FeCl_3 + PO_4^{3-} \rightarrow FePO_4 (\downarrow) + 3\ Cl^-$
2. Ferrous chloride
   $3\ FeCl_2 + 2\ PO_4^{3-} \rightarrow Fe_3(PO_4)_2 (\downarrow) + 6\ Cl^-$
3. Aluminum sulfate (alum)
   $Al_2 (SO_4)_3 \bullet 14\ H_2O + 2\ PO_4^{3-} \rightarrow 2\ AlPO_4 (\downarrow) + 3\ SO_4^{2-} + 14\ H_2O$

6.8.7.3 Chemical Phosphorous Precipitation

Phosphate precipitation with aluminum:

$$Al^3 + H_n PO_4^{3-n} \leftrightarrow AlPO_4 + nH^+$$

Phosphate precipitation with iron:

$$Fe^{3+} + H_n PO_4^{3-n} \leftrightarrow FePO_4 + nH^+$$

Phosphate precipitation with calcium:

$$10Ca^{2+} + 6PO_4^{3-} + 2OH^- \leftrightarrow Ca_{10}\left(PO_4\right)_6 (OH)_2$$

6.8.8 Solids Treatment, Handling, and Disposal

The specific gravity of all the solid matter can be computed using:

$$\frac{W_s}{S_s \rho_w} = \frac{W_f}{S_f \rho_w} + \frac{W_v}{S_v \rho_w}$$

where

W_s = weight of solids (lbf)

S_s = specific gravity of solids

ρ_w = density of water (lbf-sec^2/ft^4)

W_f = weight of fixed solids, mineral matter (lbf)

S_f = specific gravity of fixed solids

W_v = weight of volatile solids (lbf)

S_v = specific gravity of volatile solids

Treatment Processes Typical Solids Properties

Treatment Operation or Process	Specific Gravity of Solids	Specific Gravity of Sludge	Dry Solids ($lbf/10^3$ gal)		Dry Solids ($kg/10^3$ m^3)	
			Range	Typical	Range	Typical
Primary sedimentation	1.4	1.02	0.9–1.4	1.25	110–170	150
Activated sludge (waste biosolids)	1.25	1.005	0.6–0.8	0.7	70–100	80
Trickling filter (waste biosolids)	1.45	1.025	0.5–0.8	0.6	60–100	70
Extended aeration (waste biosolids)	1.30	1.015	0.7–1.0	0.8[a]	80–120	100[a]
Aerated lagoon (waste biosolids)	1.30	1.01	0.7–1.0	0.8[a]	80–120	100[a]
Filtration	1.20	1.005	0.1–0.2	0.15	12–24	20
Algae removal	1.20	1.005	0.1–0.2	0.15	12–24	20
Chemical addition to primary tanks for phosphorus removal						
Low lime (350–500 mg/L)	1.9	1.04	2.0–3.3	2.5[b]	240–400	300[b]
High lime (800–1,600 mg/L)	2.2	1.05	5.0–11.0	6.6[b]	600–1,300	800[b]
Suspended growth nitrification	–	–	–	–	–	–[c]
Suspended growth denitrification	1.20	1.005	0.1–0.25	0.15	12–30	18
Roughing filters	1.28	1.02	–	–[d]	–	–[d]

[a] Assuming no primary treatment
[b] Solids in addition to that normally removed by primary sedimentation
[c] Negligible
[d] Included in biosolids production from secondary treatment process

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6.8.9 Digestion

6.8.9.1 Aerobic Design

Typical Design Criteria for Aerobic Digesters

Design criteria for aerobic digesters

Parameter	Value
Sludge retention time (day)	
At 20°C	40
At 15°C	60
Solids loading (lb volatile solids/ft^3-day)	0.1–0.3
Oxygen requirements (lb O_2/lb solids destroyed)	
Cell tissue	~2.3
BOD_5 in primary sludge	1.6–1.9
Energy requirements for mixing	
Mechanical aerators (hp/10^3 ft^3)	0.7–1.50
Diffused-air mixing (ft^3/10^3 ft^3-min)	20–40
Dissolved-oxygen residual in liquid (mg/L)	1–2
Reduction in volatile suspended solids (VSS) (%)	40–50

Source: Republished with permission of McGraw-Hill, from *Wastewater Engineering: Treatment, Disposal, and Reuse*, Tchobanoglous, George, Metcalf & Eddy, 1991, Table 12-24, p. 837; permission conveyed through Copyright Clearance Center, Inc.

Tank Volume

$$V = \frac{Q_i\left(X_i + FS_i\right)}{X_d\left(k_d P_v + \frac{1}{\theta_c}\right)}$$

where

V = volume of aerobic digester (ft^3)

Q_i = influent average flow rate to digester (ft^3/day)

X_i = influent suspended solids (mg/L)

F = fraction of the influent BOD_5 consisting of raw primary sludge (expressed as a decimal)

S_i = influent BOD_5 (mg/L)

X_d = digester suspended solids (mg/L); typically $X_d = 0.7X_i$

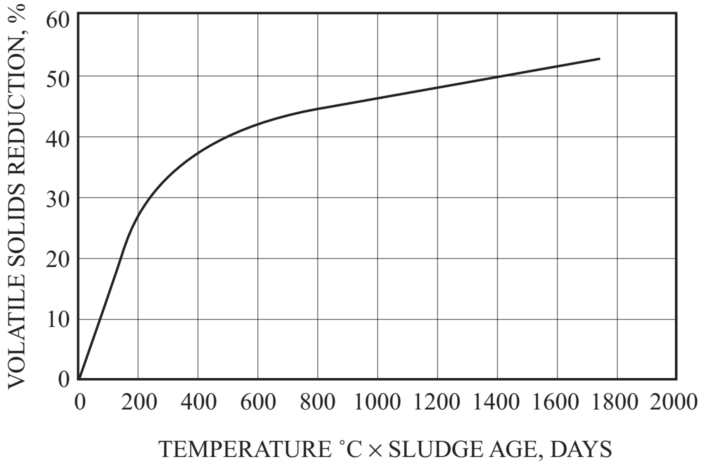
k_d = reaction-rate constant (day^{-1})

P_v = volatile fraction of digester suspended solids (expressed as a decimal)

θ_c = solids residence time (sludge age) (day)

FS_i can be neglected if primary sludge is not included on the sludge flow to the digester.

Sludge Reduction, Temperature and Age



VOLATILE SOLIDS REDUCTION IN AN AEROBIC DIGESTER AS A FUNCTION OF DIGESTER LIQUID TEMPERATURE AND DIGESTER SLUDGE AGE

Source: Republished with permission of McGraw-Hill, from *Wastewater Engineering: Treatment and Reuse*, Tchobanoglous, George, Burton, Franklin L., Stensel, H. David., Metcalf & Eddy, 2003, fig. 14-31, pp. 1537-1538; permission conveyed through Copyright Clearance Center, Inc.

6.8.9.2 Anaerobic Digestion

Design parameters for anaerobic digesters

Parameter	Standard-rate	High-rate
Solids residence time (day)	30–90	10–20
Volatile solids loading (kg/m^3/day)	0.5–1.6	1.6–6.4
Digested solids concentration (%)	4–6	4–6
Volatile solids reduction (%)	35–50	45–55
Gas production (m^3/kg VSS added)	0.5–0.55	0.6–0.65
Methane content (%)	65	65

Source: Republished with permission of McGraw-Hill, from *Environmental Engineering*, Tchobanoglous, George, Rowe, Donald R., Peavy, Howard S., 1984, Table 5-13, p. 288; permission conveyed through Copyright Clearance Center, Inc.

Standard Rate

$$\text{Reactor Volume} = \frac{V_1 + V_2}{2} t_r + V_2 t_s$$

High Rate

First stage

$$\text{Reactor Volume} = V_1 t_r$$

Second stage

$$\text{Reactor Volume} = \frac{V_1 + V_2}{2} t_t + V_2 t_s$$

where

V_1 = raw sludge input (volume/day)

V_2 = digested sludge accumulation (volume/day)

t_r = time to react in a high-rate digester = time to react and thicken in a standard-rate digester

t_t = time to thicken in a high-rate digester

t_s = storage time

Methane (CH_4) Production

$$V_{CH_4} = 0.35 \frac{L_{CH_4}}{g_{BOD}} \left[\left(S_0 - S \right) Q - 1.42 \frac{g_{BOD}}{g_{biomass}} P_x \right]$$

where

V_{CH4} = volume of methane produced (volume/day)

S_0 = influent BOD (mass/volume)

S = effluent BOD (mass/volume)

Q = volumetric flow rate of sludge (volume/time)

P_x = net mass of cell tissue produced (mass/day)

6.8.10 Disinfection

6.8.10.1 Ultraviolet Light

Refer to Drinking Water Distribution and Treatment – Ultraviolet Light (UV).

6.8.10.2 Chlorination

$NH_3 + HOCl \rightarrow H_2O + NH_2Cl$ (Monochloramine)

$NH_3 + 2HOCl \rightarrow 2H_2O + NHCl_2$ (Dichloramine)

$NH_3 + 3HOCl \rightarrow 3H_2O + NCl_3$ (Nitrogen Trichloride)

6.8.10.3 Dechlorination

Sodium sulfite:

$$Na_2SO_3 + NH_2Cl + H_2O \rightarrow Na_2SO_4 + Cl^- + NH_4^+$$
$$Na_2SO_3 + Cl_2 + H_2O \rightarrow Na_2SO_4 + 2HCl$$

Sodium bisulfite:

$$NaHSO_3 + Cl_2 + H_2O \rightarrow NaHSO_4 + 2HCl$$
$$NaHSO_3 + NH_2Cl + H_2O \rightarrow NaHSO_4 + Cl^- + NH_4^+$$

Sodium metabisulfite:

$$Na_2S_2O_5 + Cl_2 + 3H_2O \rightarrow 2NaHSO_4 + 4HCl$$
$$Na_2S_2O_5 + 2NH_2Cl + 3H_2O \rightarrow Na_2SO_4 + H_2SO_4 + 2Cl^- + 2NH_4^+$$

Sulfur dioxide:

$$SO_2 + HOCl + H_2O \rightarrow Cl^- + SO_4^{2-} + 3H^+$$

6.8.11 Advanced Treatment

6.8.11.1 Cascade Aeration

$$H = \frac{R - 1}{0.11ab(1 + 0.046T)}$$

where

H = height through which water falls (ft)

R = deficit ratio = $\dfrac{C_s - C_0}{C_s - C}$

where

C_s = wastewater dissolved oxygen saturation concentration at temperature T (mg/L)

C_0 = post-aeration influent dissolved oxygen concentration (mg/L)

C = required final dissolved oxygen concentration after cascade

T = water temperature (°C)

a = water quality parameter = 0.8 for effluent wastewater

b = weir geometry parameter

= 1.0 for weirs

= 1.1 for steps

= 1.2 for step weirs

6.9 Drinking Water Distribution and Treatment

6.9.1 Drinking Water Distribution Systems

Fire System Demands

National Board of Fire Underwriters:

$$Q = 1{,}020\sqrt{P}\left(1 - 0.01\sqrt{P}\right)$$

where

Q = needed fire flow (gal/min)

P = population served (thousands)

Insurance Services Office (ISO):

To estimate the amount of water needed to fight a fire in an individual, nonsprinklered building, ISO uses the formula:

$$\text{NFF}_i = (C_i)(O_i)[(1.0 + (X + P)_i]$$

where

NFF_i = needed fire flow (gpm)

C_i = a factor related to the type of construction

$C_i = 18F\,(A_i)^{0.5}$

F = coefficient related to the class of construction

F = 1.5 for Construction Class 1 (wood frame construction)

= 1.0 for Construction Class 2 (joisted masonry construction)

= 0.8 for Construction Class 3 (noncombustible construction) and Construction Class 4 (masonry noncombustible construction)

= 0.6 for Construction Class 5 (modified fire-resistive construction) and Construction Class 6 (fire-resistive construction)

A_i = effective area

O_i = a factor related to the type of occupancy

X = a factor related to the exposure buildings

P = a factor related to the communication between buildings

6.9.2 Drinking Water Treatment Process

6.9.2.1 Coagulation

Insoluble products are shown in italics

1. Aluminum sulfate in natural alkaline water

$$\mathrm{Al_2(SO_4)_3 + 3Ca(HCO_3)_2 \leftrightarrow} 2Al(OH)_3 \mathrm{+ 3CaSO_4 + 6CO_2}$$

2. Aluminum sulfate plus soda ash

$$\mathrm{Al_2(SO_4)_3 + 3NaCO_3 + 3H_2O \leftrightarrow} 2Al(OH)_3 \mathrm{+ 3NaSO_4 + 3CO_2}$$

3. Ferric sulfate

$$\mathrm{Fe_2(SO_4)_3 + 3Ca(HCO_3)_2 \leftrightarrow} 2Fe(OH)_3 \mathrm{+ 3CaSO_4 + 6CO_2}$$

4. Ferric chloride

$$\mathrm{2FeCl_3 + 3Ca(HCO_3)_2 \leftrightarrow} 2Fe(OH)_3 \mathrm{+ 3CaCl_2 + 6CO_2}$$

6.9.2.2 Mixing

Rapid Mix and Flocculator Design

$$\mathrm{G} = \sqrt{\frac{P}{\mu V}} = \sqrt{\frac{\gamma H_L}{t\mu}}$$

$$Gt = 10^4 \text{ to } 10^5$$

where

G = velocity gradient (mixing intensity) [ft/(sec-ft) or m/(s•m)]

P = power to the fluid (ft-lbf/sec)

V = volume (ft^3 or m^3)

μ = dynamic viscosity (lbf-sec/ft^2 or Pa•s)

γ = specific weight of water (lbf/ft^3 or N/m^3)

H_L = head loss (ft or m)

t = time (sec or s)

Reel and Paddle

$$P = \frac{C_D A_P \rho_f v_r^3}{2}$$

where

P = power dissipated (ft-lbf/sec)

C_D = drag coefficient = 1.8 for a flat blade with a L:W > 20:1

A_P = area of blade perpendicular to the direction of travel through the water (ft^2)

ρ_f = density of water ($lbf\text{-}sec^2/ft^4$)

v_r = relative or effective paddle velocity (ft/sec)

v_r = v_p × slip coefficient

v_p = velocity of paddle (ft/sec)

slip coefficient = 0.5 to 0.75

$$F = 0.5 C_D \rho_f A v_p^2$$

where

F = drag force (lbf)

v_p = $2\pi r N$

where

r = distance from shaft to center of paddle (ft or m)

N = rotational speed (rev/sec)

Turbulent Flow Impeller Mixer Power

$$P = K_T (n)^3 (D_i)^5 \rho_f$$

where

K_T = impeller constant (see table below)

n = rotational speed (rev/sec)

D_i = impeller diameter (m)

ρ_f = density of H_2O (kg/m^3)

Values of the Impeller Constant (Assuming Turbulent Flow)

Values of the Impeller Constant K_T
(Assume Turbulent Flow)

Type of Impeller	K_T
Propeller, pitch of 1, 3 blades	0.32
Propeller, pitch of 2, 3 blades	1.00
Turbine, 6 flat blades, vaned disc	6.30
Turbine, 6 curved blades	4.80
Fan turbine, 6 blades at 45°	1.65
Shrouded turbine, 6 curved blades	1.08
Shrouded turbine, with stator, no baffles	1.12

Note: Constant assumes baffled tanks having four baffles at the tank wall with a width equal to 10% of the tank diameter.

Source: Reprinted with permission from Industrial & Engineering Chemistry, "Mixing of Liquids in Chemical Processing" by J. Henry Rushton, v. 44, no. 12, p. 2934. Copyright 1952 American Chemical Society.

6.9.2.3 Flocculation

$$G = \sqrt{\frac{Q\gamma H}{\mu V}} = \sqrt{\frac{62.4H}{\mu t}} = \sqrt{\frac{P}{\mu V}}$$

where

G = mean velocity gradient (sec^{-1})

Q = flow rate (ft^3/sec)

γ = specific weight of water (lbf/ft^3)

H = head loss due to friction (ft)

μ = dynamic/absolute viscosity (lbf-sec/ft^2)

V = volume of flocculator (ft^3)

t = detention time (sec)

P = input power (ft-lbf/sec)

Typical Diffuser Wall Guidelines for Flocculation Basins

Parameter	Unit	Guideline
Opening area	Percent of flow cross section	2–5
Velocity through orifice		
Dividing first and second floc basins	m/s	0.55
Dividing floc and sedimentation basins	m/s	0.35
Head loss across baffle		
Dividing first and second floc basins	mm	7–9
Dividing floc and sedimentation basins	mm	3–4
Submergence of highest port	mm	15
Clearance below baffle for sludge	mm	25

Source: Source: Adapted in part from Kawamura, Susumu. *Integrated Design and Operation of Water Treatment Facilities*. 2nd ed. New York: John Wiley and Sons, 2000. As found in Crittenden, et. al. MWH *Water Treatment Principles and Design*. 3rd ed. New York: John Wiley and Sons, 2012, Fig. 9-16, p. 633.

6.9.3 Activated Carbon Adsorption

6.9.3.1 Freundlich Isotherm

$$q_e = K_f C_e^{1/n} = \frac{\left(C_0 - C_e\right)V}{w}$$

where

q_e = equilibrium loading on the activated carbon (mg chemical/g activated carbon)

K_f = Freundlich adsorption capacity at unit concentration (mg/g)(L/mg)$^{1/n}$

n = Freundlich strength of adsorption (dimensionless)

C_0 = initial adsorbate concentration in water (mg chemical/L)

C_e = equilibrium concentration in water after adsorption has occurred (mg chemical/L)

V = volume of liquid in reactor (L)

w = mass of dry activated carbon used (g)

Linearized Form:

$$\log q_e = \log \frac{w}{V} = \log K_f + \frac{1}{n} \log C_e$$

$$q_e = \frac{q_{\max} b C_e}{1 + b C_e}$$

where

$q_{\max}$ = ultimate adsorption capacity (mg chemical/g activated carbon)

b = relative energy of adsorption (L/mg)

6.9.3.2 Langmuir Isotherm

$$q_e = \frac{q_m K_L C_e}{1 + K_L C_e} = \frac{w}{V}$$

where

q_e = equilibrium loading on the activated carbon (mg chemical/g activated carbon)

K_L = Langmuir equilibrium constant (L/g)

q_m = theoretical monolayer saturation capacity (mg chemical/g activated carbon)

C_0 = initial adsorbate concentration in water (mg chemical/L)

C_e = equilibrium concentration in water after adsorption has occurred (mg chemical/L)

V = volume of liquid in reactor (L)

w = mass of dry activated carbon used (g)

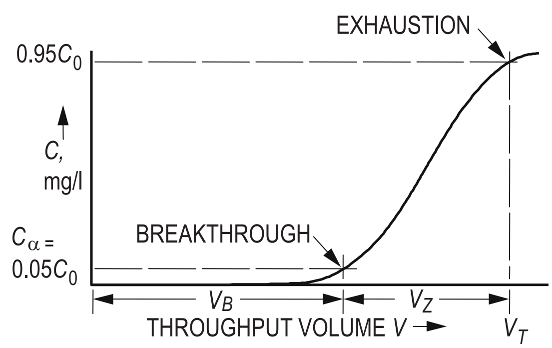
R_L = separation factor or equilibrium factor (dimensionless)

$R_L > 1$	(unfavorable)
$R_L = 1$	(linear)
$0 < R_L < 1$	(favorable)
$R_L = 0$	(irreversible)

Linearized Form:

$$\frac{1}{q_e} = \frac{1}{q_m K_L C_e} + \frac{1}{q_m}$$

$$R_L = \frac{1}{1 + K_L C_0}$$



6.9.3.3 Depth of Sorption Zone

$$Z_s = Z\left(\frac{V_Z}{V_T - 0.5V_Z}\right)$$

where

$V_Z = V_T - V_B$

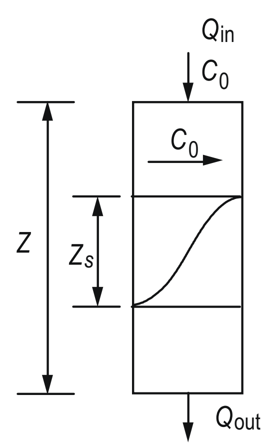
Z_s = depth of sorption zone

Z = total carbon depth

V_T = total volume treated at exhaustion ($C = 0.95\ C_0$)

V_B = total volume at breakthrough ($C = C_\alpha = 0.05\ C_0$)

C_0 = concentration of contaminant in influent



6.9.4 Air Stripping

6.9.4.1 Mass Balance

$$QC_b(z) + Q_a y_0 = QC_e + Q_a y_b(z)$$

where

Q = water flow rate (ft^3/sec)

$C_b(z)$ = bulk liquid-phase concentration at axial position along tower (mg/L)

Q_a = air flow rate (ft^3/sec)

y_0 = air-phase concentration entering tower (mg/L)

C_e = effluent liquid-phase concentration (mg/L)

$y_b(z)$ = bulk air-phase concentration at axial position z along tower (mg/L)

6.9.4.2 Henry's Law

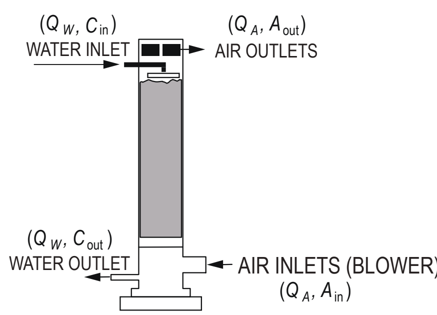
$$P_i = HC_i$$

where

P_i = partial pressure component i (varies)

H = Henry's Law constant (varies)

C_i = concentration of component i in solvent (varies)



$$A_{out} = H'C_{in}$$

$$Q_W \bullet C_{in} = Q_A H'C_{in}$$

$$Q_W = Q_A H'$$

$$H'(Q_A / Q_W) = 1$$

where

A_{out} = concentration in the effluent air ($kmol/m^3$); in this formulation of the equation A_{in} and C_{out} are assumed to be negligible, for simplicity

Q_W = water flow rate (m^3/s)

Q_A = air flow rate (m^3/s)

A_{in} = concentration of contaminant in air ($kmol/m^3$)

C_{out} = concentration of contaminants in effluent water (kmol/m^3)

C_{in} = concentration of contaminants in influent water (kmol/m^3)

6.9.4.3 Air Stripper Packing Height

$$Z = HTU \times NTU$$

where Z = stripper packing height (ft)

Assuming rapid equilibrium:

$$NTU = \left(\frac{R_s}{R_s - 1}\right) \ln\left[\frac{(C_{in}/C_{out})(R_s - 1) + 1}{R_s}\right]$$

where

NTU = number of transfer units

H = Henry's Law constant (varies)

H' = H/RT = dimensionless Henry's Law constant

T = temperature (K)

R = universal gas constant (atm•m^3/kmol•K)

R_s = stripping factor

C_{in} = concentration in the influent water (kmol/m^3)

C_{out} = concentration in the effluent water (kmol/m^3)

$$HTU = \frac{L}{K_L a\, C}$$

where

HTU = height of transfer units (ft)

L = molar flux rate of contaminant in liquid phase (kmol/m^2•s)

C = molar density of water (55.6 kmol/m^3)

$K_L a$ = overall transfer rate constant (s^{-1})

6.9.4.4 Minimum Air to Water Ratio

$$\left(\frac{Q_a}{Q}\right)_{min} = \frac{C_0 - C_e}{HC_0}$$

where

$\left(\frac{Q_a}{Q}\right)_{min}$ = minimum air-to-water ratio (dimensionless)

C_0 = influent liquid-phase concentration (mg/L)

C_e = treatment objective (mg/L)

6.9.4.5 Stripping Factor

When

$$C_e << C_0 \text{ and } \left(\frac{Q_a}{Q}\right)_{\min} = \frac{C_0 - C_e}{H'C_0} \approx \frac{1}{H'}$$

the stripping factor S is then defined as:

$$S = \frac{(Q_a/Q)}{(Q_a/Q)_{\min}} = \frac{(Q_a/Q)}{(1/H')} = \left(\frac{Q_a}{Q}\right)H'$$

where

S = stripping factor (dimensionless)

Q_a/Q = air-to-water ratio

Q_a = air flow rate (varies)

Q = water flow rate (varies)

H' = dimensionless Henry's Law constant

Gas Phase Equilibrium

$$y = H'C$$

where

y = gas-phase concentration (mg/L)

C = liquid-phase concentration in equilibrium with gas-phase concentration y (mg/L)

H' = dimensionless Henry's Law constant

6.9.5 Hardness and Softening

6.9.5.1 Lime-Soda Softening Equations

1. Carbon dioxide removal

   $$CO_2 + Ca(OH)_2 \rightarrow CaCO_3(s) + H_2O$$

2. Calcium carbonate hardness removal

   $$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3(s) + 2H_2O$$

3. Calcium noncarbonate hardness removal

   $$CaSO_4 + Na_2CO_3 \rightarrow CaCO_3(s) + 2Na+ + SO_4^{-2}$$

4. Magnesium carbonate hardness removal

   $$Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3(s) + Mg(OH)_2(s) + 2H_2O$$

5. Magnesium noncarbonate hardness removal:

   $$MgSO_4 + Ca(OH)_2 + Na_2CO_3 \rightarrow CaCO_3(s) + Mg(OH)_2(s) + 2Na^+ + SO_4^{2-}$$

6. Destruction of excess alkalinity

   $$2HCO_3^- + Ca(OH)_2 \rightarrow CaCO_3(s) + CO_3^{2-} + 2H_2O$$

7. Recarbonation

   $$Ca^{2+} + 2OH^- + CO_2 \rightarrow CaCO_3(s) + H_2O$$

6.9.5.2 Common Water Softening Compounds and Molecular Properties

Molecular Formulas	Molecular Weight	*n* # Equiv per mole	Equivalent Weight
CO_3^{2-}	60.0	2	30.0
CO_2	44.0	2	22.0
$Ca(OH)_2$	74.1	2	37.1
$CaCO_3$	100.1	2	50.0
$Ca(HCO_3)_2$	162.1	2	81.1
$CaSO_4$	136.1	2	68.1
Ca^{2+}	40.1	2	20.0
H^+	1.0	1	1.0
HCO_3^-	61.0	1	61.0
$Mg(HCO_3)_2$	146.3	2	73.2
$Mg(OH)_2$	58.3	2	29.2
$MgSO_4$	120.4	2	60.2
Mg^{2+}	24.3	2	12.2
Na^+	23.0	1	23.0
Na_2CO_3	106.0	2	53.0
OH^-	17.0	1	17.0
SO_4^{2-}	96.1	2	48.0

6.9.5.3 Total Softener Hardness

$$TH_s = TH_r - \left(\text{NaOH}_{\text{cal}} - f\right) \times 100/40$$

where

TH_s = total softener hardness (mg/L as $CaCO_3$)

TH_r = raw water total hardness (mg/L as $CaCO_3$)

NaOH_{cal} = calculated average NaOH dose (mg/L)

f = correction factor (20 mg/L NaOH)

molecular weight of NaOH = 40 g/mol

molecular weight of $CaCO_3$= 100 g/mol

6.9.6 Settling and Sedimentation

Also refer to Wastewater Collection and Treatment—Settling.

6.9.6.1 General Spherical Settling

$$v_t = \sqrt{\frac{4g\left(\rho_p - \rho_f\right)d}{3C_d\rho_f}}$$

where

C_d = drag coefficient (dimensionless)

= $24/Re$ (Laminar; $Re \le 1.0$)

= $24/Re + 3/(Re^{1/2}) + 0.34$ (Transitional)

= 0.4 (Turbulent; $Re \ge 10^4$)

Re = Reynolds number = $\frac{v_t \rho d}{\mu}$

g = acceleration due to gravity (32.2 ft/sec^2)

ρ_p = mass density of particle (lbf-sec^2/ft^4)

ρ_f = mass density of fluid (lbf-sec^2/ft^4)

d = diameter of sphere (ft)

μ = bulk viscosity of liquid = absolute viscosity (lbf-sec/ft^2)

v_t = terminal settling velocity (ft/sec)

$$v_t = \sqrt{\frac{2\left(\rho_\rho - \rho_w\right)V_p g}{C_d A \rho_w}}$$

where

v_t = terminal settling velocity (ft/sec)

C_d = drag coefficient (dimensionless)

g = acceleration due to gravity = 32.2 ft/sec^2

d = diameter of particle (ft)

ρ_p = mass density of the particle (lbm/ft^3)

ρ_w = mass density of the water (lbm/ft^3)

A = particle cross-sectional area (ft^2)

V_p = volume of the particle (ft^3)

6.9.6.2 Stokes's Law

Where $N_{Re} < 1$,

$$v_t = \frac{g\left(\rho_p - \rho_w\right)d^2}{18\mu} = \frac{g\rho_w\left(SG - 1\right)d^2}{18\mu}$$

Approach velocity = horizontal velocity = $\frac{Q}{A_x}$

Hydraulic loading rate = $\frac{Q}{A}$

Hydraulic residence time = $\frac{V}{Q} = \theta$

where

Q = flow rate (ft^3/sec)

A_x = cross-sectional area (ft^2)

A = surface area, plan view (ft^2)

V = tank volume (ft^3)

SG = specific gravity

v_t = terminal settling velocity (ft/sec)

g = acceleration due to gravity = 32.2 ft/sec^2

ρ_p = mass density of the particle (lbf-sec^2/ft^4)

ρ_w = mass density of the water (lbf-sec^2/ft^4)

d = particle diameter (ft)

μ = absolute viscosity of the fluid (lbf-sec/ft^2)

6.9.6.3 Type 1 Settling–Discrete Particle Settling

$$v_t \geq v_0$$

where

v_t = terminal settling velocity (ft/sec)

v_0 = overflow rate (ft/sec)

$$v_0 = \frac{Q}{A} = \frac{Q}{WL} = \frac{g(\rho_s - \rho)d^2}{18\mu}$$

where

Q = flow rate (ft^3/sec)

A = surface area of the settling zone (ft^2)

W, L = width and length of the basin (ft)

g = acceleration due to gravity = 32.2 ft/sec^2

ρ_s = mass density of the particle (lbf-sec^2/ft^4)

ρ = mass density of water (lbf-sec^2/ft^4)

μ = absolute viscosity of the fluid (lbf-sec/ft^2)

d = particle diameter (ft)

Basin Depth:

$$H = v_0 t$$

where t = detention time (sec)

Ideal Rectangular Basin:

$$t_0 = \frac{H}{v_t} = \frac{L}{v_H}$$

where

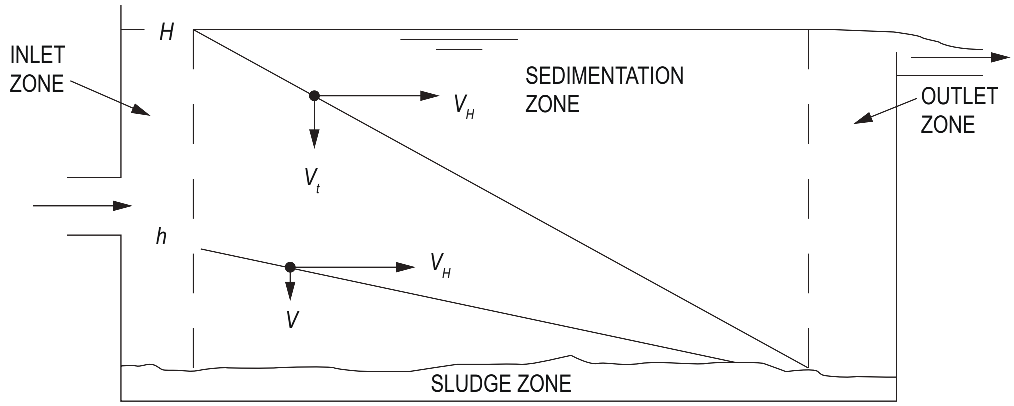
t_0 = minimum detention time required (sec)

H = height/depth of water in basin (ft)

L = length of basin (ft)

v_H = horizontal water velocity (ft/sec)

v_t = particle terminal settling velocity (ft/sec)



Flow-through Velocity:

$$v_t = \frac{Q}{HW}$$

Removal Ratio:

$$r = \frac{h}{H} = \frac{v_t t}{v_0 t} = \frac{v_t}{v_0}$$

6.9.6.4 Type 2 Settling–Flocculent Particle Settling

$$v_c = \frac{H}{t_c}$$

where

v_c = settling velocity (ft/sec)

H = height of settling column (ft)

t_c = time required for a given degree of removal to be achieved (min)

$$R = \sum_{h=1}^{n} \left(\frac{\Delta h_n}{H}\right)\left(\frac{R_n + R_{n+1}}{2}\right)$$

where

R = TSS removal (%) based on plotted percent removal as a number against time and depth

n = number of equal percent removal curve

Δh_n = distance between curves of equal percent removal (ft)

H = total height of settling column (ft)

R_n = equal percent removal curve number n

R_{n+1} = equal percent removal curve number $n + 1$

6.9.6.5 Type 3 Settling–Hindered Settling

Re < 2

$$\frac{v_h}{v} = \left(1 - C_v\right)^{4.65}$$

where

v_h = hindered settling velocity (ft/sec)

v = free settling velocity (ft/sec)

C_v = volume of particles divided by the volume of the suspension

Clarification Area:

$$A = \frac{Q}{v_s}$$

where

A = surface area of the settling zone (ft^2)

Q = overflow rate (ft^3/sec)

v_s = subsidence rate in the zone of hindering settling (ft/sec)

6.9.6.6 Type 4 Settling–Compression Settling

$$\frac{dH}{dt} = i\left(H - H_\infty\right)$$

where

H = sludge height at time t (ft)

i = constant for a given suspension

H_∞ = final sludge height (ft)

6.9.6.7 Typical Sizing and Loadings

Weir Loadings

Water Treatment—weir overflow rates should not exceed 20,000 gpd/ft

Horizontal Velocities

Water Treatment—horizontal velocities should not exceed 0.5 ft/min

Dimensions

1. Rectangular Tanks
   a. Length-to-width ratio = 3:1 to 5:1
   b. Basin width is determined by the scraper width (or multiples of the scraper width)
   c. Bottom slope is set at 1%
2. Circular Tanks
   a. Diameters up to 200 ft
   b. Diameters must match the dimensions of the sludge scraping mechanism
   c. Bottom slope is less than 8%

6.9.6.8 Clarifiers and Sedimentation Basins

Overflow rate = Hydraulic loading rate = $v_0 = Q/A_{surface}$

where

v_0 = critical settling velocity

= terminal settling velocity of smallest particle that is 100% removed

Weir loading = weir overflow rate (*WOR*)

$WOR = Q/\text{Weir Length}$

Horizontal velocity = approach velocity = $v_h = Q/A_{cross\text{-}section} = Q/A_x$

Hydraulic residence time = $V/Q = \theta$

where

Q = flow rate

A_x = cross-sectional area

A = surface area, plan view

V = tank volume

Typical Design Criteria for Sedimentation Basins

Type of Basin	Overflow Rate				Hydraulic Residence Time (hr)	Depth (ft)
	Average		Peak			
	(gpd/ft^2)	(m^3/m^2•day)	(gpd/ft^2)	(m^3/m^2•day)		
Water Treatment						
Clarification following coagulation and flocculation:						
Alum coagulation	350–550	14–22			4–8	12–16
Ferric coagulation	550–700	22–28			4–8	12–16
Upflow clarifiers						
Groundwater	1,500–2,200	61–90			1	
Surface water	1,000–1,500	41–61			4	
Clarification following lime-soda softening						
Conventional	550–1,000	22–41			2–4	
Upflow clarifiers						
Groundwater	1,000–2,500	41–102			1	
Surface water	1,000–1,800	41–73			4	

6.9.7 Taste and Odor Control

6.9.7.1 Threshold Odor Number (TON)

$$TON = \frac{A + B}{A}$$

where

TON = threshold odor number

A = mL of sample

B = mL of odor-free water

6.9.7.2 Media Filtration

Loading Rate

$$\text{Loading Rate} = \frac{Q}{A}$$

Sizing

Filter bay length-to-width ratio = 1.2:1 to 1.5:1

Effective size = d_{10}

Uniformity coefficient = d_{60}/d_{10}

where

d_x = diameter of particle class for which x% of sample is less than (m or ft)

Filter equations can be used with any consistent set of units.

Head Loss Through a Clean Filter Bed

Rose Equation

Monosized Media:

$$h_f = \frac{1.067 (v_s)^2 L C_D}{\psi g \eta^4 d}$$

Multisized Media:

$$h_f = \frac{1.067 (v_s)^2 L}{\psi g \eta^4} \sum \frac{C_{D_{ij}} x_{ij}}{d_{ij}}$$

Carman-Kozeny Equation

Monosized Media:

$$h_f = \frac{f' L (1 - \eta) v_s^2}{\eta^3 g d}$$

Multisized Media:

$$h_f = \frac{L (1 - \eta) v_s^2}{\eta^3 g} \sum \frac{f'_{ij} x_{ij}}{d_{ij}}$$

$$f' = \text{friction factor} = 150 \left(\frac{1 - \eta}{Re} \right) + 1.75$$

where

h_f = head loss through the clean bed (ft or m)

L = depth of filter media (ft or m)

η = porosity of bed = void volume/total volume

v_s = filtration rate = empty bed approach velocity (ft/sec or m/s)

v_s = Q/A_{plan}

g = gravitational acceleration (ft/sec^2 or m/s^2)

Re = Reynolds number = $\dfrac{v_s \rho d}{\mu}$

d_{ij}, d = diameter of filter media particles; arithmetic average of adjacent screen openings (m)

i = filter media (sand, anthracite, garnet)

j = filter media particle size (ft or m)

x_{ij} = percent of media retained on a sieve/screen at size j (%)

f'_{ij} = friction factors for each media fraction

C_D = drag coefficient as defined in settling velocity equations

Ψ = grain sphericity or shape factor

Bed Expansion

Monosized:

$$L_f = \frac{L_0(1-\eta_0)}{1-\left(\frac{v_B}{v_t}\right)^{0.22}}$$

Multisized:

$$L_f = L_0(1-\eta_0)\sum\frac{x_{ij}}{1-\left(\frac{v_B}{v_{t,i,j}}\right)^{0.22}}$$

$$\eta_f = \left(\frac{v_B}{v_t}\right)^{0.22}$$

where

L_f = depth of fluidized filter media (ft or m)

v_B = backwash velocity (ft/sec or m/s) = Q_B/A_{plan}

Q_B = backwash flow rate

v_t = terminal setting velocity (ft/sec or m/s)

η_f = porosity of fluidized bed

L_0 = initial bed depth (ft or m)

η_0 = initial bed porosity

6.9.8 Membrane Filtration

6.9.8.1 Operations

For cross-flow mode of operation, the transmembrane pressure is:

$$P_{tm} = \left[\frac{P_f + P_c}{2}\right] - P_p$$

where

P_{tm} = transmembrane pressure gradient (kPa)

P_f = inlet pressure of feed stream (kPa)

P_c = pressure of concentrate stream (kPa)

P_p = pressure of permeate stream (kPa)

Overall pressure drop across the filtration module is:

$$P = P_f - P_p$$

where

P = pressure drop across module (kPa)

P_f and P_p as defined above

For direct-feed mode of operation, the transmembrane pressure is:

$$P_{tm} = P_f - P_p$$

where

P_{tm} = transmembrane pressure gradient (kPa)

P_f and P_p as defined above

Total permeate flow:

$$Q_p = F_w A$$

where

Q_p = permeate stream flow rate (kg/s)

F_w = transmembrane water flux rate (kg/m^2•s)

A = membrane area (m^2)

Recovery rate (r):

$$r\,(\%) = \frac{Q_p}{Q_f} \times 100$$

where

Q_p = permeate stream flow rate (kg/s)

Q_f = feed stream flow rate (kg/s)

Rate of rejection (R):

$$R\,(\%) = \frac{C_f - C_p}{C_f} \times 100 = 1 - \frac{C_p}{C_f} \times 100$$

where

ΔC_i = solute concentration gradient (kg/m^3)

$\Delta C_i = \left[\frac{C_f + C_c}{2}\right] - C_p$

C_f = solute concentration in feed stream (kg/m^3)

C_c = solute concentration in concentrate stream (kg/m^3)

C_p = solute concentration in permeate stream (kg/m^3)

Silt Density Index:

$$SDI = \left(1 - \frac{t_i}{t_f}\right)\frac{100}{15}$$

where

SDI = silt density index

t_i = time initially needed to filter 500 mL of sample

t_f = time needed to filter 500 mL at the end of the 15-minute test period

6.9.8.2 Reverse Osmosis

Osmotic Pressure of Solutions of Electrolytes

$$\Pi = \phi v \frac{n}{V} RT$$

where

Π = osmotic pressure (Pa)

ϕ = osmotic coefficient

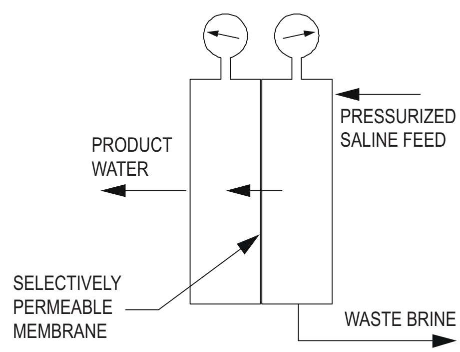
v = number of ions formed from one molecule of electrolyte

n = number of moles of electrolyte

V = specific volume of solvent (m^3/kmol)

R = universal gas constant [Pa•m^3/(kmol•K)]

T = absolute temperature (K)



A CONTINUOUS-FLOW REVERSE OSMOSIS UNIT

Salt Flux through the Membrane

$$J_s = (D_s K_s / \Delta Z)(C_{\text{in}} - C_{\text{out}})$$

where

J_s = salt flux through the membrane [kmol/(m^2•s)]

D_s = diffusivity of the solute in the membrane (m^2/s)

K_s = solute distribution coefficient (dimensionless)

C = concentration (kmol/m^3)

ΔZ = membrane thickness (m)

J_s = $K_p\,(C_{in} - C_{out})$

K_p = membrane solute mass-transfer coefficient (m/s)

$= \dfrac{D_s K_s}{\Delta Z}$

Water Flux

$$J_w = W_p\,(\Delta P - \Delta\pi)$$

where

J_w = water flux through the membrane [kmol/(m^2•s)]

W_p = coefficient of water permeation, a characteristic of the particular membrane [kmol/(m^2•s•Pa)]

ΔP = pressure differential across membrane (Pa) = $P_{in} - P_{out}$

$\Delta\pi$ = osmotic pressure differential across membrane (Pa) = $\pi_{in} - \pi_{out}$

6.9.9 Ultrafiltration

$$J_w = \frac{\varepsilon r^2 \int \Delta P}{8\mu\delta}$$

where

ε = membrane porosity

r = membrane pore size

ΔP = net transmembrane pressure

μ = absolute viscosity

δ = membrane thickness

J_w = volumetric flux (m/s)

6.9.10 Disinfection, Including Disinfection Byproducts

6.9.10.1 Chlorine Reactions in Water

Chlorine Added to Water

$$Cl_2 + H_2O \leftrightarrow H^+ + Cl^- + HOCl$$

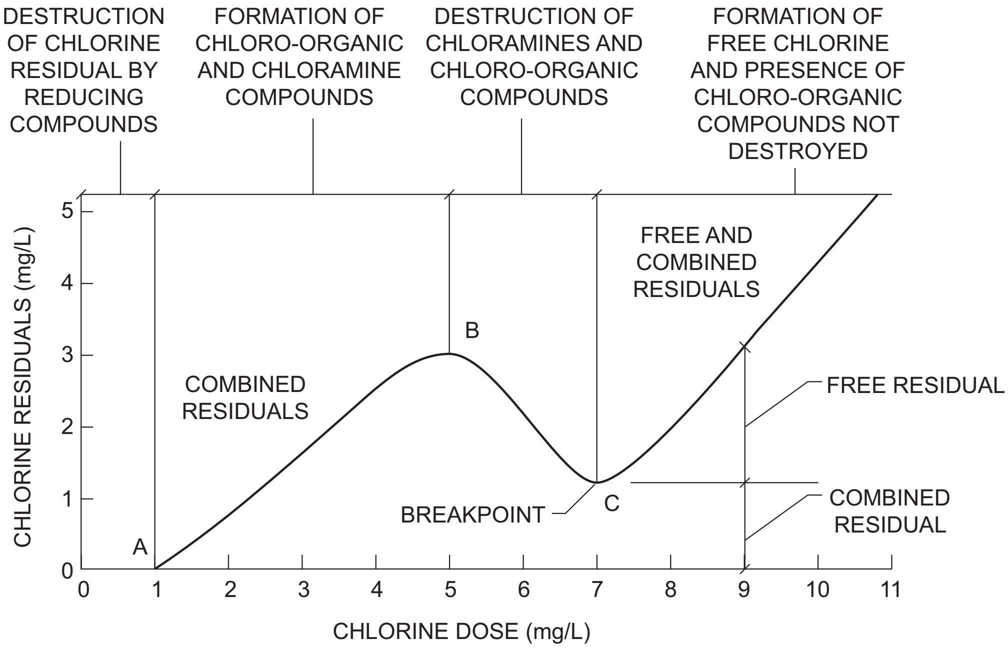
Hypochlorous Acid Ionization

$$HOCl \leftrightarrow H^+ + OCl^-$$

K_i = ionization constant (mole/L)

$$K_i = \frac{[H^+][OCl^-]}{[HOCl]} = 3 \times 10^{-8}\,\text{mole/L at } 25^\circ\text{C}$$

Chlorination Chart

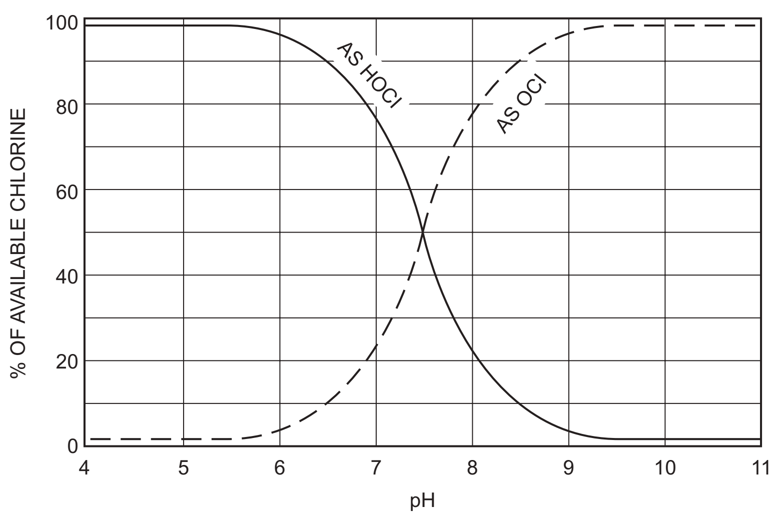


Source: Republished with permission of McGraw-Hill, from *Wastewater Engineering: Treatment and Reuse*, Tchobanoglous, George, Burton, Franklin L., Stensel, H. David., Metcalf & Eddy, 2003, Fig. 12-6, p. 1238; permission conveyed through Copyright Clearance Center, Inc.

6.9.10.2 Hypochlorite Reactions in Water

$$Ca(OCl)_2 + 2H_2O \leftrightarrow 2HOCl + Ca(OH)_2$$

$$NaOCl + H_2O \rightarrow HOCl + NaOH$$



Hypochlorite to Available Chlorine in Water Relationship

6.9.10.3 Chlorine Reactions with Ammonia

$$NH_4^+ + 1.5HOCl \rightarrow 0.5N_2 + 1.5H_2O + 2.5H^+ + 1.5Cl^-$$

6.9.10.4 Chlorine Contact Chambers

Disinfection

Chlorine contact chamber length-to-width ratio = 20:1 to 50:1

$$CT_{calc} = C \times t_{10}$$

where

CT_{calc} = calculated CT value (mg•mm/L)

C = residual disinfectant concentration measured during peak hourly flow (mg/L)

t_{10} = time it takes 10% of the water to flow through the reactor measured during peak hourly flow (min)

= can be determined from traces study data or the following relationship, $t_{10(approx)} = \theta \times BF$

where

θ = hydraulic residence time (min)

BF = baffling factor

Baffling Factors

Baffling Condition	Baffling Factor	Baffling Description
Unbaffled (mixed flow)	0.1	None, agitated basin, very low length to width ratio, high inlet and outlet flow velocities.
Poor	0.3	Single or multiple unbaffled inlets and outlets, no intra-basin baffles.
Average	0.5	Baffled inlet or outlet with some intra-basin baffles.
Superior	0.7	Perforated inlet baffle, serpentine or perforated intra-basin baffles, outlet weir or perforated launders.
Perfect (plug flow)	1.0	Very high length to width ratio (pipeline flow), perforated inlet, outlet, and intra-basin baffles.

Source: U.S. Environmental Protection Agency. LT1ESWTR *Disinfection Profiling and Benchmarking: Technical Guidance Manual*. Washington, DC: EPA, 2003, Table 4-2, p. 32. https://nepis.epa.gov/Exe/ZyPDF.cgi?Dockey=20002649.txt.

6.9.11 Removal and Inactivation Requirements

Microorganism	Required Log Reduction	Treatment
Giardia	3-log (99.9%)	Removal and/or inactivation
Virsuses	4-log (99.99%)	Removal and/or inactivation
Cryptosporidium	2-log (99%)	Removal

Source: U.S. Environmental Protection Agency. LT1ESWTR *Disinfection Profiling and Benchmarking: Technical Guidance Manual*. Washington, DC: EPA, 2003, Table 7-1, p. 62. https://nepis.epa.gov/Exe/ZyPDF.cgi?Dockey=20002649.txt.

6.9.12 Typical Removal Credits and Inactivation Requirements for Various Treatment Technologies

Process	Typical Log Removal Credits		Resulting Disinfection Log Inactivation Requirements	
	Giardia	**Viruses**	***Giardia***	**Viruses**
Conventional Treatment	2.5	2.0	0.5	2.0
Direct Filtration	2.0	1.0	1.0	3.0
Slow Sand Filtration	2.0	2.0	1.0	2.0
Diatomaceous Earth Filtration	2.0	1.0	1.0	3.0
Unfiltered	0	0	3.0	4.0

Source: U.S. Environmental Protection Agency. LT1ESWTR *Disinfection Profiling and Benchmarking: Technical Guidance Manual*. Washington, DC: EPA, 2003, Table 7-2, p. 62. https://nepis.epa.gov/Exe/ZyPDF.cgi?Dockey=20002649.txt.

CT Values* For 3-LOG Inactivation of *Giardia* Cysts by Free Chlorine

Chlorine Concentration (mg/L)	Temperature <= 0.5°C							Temperature = 5°C							Temperature = 10°C						
	pH							pH							pH						
	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0
<=0.4	137	163	195	237	277	329	390	97	117	139	166	198	236	279	73	88	104	125	149	177	209
0.6	141	168	200	239	286	342	407	100	120	143	171	204	244	291	75	90	107	128	153	183	218
0.8	145	172	205	246	295	354	422	103	122	146	175	210	252	301	78	92	110	131	158	189	226
1.0	148	176	210	253	304	365	437	105	125	149	179	216	260	312	79	94	112	134	162	195	234
1.2	152	180	215	259	313	376	451	107	127	152	183	221	267	320	80	95	114	137	166	200	240
1.4	155	184	221	266	321	387	464	109	130	155	187	227	274	329	82	98	116	140	170	206	247
1.6	157	189	226	273	329	397	477	111	132	158	192	232	281	337	83	99	119	144	174	211	253
1.8	162	193	231	279	338	407	489	114	135	162	196	238	287	345	86	101	122	147	179	215	259
2.0	165	197	236	286	346	417	500	116	138	165	200	243	294	353	87	104	124	150	182	221	265
2.2	169	201	242	297	353	426	511	118	140	169	204	248	300	361	89	105	127	153	186	225	271
2.4	172	205	247	298	361	435	522	120	143	172	209	253	306	368	90	107	129	157	190	230	276
2.6	175	209	252	304	368	444	533	122	146	175	213	258	312	375	92	110	131	160	194	234	281
2.8	178	213	257	310	375	452	543	124	148	178	217	263	318	382	93	111	134	163	197	239	287
3.0	181	217	261	316	382	460	552	126	151	182	221	268	324	389	95	113	137	166	201	243	292

Chlorine Concentration (mg/L)	Temperature = 15°C							Temperature = 20°C							Temperature = 25°C						
	pH							pH							pH						
	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0
<=0.4	49	59	70	83	99	118	140	36	44	52	62	74	89	105	24	29	35	42	50	59	70
0.6	50	60	72	86	102	122	146	38	45	54	64	77	92	109	25	30	36	43	51	61	73
0.8	52	61	73	88	105	126	151	39	46	55	66	79	95	113	26	31	37	44	53	63	75
1.0	53	63	75	90	108	130	156	39	47	56	67	81	98	117	26	31	37	45	54	65	78
1.2	54	64	76	92	111	134	160	40	48	57	69	83	100	120	27	32	38	46	55	67	80
1.4	55	65	78	94	114	137	165	41	49	58	70	85	103	123	27	33	39	47	57	69	82
1.6	56	66	79	96	116	141	169	42	50	59	72	87	105	126	28	33	40	48	58	70	84
1.8	57	68	81	98	119	144	173	43	51	61	74	89	106	129	29	34	41	49	60	72	86
2.0	58	69	83	100	122	147	177	44	52	62	75	91	110	132	29	35	41	50	61	74	88
2.2	59	70	85	102	124	150	181	44	53	63	77	93	113	135	30	35	42	51	62	75	90
2.4	60	72	86	105	127	153	184	45	54	65	78	95	115	138	30	36	43	52	63	77	92
2.6	61	73	88	107	129	156	188	46	55	66	80	97	117	141	31	37	44	53	65	78	94
2.8	62	74	89	109	132	159	191	47	56	67	81	99	119	143	31	37	45	54	66	80	96
3.0	63	76	91	111	134	162	195	47	57	68	83	101	122	146	32	38	46	55	67	81	97

*Although units did not appear in the original tables, units are min•mg/L

Source: U.S. Environmental Protection Agency. LT1ESWTR *Disinfection Profiling and Benchmarking: Technical Guidance Manual*. Washington, DC: EPA, 2003, Table B-1, p. 103. https://nepis.epa.gov/Exe/ZyPDF.cgi?Dockey=20002649.txt.
https://nepis.epa.gov/Exe/ZyPDF.cgi?Dockey=20002649.txt.

CT VALUES* FOR 4-LOG INACTIVATION OF VIRUSES BY FREE CHLORINE

Temperature (°C)	pH 6–9	pH 10
0.5	12	90
5	8	60
10	6	45
15	4	30
20	3	22
25	2	15

*Although units did not appear in the original tables, units are min•mg/L

Source: U.S. Environmental Protection Agency. *LT1ESWTR Disinfection Profiling and Benchmarking: Technical Guidance Manual*. Washington, DC: EPA, 2003, Table B-2, p. 104. https://nepis.epa.gov/Exe/ZyPDF.cgi?Dockey=20002649.txt.

CT Values (mg•min/L) for *Cryptosporidium* Inactivation by Chlorine Dioxide[1]

Log Credit	Water Temperature, °C										
	< –0.5	1	2	3	5	7	10	15	20	25	30
(i) 0.25	159	153	140	128	107	90	69	45	29	19	12
(ii) 0.5	319	305	279	256	214	180	138	89	58	38	24
(iii) 1.0	637	610	558	511	429	360	277	179	116	75	49
(iv) 1.5	956	915	839	767	643	539	415	268	174	113	73
(v) 2.0	1,275	1,220	1,117	1,023	858	719	553	357	232	150	98
(vi) 2.5	1,594	1,525	1,396	1,278	1,072	899	691	447	289	188	122
(vii) 3.0	1,912	1,830	1,675	1,534	1,286	1,079	830	536	347	226	147

[1] Systems may use this equation to determine log credit between the indicated values:
Log credit = $(0.001506 \times (1.09116)^{Temp}) \times CT$.

Source: Copied from Environmental Protection Agency 141.720 2010 ed., p. 623; full source below.

CT Values (mg•min/L) for *Cryptosporidium* Inactivation by Ozone[1]

Log Credit	Water Temperature, °C										
	< –0.5	1	2	3	5	7	10	15	20	25	30
(i) 0.25	6.0	5.8	5.2	4.8	4.0	3.3	2.5	1.6	1.0	0.6	0.39
(ii) 0.5	12	12	10	9.5	7.9	6.5	4.9	3.1	2.0	1.2	0.78
(iii) 1.0	24	23	21	19	16	13	9.9	6.2	3.9	2.5	1.6
(iv) 1.5	36	35	31	29	24	20	15	9.3	5.9	3.7	2.4
(v) 2.0	48	46	42	38	32	26	20	12	7.8	4.9	3.1
(vi) 2.5	60	58	52	48	40	33	25	16	9.8	6.2	3.9
(vii) 3.0	72	69	63	57	47	39	30	19	12	7.4	4.7

[1] Systems may use this equation to determine log credit between the indicated values:
Log credit = $(0.0397 \times (1.09757)^{Temp}) \times CT$.

Source: Copied from Environmental Protection Agency 141.720 2010 ed., p. 623; full source below.

CFR Title 40. Chapter I. Environmental Protection Agency, Subchapter D. Water Programs, Part 141. National Primary Drinking Water Regulations, Subpart W. *Enhanced Treatment for Cryptosporidium*, Subjgrp 28. "Requirements for Microbial Toolbox Components," Section 141.720. Inactivation toolbox components, 7–1–10 edition, p. 623, www.govinfo.gov/content/pkg/CFR-2010-title40-vol22/pdf/CFR-2010-title40-vol22-sec141-720.pdf.

6.9.12.1 Ultraviolet Light (UV)

UV dose D is defined as follows:

$$D = I \times t$$

where

D = UV dose (mJ/cm^2) (Note: $mJ/cm^2 = mW \bullet s/cm^2$)

I = UV intensity (mW/cm^2)

t = exposure time (s)

For UV doses greater than 10 mJ/cm^2:

$$N_D(t) = N_D(0)e^{-kIt}$$

where

$N_D(t)$ = total number of surviving dispersed microorganisms at time t

$N_D(0)$ = total number of dispersed microorganisms prior to UV light application (at time t = 0)

k = inactivation rate coefficient (cm^2/mW•s)

I = average intensity of UV light in bulk solution (mW/cm^2)

t = exposure time (s)

Inactivation of both dispersed and particle-associated microorganisms with an applied density to the bulk liquid medium:

$$N(t) = N_D(0)e^{-kd} + \frac{N_p(0)}{kd}\left(1 - e^{-kd}\right)$$

where

$N(t)$ = total number of surviving microorganisms at time t

$N_D(0)$ = total number of dispersed microorganisms prior to application of disinfectant at time t = 0

$N_p(0)$ = total number of particles containing at least one microorganism at time t = 0

k = inactivation rate coefficient (cm^2/mJ)

d = UV dose (mJ/cm^2)

UV Dose Table for *Cryptosporidium*, *Giardia lamblia*, and Virus Inactivation Credit

Log Credit	*Cryptosporidium* UV dose (mJ/cm^2)	*Giardia lamblia* UV dose (mJ/cm^2)	Virus UV dose (mJ/cm^2)
(i) 0.5	1.6	1.5	39
(ii) 1.0	2.5	2.1	58
(iii) 1.5	3.9	3.0	79
(iv) 2.0	5.8	5.2	100
(v) 2.5	8.5	7.7	121
(vi) 3.0	12	11	143
(vii) 3.5	15	15	163
(viii) 4.0	22	22	186

Source: CFR Title 40. Chapter I. Environmental Protection Agency, Subchapter D. Water Programs, Part 141. National Primary Drinking Water Regulations, Subpart W. *Enhanced Treatment for Cryptosporidium*, Subjgrp 28. "Requirements for Microbial Toolbox Components," Section 141.720. Inactivation toolbox components, 7–1–10 edition, p. 624, www.govinfo.gov/content/pkg/CFR-2010-title40-vol22/pdf/CFR-2010-title40-vol22-sec141-720.pdf.

Physical Properties of Air at Standard Atmospheric Pressure

Temperature (°F)	Density, ρ (slugs/ft^3)	Specific Weight, γ (lbf/ft^3)	Dynamic Viscosity, μ (lbf-sec/ft^2)	Kinematic Viscosity, ν (ft^2/sec)	Specific Heat Ratio, *k* (---)	Speed of Sound, *c* (ft/sec)
–40	2.939×10^{-3}	9.456×10^{-2}	3.29×10^{-7}	1.12×10^{-4}	1.401	1,004
–20	2.805	9.026	3.34	1.19	1.401	1,028
0	2.683	8.633	3.38	1.26	1.401	1,051
10	2.626	8.449	3.44	1.31	1.401	1,062
20	2.571	8.273	3.50	1.36	1.401	1,074
30	2.519	8.104	3.58	1.42	1.401	1,085
40	2.469	7.942	3.60	1.46	1.401	1,096
50	2.420	7.786	3.68	1.52	1.401	1,106
60	2.373	7.636	3.75	1.58	1.401	1,117
70	2.329	7.492	3.82	1.64	1.401	1,128
80	2.286	7.353	3.86	1.69	1.400	1,138
90	2.244	7.219	3.90	1.74	1.400	1,149
100	2.204	7.090	3.94	1.79	1.400	1,159
120	2.128	6.846	4.02	1.89	1.400	1,180
140	2.057	6.617	4.13	2.01	1.399	1,200
160	1.990	6.404	4.22	2.12	1.399	1,220
180	1.928	6.204	4.34	2.25	1.399	1,239
200	1.870	6.016	4.49	2.40	1.398	1,258
300	1.624	5.224	4.97	3.06	1.394	1,348
400	1.435	4.616	5.24	3.65	1.389	1,431
500	1.285	4.135	5.80	4.51	1.383	1,509
750	1.020	3.280	6.81	6.68	1.367	1,685
1,000	0.845	2.717	7.85	9.30	1.351	1,839
1,500	0.629	2.024	9.50	15.1	1.329	2,114